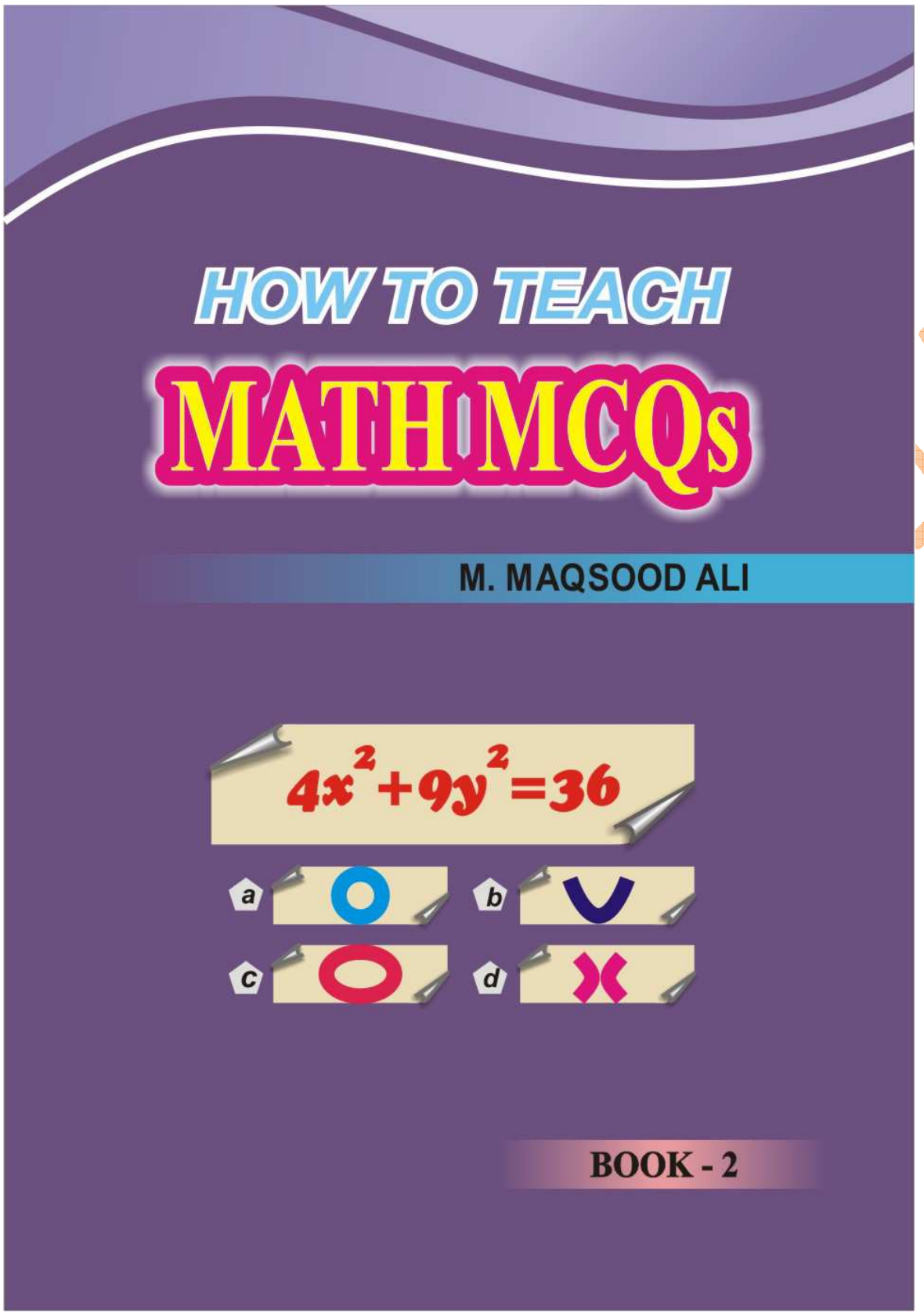
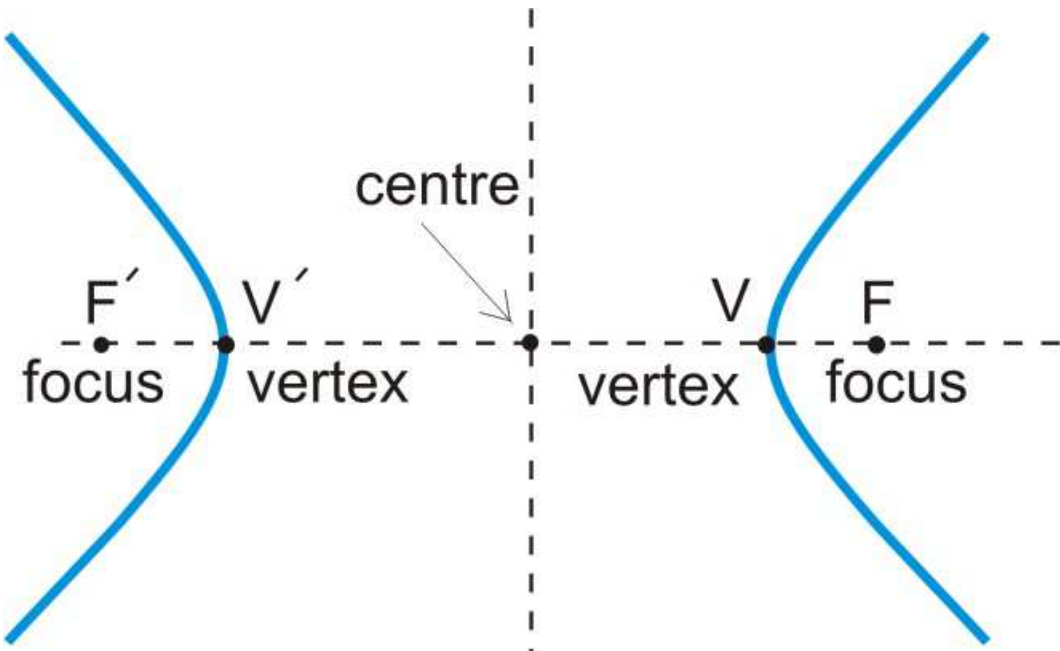
