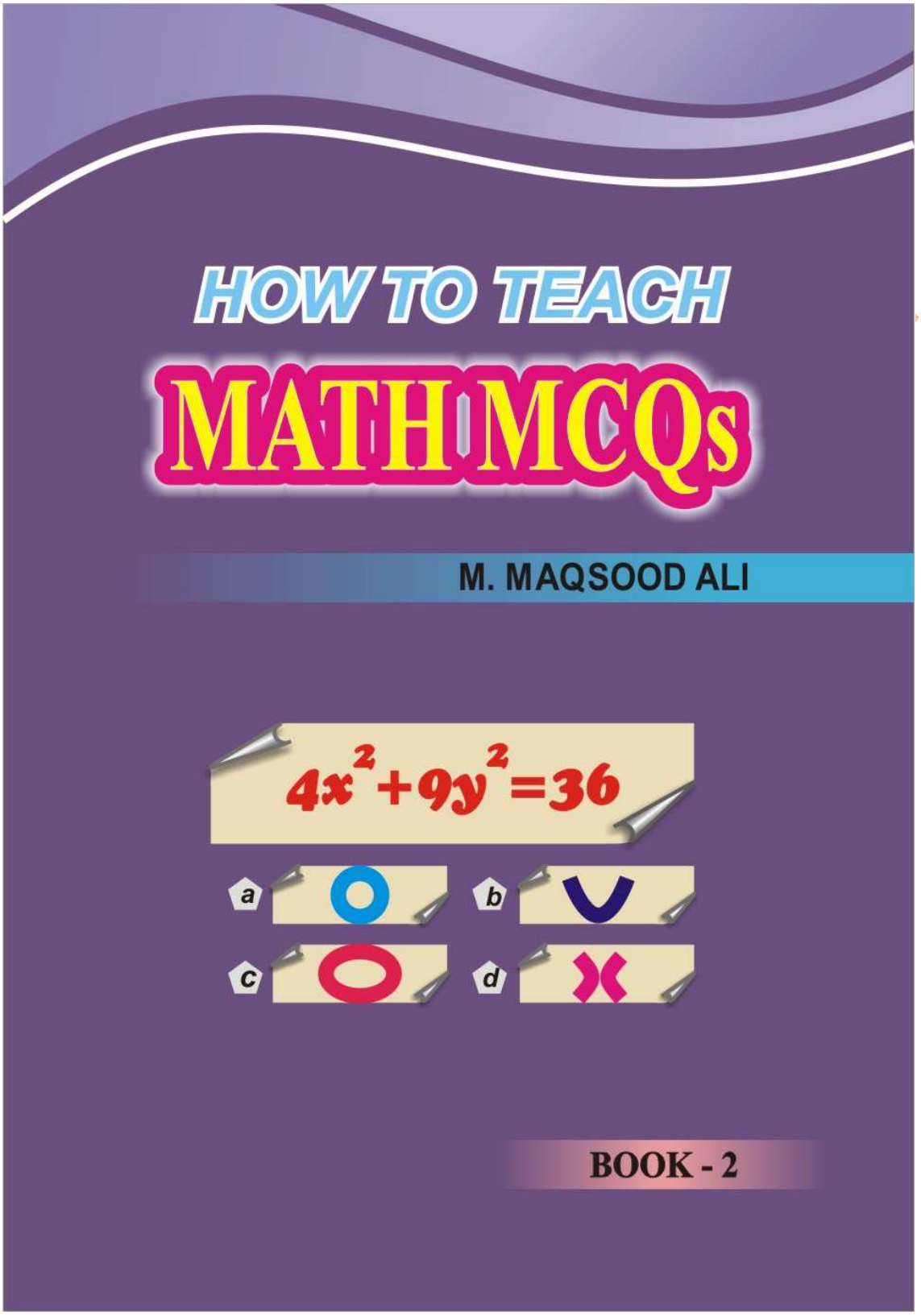

HOW TO TEACH
MATH MCQs
M. MAQSOOD ALI
$4x^2+9y^2=36$
a
b
c
d
BOOK - 2

Chapter 11

VECTORS

INITIAL AND TERMINAL POINTS OF A VECTOR

If A (a_1, b_1, c_1) and B (a_2, b_2, c_2) are initial and terminal points respectively , then vector $\overrightarrow{AB}$ is

