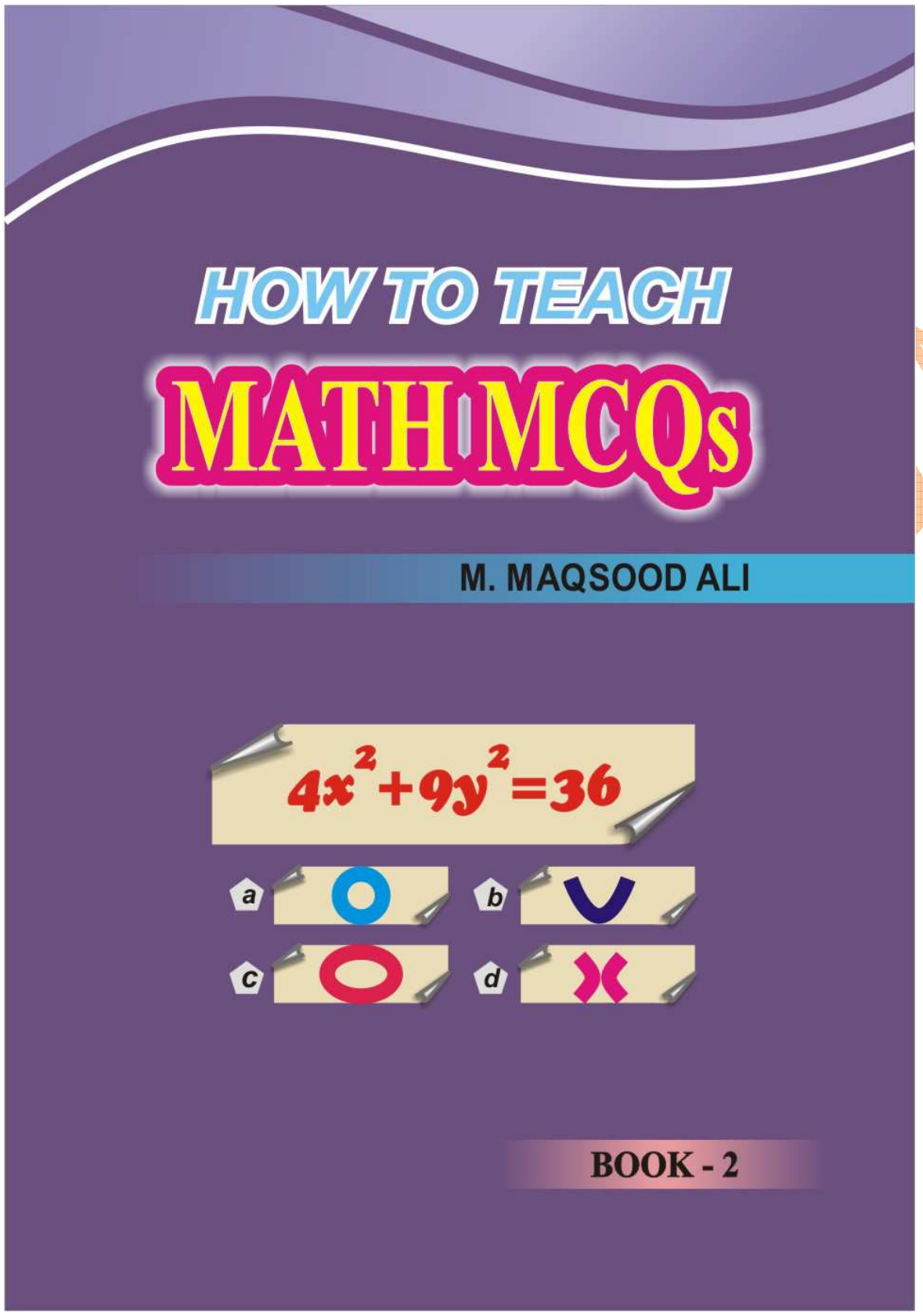
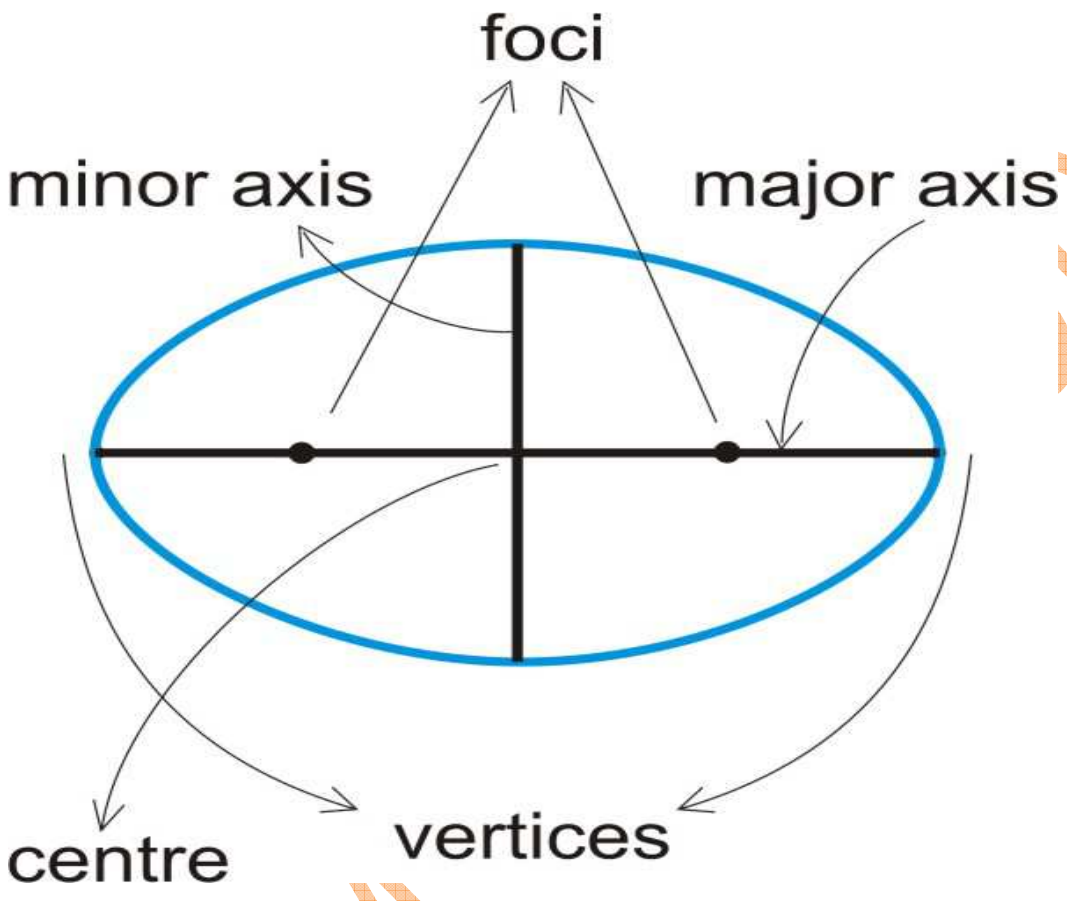
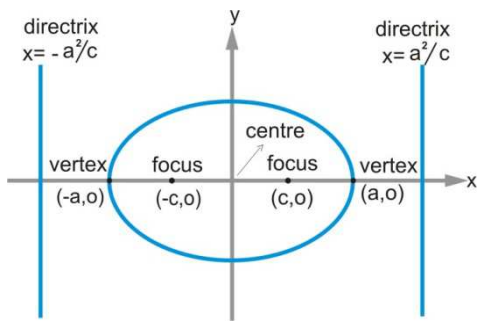
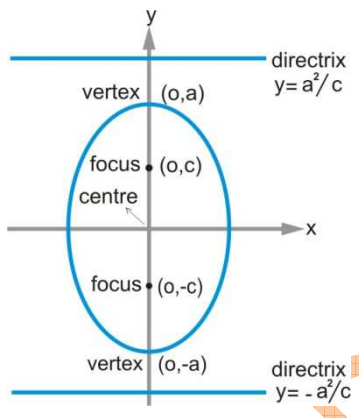