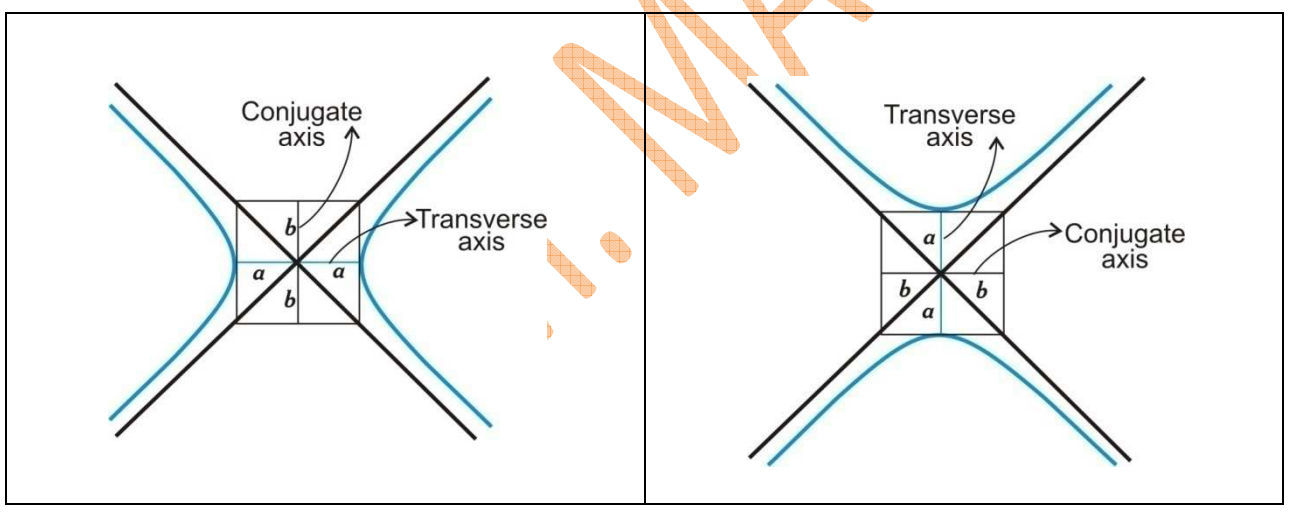