$$\begin{aligned}\overrightarrow{AB} &= (a_2, b_2, c_2) - (a_1, b_1, c_1)\\ &= (a_2 - a_1, b_2 - b_1, c_2 - c_1)\\ &= (a_2 - a_1)\, i + (b_2 - b_1)\, j + (c_2 - c_1)\, k\end{aligned}$$

MCQ- 1:

If $P(3, -5, 8)$ and $Q = (-6, 2, 7)$ are the points, then $\overrightarrow{PQ}$ =?

(a) $-9i + 7j - k$ (b) $3i - 3j + 15k$

(c) $-3i + 3j - 15k$ (d) $9i - 7j + k$

Solution:

$$\begin{aligned}\overrightarrow{PQ} &= (-6, 2, 7) - (3, -5, 8)\\ &= (-6 - 3,\ 2 + 5,\ 7 - 8)\\ &= (-9, 7, -1)\\ &= -9i + 7j - k\end{aligned}$$

The answer is (a).

MCQ-2 :

If P$(6, -4, 9)$ and Q are the points, what are the coordinates of Q if $\overrightarrow{PQ} = 12i - 2j + 5k$?

(a) $(-6, -2, 4)$ (b) $(3, -8, 12)$

(c) $(18, -6, 14)$ (d) $(6, 2, -4)$

Solution:

$$\overrightarrow{PQ} = \text{(coordinates of } Q\text{)} - \text{(coordinates of } P\text{)}$$

$$\text{(coordinates of } Q\text{)} = \text{(coordinates of } P\text{)} + \overrightarrow{PQ}$$

$$= (6, -4, 9) + (12, -2, 5)$$

$$Q\ (6\ +\ 12\ ,-\ 4\ -\ 2\ ,9\ +\ 5)$$

$$Q\ (18\ ,-6\ ,14\ )$$

The answer is (c).

MCQ-3 :

$x = ?$, if $A(3,-5\ ,2)\ ,B\ (-1\ ,x,-4)$ and $\overrightarrow{AB}\ =\ -4i + j - 6k)$?

(a) 6 (b) -4 (c) -6 (d) -5

Solution:

$$\overrightarrow{AB}\ = B - A$$

$$\overrightarrow{AB}\ =\ (-4\ ,1\ ,-6)\ ,\ \ A(3,-5\ ,2)\ \ ,\ \ B\ (-1\ ,x,-4)$$

$$1\ =\ x\ -\ (-5)$$

$$1 = x\ +\ 5$$

$$x = 1 - 5$$

$$x = -4$$

The answer is (b).

EXERCISE-1

(1) $A(2,5,-3)$ and $\overrightarrow{AB} = 6i - 2j - 3k$. What are the coordinates of B?

(a) $(4,-7,0)$ (b) $(8,3,-6)$ (c) $(8,3,-9)$ (d) $(4,3,-6)$

(2) $\overrightarrow{AB} = 2i + 3j - 3k$ and $B(5,2,4)$. What are the coordinates of A?

(a) $(3,8,-2)$ (b) $(7,5,1)$ (c) $(-3,1,-7)$ (d) $(3,-1,7)$

MAGNITUDE OF A VECTOR

$$\bar{r} = ai + bj + ck$$

is a vector.

Magnitude of vector $\bar{r}$ is

$$r = |\bar{r}| = \sqrt{a^2 + b^2 + c^2}$$

MCQ-4 :

$a = ?$, if magnitude of the vector $\bar{r} = i + aj - 2k$ is 3?

(a) -5 (b) $\sqrt{12}$ (c) -2 (d) $\sqrt{3}$

Solution:

$$r = 3 \quad , \quad \bar{r} = i + aj - 2k$$

$$r = \sqrt{1^2 + a^2 + (-2)^2}$$

$$3 = \sqrt{1 + a^2 + 4}$$

$$9 = a^2 + 5$$

$$a^2 = 4$$

$$a = \pm 2$$

The answer is (c).