HOW TO TEACH
MATH MCQs
M. MAQSOOD ALI
$4x^2+9y^2=36$
a
b
c
d
BOOK - 2

Chapter 9

ELLIPSES



ELLIPSE CENTRE AT ORIGIN

Major axis is along x-axis	Major axis is along y-axis
	
i) Equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $b^2x^2+a^2y^2=a^2b^2$	i) Equation of ellipse $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ or $a^2x^2+b^2y^2=a^2b^2$
ii) Vertices at $(\pm a, 0)$	ii) Vertices at $(0, \pm a)$
iii) Foci at $(\pm c, 0)$	iii) Foci at $(0, \pm c)$
iv) Equation of directrices $x=\pm\frac{a^2}{c}$ or $x=\pm\frac{a}{e}$	iv) Equation of directrices $y=\pm\frac{a^2}{c}$ or $y=\pm\frac{a}{e}$

Formulae: (Same for both types of ellipses)

i) $c^2 = a^2 - b^2$

ii) $b^2 = a^2(1 - e^2)$

iii) $e = {}^{c}/_{a}$

iv) $c = ae$

v) Length of latus rectum$= \frac{2b^2}{a}$

vi) Length of major axis$= 2a$

vii) Length of minor axis$= 2b$

viii) Semi major axis$= a$

ix) Semi minor axis$= b$

x) Distance between foci$= 2c$

xi) Distance between directrices$= 2(\frac{a^2}{c})$

MCQ-1 :

What are the vertices or the ellipse $9x^2 + 25y^2 = 225$?