HOW TO TEACH
MATH MCQs
M. MAQSOOD ALI
$4x^2+9y^2=36$
a
b
c
d
BOOK - 2

Chapter 10

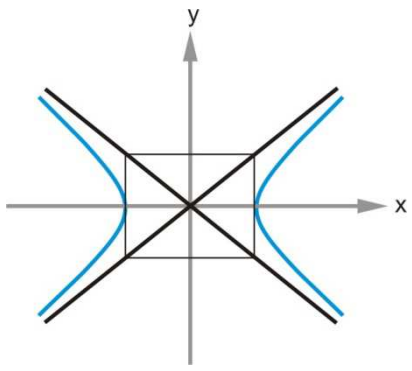
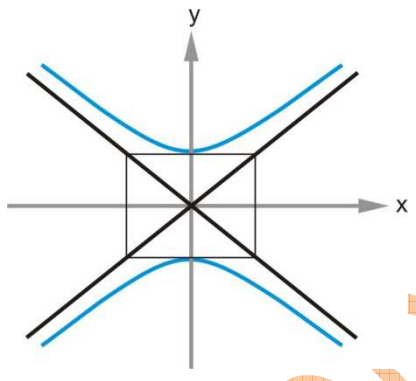
HYPERBOLAS



TRANSVERSE AND CONJUGATE AXIS



HYPERBOLAS CENTRE AT ORIGIN

Transvers axis is along x-axis	Transvers axis is along y-axis
	
i) Equation of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ or $b^2x^2-a^2y^2=a^2b^2$	i) Equation of hyperbola $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ or $b^2y^2-a^2x^2=a^2b^2$
ii) Vertices at $(\pm a, 0)$	ii) Vertices at $(0, \pm a)$
iii) Foci at $(\pm c, 0)$	iii) Foci at $(0, \pm c)$
iv) Equation of directrices $x=\pm\frac{a^2}{c}$ or $x=\pm\frac{a}{e}$	iv) Equation of directrices $y=\pm\frac{a^2}{c}$ or $y=\pm\frac{a}{e}$

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MCQ- 2:

What are the equations of directrices of hyperbola $25y^2 - 9x^2 = 225$?

(a) $y = \pm\frac{9}{\sqrt{34}}$ (b) $x = \pm\frac{3}{34}$ (c) $y = \pm{}^{5}/_{3}$ (d) $x = \pm{}^{8}/_{\sqrt{31}}$

Solution:

$$25y^2 - 9x^2 = 225$$

The coefficient of y^2 is positive ,
so the transverse axis is along y – axis .

$$a^2 = \frac{225}{25} = 9 \ , \quad b^2 = \frac{225}{9} = 25$$

$$c^2 = a^2 + b^2$$

$$c^2 = 9+25$$

$$c^2 = 34$$

$$c = \sqrt{34}$$

Equation of directrices

$$y = \pm\frac{a^2}{c}$$

$$y = \pm\frac{9}{\sqrt{34}}$$

The answer is (a).

MCQ- 3:

What is the length of latus rectum of hyperbola $36x^2 - 25y^2 = 225$?

(a) 12 (b) ${}^{24}/_{7}$ (c) 6 (d) ${}^{36}/_{5}$

Solution:

$$36x^2 - 25y^2 = 225 \rightarrow (1)$$

Coefficient of x^2 is positive , so axis of hyperbola is

along x – axis.

$$a^2 = \frac{225}{36} = \frac{25}{4} \quad , \quad b^2 = \frac{225}{25} = 9$$

$$a = \frac{5}{2} \quad , \quad b = 3$$

$$\text{Length of latus rectum} = \frac{2b^2}{a}$$

$$= \frac{2\times 9}{5/2}$$

$$= \frac{36}{5}$$

The answer is (d).