MCQ-5 :

$\bar{a} = 2i + j$ and $\bar{b} = 3i - 5j$. What is the value of $|2\bar{a} - \bar{b}|$?

(a) $\sqrt{12}$ (b) $\sqrt{42}$ (c) 6 (d) $\sqrt{38}$

Solution:

$$2\bar{a} - \bar{b} = (4i + 2j) - (3j - 5k)$$

$$= 4i - j + 5k$$

$$|2\bar{a} - \bar{b}| = \sqrt{16 + 1 + 25}$$

$$= \sqrt{42}$$

The answer is (b).

UNIT VECTOR

$$\hat{r} = \frac{\bar{r}}{|\bar{r}|}$$

is a unit vector , in the direction of vector $\bar{r}$ with magnitude 1.

MCQ- 6:

What is the unit vector in the direction of the vector $\bar{r} = 8i + 4j - k$?

(a) $\frac{8}{7}i + \frac{4}{7}j - \frac{1}{7}k$ (b) $\frac{8}{9}i + \frac{4}{9}j - \frac{1}{9}k$

(c) $\frac{8}{5}i + \frac{4}{5}j - \frac{1}{5}k$ (d) $2i + \frac{1}{2}j - \frac{1}{8}k$

Solution:

$$\bar{r} = 8i + 4j - k$$

$$r = |\bar{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{64 + 16 + 1}$$

$$= 9$$

Divide the vector $\bar{r}$ by 9

$$\hat{r} = \frac{8}{9}i + \frac{4}{9}j - \frac{1}{9}k$$

The answer is (b).

EXERCISE-2

(1) What is the magnitude of vector $2i - 3j + 5k$?

(a) $\sqrt{38}$ (b) $\sqrt{20}$ (c) $2\sqrt{3}$ (d) $5\sqrt{2}$

(2) What is the magnitude of the vector $3i - 4j$?

(a) 1 (b) -1 (c) 5 (d) 25

(3) What is the unit vector of the vector $6i - 8j$?

(a) $\frac{3}{2}i - j$ (b) $3i - 2j$ (c) $\frac{3}{5}i - \frac{4}{5}j$ (d) $i - j$

(4) What is the vector in the direction of the vector $i + j$ of magnitude 3 ?

(a) $\frac{3}{\sqrt{2}}i + \frac{3}{\sqrt{2}}j$ (b) $3i + 3j$ (c) $\frac{3}{2}i + \frac{3}{2}j$ (d) $\frac{1}{3}i + \frac{1}{3}j$

(5) What is the vector in the direction of the vector $4i - 3j$ of magnitude 2 ?

(a) $2i - \frac{3}{2}j$ (b) $\frac{4}{5}i - \frac{3}{5}j$ (c) $8i - 6j$ (d) $\frac{8}{5}i - \frac{6}{5}j$

POSITION VECTOR

If O be the origin and $P(a,b,c)$ a point, then $\overrightarrow{OP}$ is called position vector.

$$\begin{aligned}\overrightarrow{OP} &= (a,b,c) - (0,0,0)\\ &= (a-0, b-0, c-0)\\ &= (a,b,c)\\ &= ai + bj + ck\end{aligned}$$

MCQ- 7:

$x = ?$, if $\overrightarrow{OP} = 5i + 3j - xk$ is the position vector of $P\ (5, 3, -8)$?

(a) 0 (b) 6 (c) -8 (d) 8

Solution:

$$P\ (5, 3, -8)$$

$$\overrightarrow{OP} = 5i + 3j - xk$$

$$-x = -8 \quad \{\because \overrightarrow{OP} = 5i + 3j - 8k$$

$$x = 8$$

The answer is (d).

MCQ- 8:

$3\overrightarrow{PQ}$ =? If the position vector of P and Q are $2i - 3j + 6k$ and $-6i + 3j - k$ respectively.