(a) $(0, \pm 3)$ (b) $(\pm 3, 0)$ (c) $(\pm 5, 0)$ (d) $(0, \pm 5)$

Solution:

$$9x^2 + 25y^2 = 225$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

Center at origin and major axis is along x-axis $\{\because 9 < 25$

$a^2 = 25$ $\{\because a^2 = \frac{225}{9} = 25$

$a = 5$

vertices at $(\pm 5, 0)$

The answer is (c).

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$$c^2 = 36$$

$$c = 6$$

Foci at $(\pm c, 0)$

F $(\pm 6, 0)$

The answer is (d).

EXERCISE-1

(1) What is the vertices of the ellipse $25x^2 + 9y^2 = 225$?

(a) (0, ± 5) (b) (± 5, 0) (c) (0, ± 3) (d) (± 3, 0)

(2) What are the foci of the ellipse $16x^2 + 25y^2 = 400$?

(a) (± 5, 0) (b) (0, ± 3) (c) (± 3, 0) (d) (0, ± 5)

(3) What are the coordinates of end-point of minor axis of the ellipse $9x^2 + 4\,y^2 = 36$?

(a) (± 3, 0) (b) (0, ± 3) (c) (0, ± 2) (d) (± 2, 0)

(4) What are the equations of the directrices of an ellipse $4x^2 + 20\,y^2 = 80$?

(a) $y = \pm 2$ (b) $x = \pm 5$ (c) $x = \pm\frac{5}{2}$ (d) $y = \pm\frac{10}{3}$

(5) What is the eccentricity of the ellipse $25x^2 + 9y^2 = 225$?

(a) $\frac{2}{5}$ (b) $\frac{5}{4}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

(6) What is the length of latus rectum of the ellipse $16x^2 + 4y^2 = 64$?

(a) 1 (b) 2 (c) 4 (d) 8

(7) What is the length of the latus rectum of an ellipse if length of major axis and minor axis are 8 and 6 respectively?

(a) 10 (b) 64 (c) 9 (d) 4.5

(8) Semi major axis and semi minor axis of an ellipse are 5 and 3 respectively. What is the eccentricity?

(a) $\frac{9}{25}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{16}{25}$

(9) The eccentricity of an ellipse whose centre is at origin is $\frac{4}{5}$. The length of major axis is 20 and lies on y-axis. What are the foci?

(a) (0, ± 10) (b) (0, ± 25) (c) (0, ± 6) (d) (0, ± 8)

(10) e is the eccentricity of an ellipse. Which is true?

(a) $e = 1$ (b) $e > 1$ (c) $e < 1$ (d) $e = 0$

(11) e is the eccentricity of an ellipse. Which is the possible value of e?

(a) 0.8 (b) 1 (c) 2.5 (d) -0.5

EQUATION OF ELLIPSE

MCQ-4 :

What is the equation of ellipse vertices at $(\pm 6,0)$ and foci $(\pm 4,0)$?

(a) $36x^2 + 16y^2 = 576$ (b) $16x^2 + 36y^2 = 576$

(c) $20x^2 + 36y^2 = 720$ (d) None

Solution:

Vertices $(\pm 6, 0)$

$\because$ ordinate = 0

$\therefore$ centre at origin, major axis is along x-axis and

$$a = 6$$
$$a^2 = 36$$

foci $(\pm 4,0)$

$$c = 4$$
$$c^2 = 16$$

$$c^2 = a^2 - b^2$$
$$b^2 = a^2 - c^2$$

$$b^2 = 36 - 16$$
$$= 20$$

$\because$ major axis is along x-axis

coeff.of $x^2 <$ coeff.of y^2

Equation of ellipse is

$$20x^2 + 36y^2 = 720$$

The answer is (c).

EXERCISE-2

(1) What is the equation of ellipse centre at origin, vertices at $(0, \pm 4)$ and foci at $(0, \pm 2)$?