EXERCISE-1

(1) What are the vertices of the hyperbola $2y^2 - 9x^2 = 18$.

(a) $(\pm\sqrt{2}, 0)$ (b) $(0, \pm\sqrt{2})$ (c) $(\pm 3, 0)$ (d) $(0, \pm 3)$

(2) What are the foci of the hyperbola $9x^2 - 16y^2 = 144$?

(a) $(\pm 4, 0)$ (b) $(\pm 3, 0)$ (c) $(\pm 5, 0)$ (d) $(0, \pm 2)$

(3) What are the equations of directrices of the hyperbola $5y^2 - 25x^2 = 150$?

(a) $y = \pm 4$ (b) $y = \pm 5$ (c) $x = \pm 4$ (d) $x = \pm 5$

(4) The centre of a hyperbola is at origin length of transverse axis lie on y-axis is 8 and eccentricity $\frac{3}{2}$. What are the foci?

(a) $(\pm 6, 0)$ (b) $(0, \pm 6)$ (c) $(0, \pm 12)$ (d) $(0, \pm 24)$

(5) e is the eccentricity of the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$. What are the equations of directrices.

(a) $x = \pm\frac{e}{a}$ (b) $y = \pm\frac{a}{e}$ (c) $x = \pm\frac{a}{e}$ (d) $y = \pm\frac{e}{a}$

(6) e is the eccentricity of the hyperbola, which is true?

(a) $e = 1$ (b) $e > 1$ (c) $e < 1$ (d) $e = 0$

(7) Which is the possible value of eccentricity of the hyperbola?

(a) -1.5 (b) 0.5 (c) 2.5 (d) 1

EQUATION OF HYPERBOLA

MCQ- 4:

What is the equation of hyperbola centre at origin, vertices at $(0\ , \pm 4)$ and length of latus rectum is 6?

(a) $12x^2 - 16y^2 = 192$ (b) $12y^2 - 16x^2 = 192$

(c) $16y^2 - 12x^2 = 192$ (d) None

Solution:

$$vertices(0, \pm 4)$$

$$abscissa = 0$$

$$\therefore \text{ the transverse axis is along } y - axis$$

$$\therefore coeff.of\ y^2 is\ positive$$

$$a = 4 \Rightarrow a^2 = 16$$

$$length\ of\ latus\ rectum = 6$$

$$\frac{2b^2}{a} = 6$$

$$\frac{2b^2}{4} = 6$$

$$b^2 = 12$$

Equation of hyperbola

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$b^2y^2 - a^2x^2 = a^2b^2$$

$$12y^2 - 16x^2 = 192$$

The answer is (b).

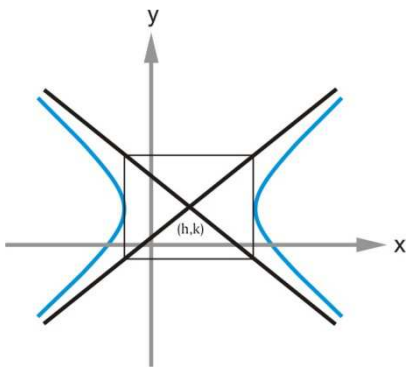
EXERCISE-2

(1) What is the equation of hyperbola centre at origin, vertices at $(\pm 3,0)$ and foci $(\pm 4,0)$?