(a) $-24i + 18j - 21k$ (b) $-12i + 15k$

(c) $-4i + 5k$ (d) $6i - 9j + 12k$

Solution:

$$3\overrightarrow{PQ} = 3(\overrightarrow{OQ} - \overrightarrow{OP})$$
$$= 3\{(-6i + 3j - k) - (2i - 3j + 6k)\}$$

$$\begin{aligned}3\overrightarrow{PQ} &= 3(-8i + 6j - 7k)\\ &= -24i + 18j - 21k\end{aligned}$$

The answer is (a).

EXERCISE-3

(1) The position vector of A is $\overrightarrow{OA} = 6i + 3j - 8k$ and $\overrightarrow{AB} = 10i + 8j + 2k$. What is the position vector of B?

(a) $16i + 11j - 10k$ (b) $16i + 11j - 6k$

(c) $4i + 5j + 10k$ (d) $4i + 5j - 6k$

(2) The position vector of B is $\overrightarrow{OB} = 8i + 2j - 5k$.and $\overrightarrow{AB} = 12i + 4j - 3k$. What is the position vector of A?

(a) $-4i - 2j - 2k$ (b) $20i + 6j - 8k$

(c) $-4i - 2j - 8k$ (d) $20i + 6j - 2k$

PARALLEL VECTORS

Two vectors $\bar{a}$and $\bar{b}$ are parallel, if

$$\bar{a} = m\,\bar{b}$$

where m is a scalar (real number).

MCQ-9 :

Which vector is parallel to the vector $2i - 6j - 10k$?

(a) $2i + 6j + 10k$ (b) $-i + 3j + 5k$

(c) $i - 2j - 5k$ (d) $-3i + 18j + 20k$

Solution:

$$-2\ \{\text{option } (b)\}$$
$$= -2\,(-i + 3j + 5k)$$
$$= 2i - 6j - 10k$$

The answer is (b).

EXERCISE-4

(1) $\bar{a} = 4i + 5j$ is parallel to $\bar{b} = 8i + mj$. What is the value of m?

(a) 10 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 20

(2) $\bar{p} = -2i + mj$ and $\bar{q} = 8i + 12j$ are parallel vectors. What is the value of m?

(a) $-\frac{1}{4}$ (b) -3 (c) 6 (d) $\frac{4}{3}$

DIRECTION COSINES

$(\cos\alpha , \cos\beta , \cos\gamma)$ are the directions cosine of a vector $\bar{r}$,

where α, β and γ are angle between the vector $\bar{r}$ and positive x , y and z axis respectively.

If $\bar{r} = xi + yj + zk$, then

$$x = r\cos\alpha, \quad y = r\cos\beta, \quad z = r\cos\gamma$$

$$\cos\alpha = \frac{x}{r} \quad , \quad \cos\beta = \frac{y}{r} \quad , \quad \cos\gamma = \frac{z}{r}$$

where r = $|\bar{r}|$

Note:

The components of a unit vector of a vector are directions cosine of the vector.

Since $(\cos\alpha, \cos\beta, \cos\gamma) = (\frac{x}{r} , \frac{y}{r} , \frac{z}{r})$

MCQ-10 :

What are the directions cosine of the vector $\bar{r} = 3i - 4k$?

(a) (3/5 , 0 , 4/5) (b) (5/3 , 0 , 4/5)

(c) (2/5 , 4/5 , 7/5) (d) (7/5 , 3/5 , 4/5)

Solution:

$$\bar{r} = 3i - 4k$$

$$x = 3 \ , \ y = 0 \ , \ z = -4$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{9 + 16} = 5$$

$$\cos\alpha = \frac{x}{r} \ , \ \cos\beta = \frac{y}{r} \ , \ \cos\gamma = \frac{z}{r}$$

$$(3/5 \ , 0 , \ 4/5)$$

The answer is (a).

MCQ- 11: What is the angle between the y-axis and the vector $3i + 2j - k$?

(a) $\cos^{-1}\frac{2}{\sqrt{7}}$ (b) $\cos^{-1}\sqrt{\frac{2}{7}}$ (c) $\cos^{-1}\frac{2}{\sqrt{3}}$ (d) $\cos^{-1}\sqrt{\frac{2}{5}}$

Solution:

$3i + 2j - k$

$x = 3\ ,\ y = 2\ ,\ z = 1$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{9 + 4 + 1}$$

$$r = \sqrt{14}$$

β *is* the angle between the vector and y-axis.

$$\cos\beta = \frac{y}{r}$$

$$= \frac{2}{\sqrt{14}}$$

$$= \sqrt{\frac{2}{7}}$$

$$\beta = \cos^{-1}\sqrt{\frac{2}{7}}$$

The answer is (b).

EXERCISE-5

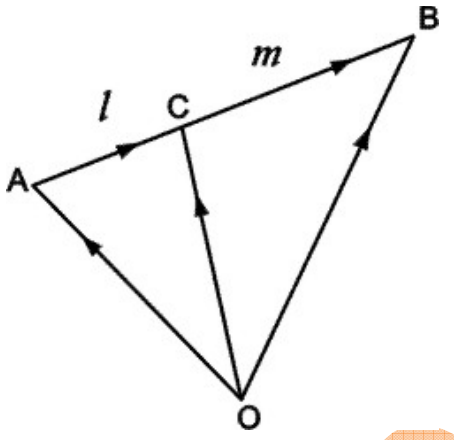
(1) What is the angle of the vector $\bar{r} = 3i + 4k$ from $z - axis$?

(a) $\cos^{-1} 1$ (b) $\cos^{-1}\frac{3}{5}$ (c) $\cos^{-1}\frac{4}{5}$ (d) $\cos^{-1} 0$

(2) What is the angle of the vector $\bar{r} = 8i - 6k$ from $y - axis$?

(a) $\cos^{-1}\frac{8}{10}$ (b) $\cos^{-1}\left(\frac{-6}{10}\right)$ (c) $\cos^{-1} 0$ (d) $\cos^{-1} 1$

POSITION VECTORS AND RATIO



are the position vectors of points A, B and C respectively.

If C divides in the ratio , then

=

MCQ-12 :

? , if , and C divides

In the ratio 2:3.

(a) – (b) – (c) – (d) –

Solution:

=

–

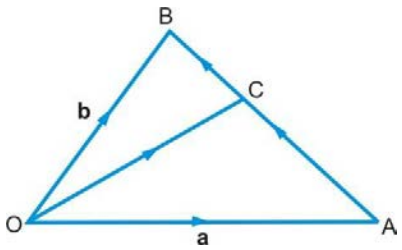
–

–

The answer is (d).

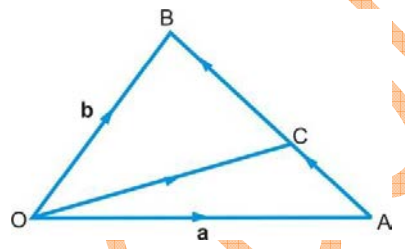
EXERCISE-6

(1) , and – What is the vector



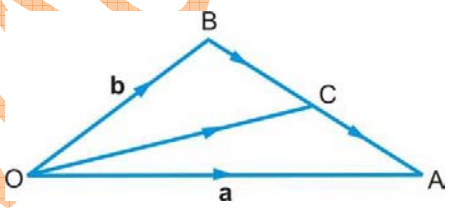
(a) – (b) – (c) – (d) –

(2) , and C divides in the ratio 2:3. What is the vector



(a) (b) – (c) – (d) –

(3) , and – What is the vector



(a) – (b) – (c) – (d) –

DOT PRODUCT

$$\bar{u} = a_1 i + b_1 j + c_1 k \quad \text{and}$$
$$\bar{v} = a_2 i + b_2 j + c_2 k$$

The dot product of $\bar{u}$ and $\bar{v}$ is

$$\bar{u}.\bar{v} = a_1\, b_2 + b_1 b_2 + c_1 c_2$$

VALUE OF COSθ:

If θ is the angle between the vectors $\bar{u}$ and $\bar{v}$ then

$$\cos\theta = \frac{\bar{u}.\bar{v}}{|\bar{u}||\bar{v}|}$$

MCQ- 13:

a =?, if $\bar{u} = 2i - 3j + 5k$, $\bar{v} = 3i + aj - k$ and $\bar{u}.\bar{v} = -29$

(a) 4 (b) 10 (c) 8 (d) 6

Solution:

$$\bar{u}.\bar{v} = (2 \times 3) + (-3 \times a) + (-1 \times 5)$$
$$-29 = 6 - 3a - 5$$
$$-3a = -30$$
$$a = 10$$

The answer is (b).

MCQ- 14: $cos\theta$ =? , if $\bar{a} = 2i - j + 3k$ and $\bar{b} = i + 3j - 2k$.

(a) $-\frac{1}{2}$ (b) $1/\sqrt{2}$ (c) $-7/8$ (d) $-7/\sqrt{14}$

Solution:

$$\bar{a}.\bar{b} = 2 - 3 - 6$$
$$= -7$$
$$a = \sqrt{4+1+9} = \sqrt{14}$$
$$b = \sqrt{1+9+4} = \sqrt{14}$$
$$\cos\theta = \frac{\bar{a}.\bar{b}}{ab} = \frac{-7}{14} = -\frac{1}{2}$$

The answer is (a).

ORTHOGONAL VECTORS

Two vectors $\bar{u}$ and $\bar{v}$ are orthogonal, if

$$\bar{u}.\,\bar{v}=0$$

MCQ-15 :

$\bar{u}=2i+3j-5k$ and $\bar{v}=6i-hj+3k$ are orthogonal. What is the value of h?

(a) -1 (b) 3 (c) -2 (d) 6

Solution:

$\bar{u}$ *and* $\bar{v}$ are orthogonal

$$\bar{u}.\,\bar{v}=0$$

$$(2i+3j-5k).(6i-hj+3k)=0$$

$$12-3h-15=0$$

$$-3h=3$$

$$h=-1$$

The answer is (a).

WORK DONE

A particle is displaced d by the force F, the work done is

$$\text{Work done} = \bar{F}.\bar{d}$$

MCQ-16 :

A particle is displaced from (2, 3, 8) to (9, 5, 1) by the force $9i+2j-6k$. What is the work done?

(a) 109 (b) 208 (c) 72 (d) 48

Solution:

$$\bar{d}=(9,5,1)-(2,3,8)$$

$$=7i+2j-7k$$

$$\text{work done} = \bar{F}.\bar{d}$$

$$= (9i + 2j - 6k) . (7i + 2j - 7k)$$

$$= 63 + 4 + 42$$

$$= 109$$

The answer is (a).

EXERCISE-7

(1) If $\bar{a} = 2i - 3j + 5k$ and $\bar{b} = 5i + j$,Then $\bar{a}.\bar{b} =$?

(a) 8 (b) -13 (c) $10i - j$ (d) 7

(2) $\bar{a} = 3i - 2k$ and $\bar{b} = mi + 5j + 9k$ are orthogonal vectors. What is the value of m?

(a) 6 (b) 2 (c) 8 (d) 3

(3) What is the work done if a particle is displaced from point (2, 8, 5) to (9, 3, 7) by the force $2i - j + k$?

(a) 15 (b) 26 (c) 21 (d) 9

(4) The magnitude of a vector $\bar{r}$ is $|\bar{r}|$. What is the value of $\bar{r}.\bar{r}$?

(a) $\frac{\bar{r}}{|\bar{r}|}$ (b) $|\bar{r}|^2$ (c) $\bar{r}^2$ (d) $|\bar{r}|\bar{r}$

(5) If $\bar{r} . \bar{r} = m^2 + 1$ and $|\bar{r}| = 5$. What is the positive value of m?

(a) 3 (b) $3\sqrt{2}$ (c) 2 (d) $2\sqrt{6}$

(6) $\bar{r} . \bar{r} = 9$ and $|\bar{r}| = m$. What is the positive value of m?