(a) $7x^2 - 9y^2 = 63$ (b) $9x^2 - 7y^2 = 63$

(c) $9x^2 - 16y^2 = 144$ (d) $16x^2 - 9y^2 = 144$

HYPERBOLA CENTRE AT (h, k)



Comparison:

The comparison of equations of hyperbolas centre at origin and centre at (h, k):

Centre at origin	Centre at (h,k)
Transverse axis is along x-axis: i) Equation of ellipse $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $b^2x^2 - a^2y^2 = a^2b^2$ ii) Vertices at $(\pm a, 0)$ iii) Foci at $(\pm c, 0)$	Transverse axis is along x-axis: i) Equation of ellipse $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $b^2(x-h)^2 - a^2(y-k)^2 = a^2b^2$ ii) Vertices at $(\pm a + h, k)$ iii) Foci at $(\pm c + h, k)$
Transverse axis is along y-axis: i) Equation of ellipse $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ or $b^2y^2 - a^2x^2 = a^2b^2$ ii) Vertices at $(0, \pm a)$ iii) Foci at $(0, \pm c)$	Transverse axis is along y-axis: i) Equation of ellipse $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ or $b^2(y-k)^2 - a^2(x-h)^2 = a^2b^2$ ii) Vertices at $(h, \pm a + k)$ iii) Foci at $(h, \pm c + k)$

Formulae: (Same for both types of hyperbolas)

i) $c^2 = a^2 + b^2$

ii) $b^2 = a^2(e^2 - 1)$

iii) $e = {}^{c}/_{a}$

iv) $c = ae$

v) Length of latus rectum$= \frac{2b^2}{a}$

vi) Length of transverse axis$= 2a$

vii) Length of conjugate axis$= 2b$

viii) Semi transverse axis$= a$

ix) Semi conjugate axis$= b$

x) Distance between foci$= 2c$

xi) Distance between directrices $= 2(\frac{a^2}{c})$

MCQ-5 :

What is the eccentricity of the hyperbola

$$11x^2 - 25y^2 - 22x - 100y - 1978 = 0 \ ?$$

a^2 and b^2 are not in fraction they are positive integers.

(a) 8/3 (b) 6/5 (c) 7/2 (d) 11/8

Solution:

$$11x^2 - 25y^2 - 22x - 100y - 1978 = 0$$

$$a^2 = 25 \quad , \quad b^2 = 11$$

$$c^2 = a^2 + b^2$$

$$c^2 = 25 + 11$$

$$c^2 = 36$$

$$c = 6$$

Eccentricity:

$$e = {}^{c}/_{a}$$

$$e = {}^{6}/_{5}$$

The answer is (b).

MCQ- 6:

What is the eccentricity of hyperbola

$$7x^2 - 9y^2 - 28x - 18y - 44 = 0\ ?$$

(a) ${}^{9}/_{11}$ (b) ${}^{16}/_{19}$ (c) ${}^{5}/_{7}$ (d) ${}^{4}/_{3}$

Solution:

The four options are

(a) $\frac{9}{11} < 1$ (b) $\frac{16}{19} < 1$ (c) $\frac{5}{7} < 1$ (d) $\frac{4}{3} > 1$

Since the eccentricity of hyperbola is greater than 1, so

The answer is (d).

CONVERTING INTO STANDARD FORM

Note: {Convert general equation into standard equation when it is not conform (or not given) that a^2 and b^2 are in fractions or positive integers.}

Case-1: a^2 and b^2 are not in fraction, they are positive integers.

The Equation of Hyperbola is

$$9x^2 - 4y^2 - 36x - 40y - 100 = 0 \rightarrow (1)$$

Coeff. of $x^2(x + \frac{coeff.of\ x}{2.\ coeff.of\ x^2})^2$ − Coeff. of $y^2(y - \frac{coeff.of\ y}{2.\ coeff.of\ y^2})^2 =$

$+100 +$ Coeff. of $x^2(\frac{coeff.of\ x}{2.\ coeff.of\ x^2})^2$ − Coeff. of $y^2(\frac{coeff.of\ y}{2.\ coeff.of\ y^2})^2$

First step: (For L.H.S) Incomplete equation

$$9\left(x - \frac{36}{2\times 9}\right)^2 - 4(y + \frac{40}{2\times 4})^2$$

Second step:

$$9(x-2)^2 - 4(y+5)^2 = 100 + 9\times 2^2 - 4\times 5^2$$
$$= 36$$

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$$\text{coeff. of } x^2 = 5 \neq b^2 = {}^{8}/_{3}$$

so that when a^2 or b^2 or both are in fraction, then in general equation of hyperbola.

$$\text{coeff. of } y^2 \neq a^2$$
$$\text{coeff. of } x^2 \neq b^2$$

In this case to find a^2 and b^2 convert the general equation into standard equation of hyperbola.