(a) $3\sqrt{3}$ (b) 3 (c) 81 (d) 9

CROSS PRODUCT

and =

The cross product of and is

×

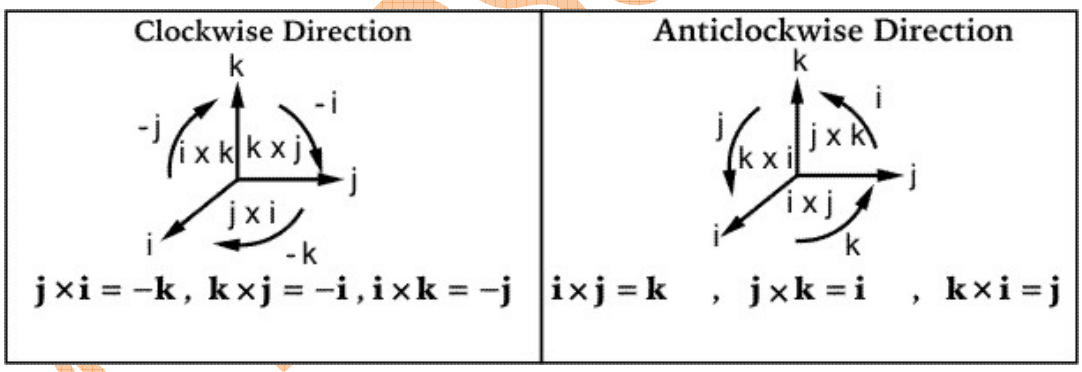
– – – –

Shortcut:

x = (coeff. of x coeff. of – coeff. of x coeff. of)
(coeff. of x coeff. of – coeff. of x coeff. of)
(coeff. of x coeff. of – coeff. of x coeff. of)

CLOCKWISE AND ANTICLOCKWISE DIRECTIONS:

are unit vectors in the directions of x, y and z-axis respectively.



MCQ-17 :

if and

(a) – (b) –

(c) – (d) –

Solution:

=

= – –

$$\bar{a} \times \bar{b} = i(-12-12) - j(-18+20) + k(-9-10)$$
$$= -24i - 2j - 19k$$

The answer is (c).

MCQ-18 :

$\bar{a} \times \bar{b} = ?$, if $\bar{a} = 5i + 2j - 4k$ and $\bar{b} = 6i - 3j + 7k$?

(a) $2i + 8j - 12k$ (b) $2i - 59j - 27k$

(c) $6i + 12j - 35k$ (d) None

Solution:

$$\bar{a} = 5i + 2j - 4k$$
$$\bar{b} = 6i - 3j + 7k$$

Note: Due to the option (d) None, find $\bar{a} \times \bar{b}$ completely.

$$\bar{a} \times \bar{b} = i(14-12) - j(35+24) + k(-15-12)$$
$$= 2i - 59j - 27k$$

The answer is (b).

MCQ-19 :

$|\bar{a} \times \bar{b}| = ?$, $\bar{a} = 2i - j + k$, $\bar{b} = i + 2j - k$

(a) $\sqrt{47}$ (b) $\sqrt{12}$ (c) $\sqrt{26}$ (d) $\sqrt{35}$

Solution:

$$\bar{a} = 2i - j + k$$
$$\bar{b} = i + 2j - k$$

$$\bar{a} \times \bar{b} = i(1-2) - j(-2-1) + k(4+1)$$
$$= -i + 3j + 5k$$
$$|\bar{a} \times \bar{b}| = \sqrt{1+9+25}$$
$$= \sqrt{35}$$

The answer is (d).

MCQ-20 :

$\bar{a} \times \bar{b}$ = ? , $\bar{a} = i - 2j + 5k$, $\bar{b} = 3i + 4j - 2k$.

(a) $-16i + 17j + 10k$ (b) $-16i - 5j + 10k$

(c) $-16i + 8j + 10k$ (d) $-16i - 12j + 10k$

Solution:

$$\bar{a} = i - 2j + 5k$$
$$\bar{b} = 3i + 4j - 2k$$

Note: First and last term in each options are same, so check only second term.