MCQ- 7:

What are the vertices of hyperbola

$7\,(y\,-2)^2 - 9\,(x\,+\,1)^2 = 63$?

(a) $(-1,-1)\,,\ (-1,5)$ (b) $(-1,8)\,,(-1,2)$

(c) $(2,-1)\,,(2,5)$ (d) $(2,6)\,,(2,9)$

Solution:

$$7\,(y\,-2)^2 - 9\,(x\,+\,1)^2 = 63 \ \rightarrow (1)$$

Transverse axis is parallel to y – axis.

Standard equation of hyperbola

$$b^2(y-\mathrm{k})^2 - a^2(x\,-\,h)^2 = a^2b^2 \ \rightarrow (2)$$

$$a^2 = \frac{63}{7} = 9 \quad , \quad b^2 = \frac{63}{9} = 7 \ , \quad h = -1 \ , \quad k = 2$$

$$\Rightarrow a = 3$$

Vertices

$$(h, \pm a + k),$$
$$= (-1\,, \pm 3 + 2)$$
$$= (-1\,, 5)\,, (-1, -1)$$

The answer is (a).

MCQ-8 :

What are the foci of hyperbola

$$\frac{x^2}{5} - \frac{(y-1)^2}{4} = 1 \text{ ?}$$

(a) $(4,1),(-4,1)$ (b) $(-3,1),(5,1)$

(c) $(-3,1),(3,1)$ (d) $(-2,3),(2,3)$

Solution:

$$\frac{x^2}{5} - \frac{(y-1)^2}{4} = 1$$

$$h = 0 \quad , \quad k = 1$$
$$a^2 = 5 \quad , \quad b^2 = 4$$

$$c^2 = a^2 + b^2$$
$$c^2 = 5 + 4$$
$$c^2 = 9$$
$$c = 3$$

Foci , when centre at origin are

$$(\pm c\ ,0)$$

so ,foci when centre at $(h\ ,k)$

$$(\pm c + h\ ,k)$$

$$= (\pm 3 + 0\ ,1)$$
$$= (-3\ ,1) \quad , \quad (3\ ,1)$$

The answer is (c).

CENTRE OF HYPERBOLA

Equation of hyperbola centre at (h , k)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$b^2(x-h)^2 - a^2(y-k)^2 = a^2b^2$$

$$b^2(x^2 - 2hx + h^2) - a^2(y^2 - 2ky + k^2) = a^2b^2$$

$$b^2x^2 - 2b^2hx + b^2h^2 - a^2y^2 + 2a^2ky + a^2k^2 - a^2b^2 = 0$$

$$b^2x^2 - a^2y^2 - 2b^2hx + 2a^2ky + b^2h^2 + a^2k^2 - a^2b^2 = 0$$

$$b^2x^2 - a^2y^2 - 2b^2hx + 2a^2ky + b^2h^2 + a^2k^2 - a^2b^2 = 0$$

so that

$$h = -\frac{-2hb^2}{2b^2} \quad , \quad k = -\frac{2ka^2}{2a^2}$$

$$h = -\frac{(coeff.ofx)}{2(coeff.ofx^2)} \quad , \quad k = -\frac{coeff.ofy}{2(coeff.ofy^2)}$$

MCQ- 9:

What is the centre of the hyperbola

$$8y^2 - 5x^2 + 10x + 80y + 155 = 0 \, ?$$

(a) $(5 , -8)$ (b) $(2 , -8)$ (c) $(3 , -6)$ (d) $(1, -5)$

Solution:

$$h = -\frac{coeff\ .of\ x}{2(coeff\ .of x^2)} \quad , \quad k = -\frac{coeff\ .of\ y}{2(coeff\ .of y^2)}$$

$$h = -\frac{10}{2(-5)} \quad , \quad k = -\frac{80}{2(8)}$$

$$= 1 \quad , \quad = -5$$

Centre at $(1 , -5)$.

EXERCISE-3

(1) What is the vertex of the hyperbola $5(x-1)^2 - 16(y-3)^2 = 80$?

(a) (2, 5) (b) (1, 7) (c) (3, 5) (d) $(-3, 3)$

(2) What is the vertex of the hyperbola $4(y-3)^2 - 16x^2 = 64$?

(a) (3, 2) (b) (0, 7) (c) (4, 3) (d) (0, 3)

(3) What is a focus of the hyperbola $10y^2 - 15(x-2)^2 = 90$?

(a) $(2, \sqrt{10})$ (b) $(2, -5)$ (c) (0, 5) (d) $(0, \sqrt{15})$

(4) What is an equation of directrix of hyperbola $6y^2 - 15(x-2)^2 = 90$?

(a) $x = 6$ (b) $y = -3$ (c) $y = 5$ (d) $x = 7$

(5) What is an equation of directrix of hyperbola

$$10(x+2)^2 - 10(y-3)^2 = 60?$$

(a) $x = 0.5$ (b) $x = 4.5$ (c) $y = 2.5$ (d) $y = 3$

(6) What is the length of latus rectum of the hyperbola

$9(y-2)^2 - 36(x-1)^2 = 36 \times 9$?

(a) 5 (b) 9 (c) 3 (d) 6

EQUATION OF TANGENT TO HYPERBOLA

Equation of tangent to the hyperbola at point (x_1, y_1).

S.NO.	EQUATION OF HYPERBOLA	EQUATION OF TANGENT
i	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$
ii	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$\frac{yy_1}{a^2} - \frac{xx_1}{b^2} = 1$
	or	
i	$b^2x^2 - a^2y^2 = a^2b^2$	$b^2xx_1 - a^2yy_1 = a^2b^2$
ii	$b^2y^2 - a^2x^2 = a^2b^2$	$b^2yy_1 - a^2xx_1 = a^2b^2$

MCQ- 10:

What is the equation of tangent to the hyperbola $x^2 - y^2 = 16$ at point $(-5, 3)$?

(a) $3x - y + 18 = 0$ (b) $5x + 3y + 9 = 0$

(c) $2x - 5y + 25 = 0$ (d) $5x + 3y + 16 = 0$

Solution:

$x^2 - y^2 = 16$

The equation of tangent

$$xx_1 - yy_1 = 16$$

Point $(-5, 3)$

$$-5x - 3y = 16$$

$$5x + 3y + 16 = 0$$

The answer is (d).

TANGENTS AT VERTICES

The tangents at the vertices are parallel to x-axis or y-axis.

Case-1: Transverse axis is along x-axis:

$$5y = kx + 9$$

$$y = \frac{k}{5}x + \frac{9}{5}$$

$$m = \frac{k}{5} \quad , \quad c = \frac{9}{5}$$

Conduction of tangency

$$c^2 = -b^2m^2 + a^2$$

$$\frac{81}{25} = -9 \times \frac{k^2}{25} + 9$$

$$\frac{9k^2}{25} = 9 - \frac{81}{25}$$

$$k^2 = 16$$

$$k = \pm 4$$

The answer is (c).

EXERCISE-5

(1) $k =?$, if the line $y = kx + 3$ is tangent to the hyperbola $4x^2 - 9y^2 = 36$?

(a) $\frac{\sqrt{7}}{2}$ (b) $\frac{5}{2}$ (c) $\frac{\sqrt{5}}{3}$ (d) $\frac{\sqrt{13}}{3}$

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