$= -\text{j}\{1 \times (-2) - 3 \times 5\}$

$= -j\,(-2 - 15)$

$= 17j$

The answer is (a).

MCQ-21 :

$\bar{a} \times \bar{b}$ = ? , $\bar{a} = 3i - 2j + 4k$, $\bar{b} = 6i - 5j + 7k$.

(a) $3i - 2j + 8k$ (b) $4i + 6j - 7k$

(c) $6i + 3j - 3k$ (d) $9i - 10j + 12k$

Solution:

$$\bar{a} = 3i - 2j + 4k$$
$$\bar{b} = 6i - 5j + 7k$$

Note: Four options have different answers, so check any one term.

$= i(-14 + 20)$

$= 6i$

The answer is (c).

PERPENDICULAR VECTOR

$$\bar{u} \times \bar{v}$$

is a vector perpendicular to both the vectors $\bar{u}$ and $\bar{v}$.

UNIT PERPENDICULAR VECTOR

$$\frac{\bar{u} \times \bar{v}}{|\bar{u} \times \bar{v}|}$$

is a unit vector perpendicular to both the vectors $\bar{u}$ and $\bar{v}$.

MCQ- 22:

$\bar{u} = 3i - 2j + 4k$ and $\bar{v} = 5i + 3j - k$ are two vectors.What is the vector perpendicular to both the vectors $\bar{u}$ and $\bar{v}$?

(a) $2i + 5j - 5k$ (b) 15i – 6j – 4k

(c) $-10i + 23j + 29k$ (d) None

Solution:

$$\bar{u} = 3i - 2j + 4k$$
$$\bar{v} = 5i + 3j - k$$

$$\bar{u} \times \bar{v} = (2 - 12)i - (-3 - 20)j + (9 + 10)k$$
$$= -10i + 23j + 19k$$

The answer is (d).

EXERCISE-8

(1) If $\bar{a} = 2i + 3k$ and $\bar{b} = 5j$, then $\bar{a} \times \bar{b} =$?

(a) 0 (b) $-15i + 10k$ (c) $2i + 5j + 3k$ (d) $10i + 15k$

(2) $\bar{a} = 6i$ and $\bar{b} = 2k$. What is the vector perpendicular to both $\bar{a}$ and $\bar{b} =$?

(a) $-12j$ (b) $12j$ (c) 0 (d) $3j$

TRIPLE PRODUCT

are three vectors.

The triple product of the vectors

is defined as

where

ACCORDING TO THE PROPERTIES OF DETERMINANT:

(i)

Similarly

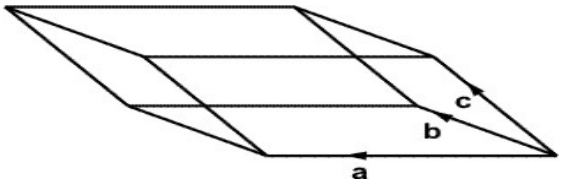
So that the value of is equal to zero if .

(ii) Interchange two vectors:

(iii) Taking common a scalar from a column:

(iv)

VOLUME OF PARALLELOPIPED:



Volume of Parallelopiped

EXERCISE-9

(1) $i.j \times k = ?$

(a) 1 (b) 0 (c) i^2 (d) j

(2) $k.j \times i = ?$

(a) -1 (b) 1 (c) 0 (d) k^2

(3) $i \times k.j = ?$

(a) 1 (b) 0 (c) -1 (d) j^2

(4) $\bar{a}\,.\bar{b} \times \bar{a} = ?$

(a) 1 (b) a^2b (c) $a^2 \times b$ (d) 0

(5) What is the volume of parallelepiped whose three adjacent adges are represented by the vectors $\bar{a}$, $\bar{b}$, $2\bar{a}$?

(a) abc (b) 1 (c) 2 (d) 0

(6) What is the volume of parallelepiped whose three adjacent adges are represented by the vectors $3i, 2j, k$?

(a) 6 (b) 1 (c) 0 (d) $\frac{3}{2}$

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