(a) $16x^2 + 4y^2 = 32$ (b) $4x^2 + 16y^2 = 32$

(c) $12x^2 + 16y^2 = 192$ (d) $16x^2 + 12y^2 = 192$

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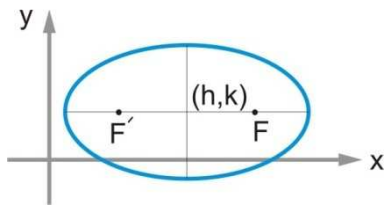
ELLIPSE CENTRE AT (h, k)



Comparison:

The comparison of equations of ellipses "centre at origin" and "centre at (h, k)":

Centre at origin	Centre at (h, k)
Major axis is along x-axis: i) Equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $b^2x^2+a^2y^2=a^2b^2$ ii) Vertices at $(\pm a, 0)$ iii) Foci at $(\pm c, 0)$	Major axis is along x-axis: i) Equation of ellipse $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ or $b^2(x-h)^2+a^2(y-k)^2=a^2b^2$ ii) Vertices at $(\pm a+h, k)$ iii) Foci at $(\pm c+h, k)$
Major axis is along y-axis: i) Equation of ellipse $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ or $a^2x^2+b^2y^2=a^2b^2$ ii) Vertices at $(0, \pm a)$ iii) Foci at $(0, \pm c)$	Major axis is along y-axis: i) Equation of ellipse $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ or $a^2(x-h)^2+b^2(y-k)^2=a^2b^2$ ii) Vertices at $(h, \pm a+k)$ iii) Foci at $(h, \pm c+k)$

Formulae: (Same for both types of ellipses)

i) $c^2 = a^2 - b^2$

ii) $b^2 = a^2(1 - e^2)$

iii) $e = {}^{c}/_{a}$

iv) $c = ae$

v) Length of latus rectum$= \frac{2b^2}{a}$

vi) Length of major axis$= 2a$

vii) Length of minor axis$= 2b$

viii) Semi major axis$= a$

ix) Semi minor axis$= b$

x) Distance between foci$= 2c$

xi) Distance between directrices$= 2(\frac{a^2}{c})$

MCQ-5 : What are the vertices of the ellipse

$$4(x-6)^2 + 9(y+5)^2 = 36\ ?$$

(a) $(3,-5),(9,-5)$ (b) $(-2,9)\ ,(8,6)$

(c) $(-4,6),(8,6)$ (d) $(9,-5),(3,-5)$

Solution:

$\because$ Coeff. of $(x-6)^2$ $<$ Coeff. of $(y+5)^2$

$\therefore$Major axis is along x-axis

$$a^2 = \frac{36}{4} = 9 \quad \Rightarrow \quad a = 3$$

$$b^2 = \frac{36}{9} = 4 \quad \Rightarrow \quad b = 2,$$

$$(x-6)^2 \quad \Rightarrow \quad h = 6$$

$$(y+5)^2 \quad \Rightarrow \quad k = -5$$

Vertices at $(\pm a + h, k)$

$$= (\pm 3 + 6, -5)$$

$$= (9,-5),(3,-5)$$

The answer is (d).

TO FIND CENTRE OF THE ELLIPSE

If the given equation of ellipse is in general form.

Note: $\{a^2$ and b^2 are not in fraction they are positive integers.

So that

(i) Coefficient of $x^2 = a^2 \ \ or \ \ b^2$

(ii) Coefficient of $y^2 = b^2 \ \ or \ \ a^2 \}$

Standard equation of the ellipse centre at (h,k)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$b^2(x-h)^2 + a^2(y-k)^2 = a^2b^2$$

$$b^2(x^2 - 2hx + h^2) + a^2(y^2 - 2ky + k^2) = a^2b^2$$

$$b^2x^2 - 2hb^2x + b^2h^2 + a^2y^2 - 2ka^2y + a^2k^2 = a^2b^2$$

$$b^2x^2 + a^2y^2 - 2hb^2x - 2ka^2y + b^2h^2 + a^2k^2 - a^2b^2 = 0$$

$$b^2x^2 + a^2y^2 + 2(-h)b^2x + 2(-k)a^2y + b^2h^2 + a^2k^2 - a^2b^2 = 0$$

is the general equation of ellipse.

$$h = -\frac{2(-h)b^2}{2b^2} \quad , \quad k = -\frac{2(-k)a^2}{2a^2}$$

$$h = -\frac{coeff.ofx}{2(coeff.ofx^2)} \quad , \quad k = -\frac{coeff.ofy}{2(coeff,ofy^2)}$$

MCQ- 6:

What is the centre of the ellipse

$$6x^2 + 10y^2 + 36x - 160y + 634 = 0\ ?$$

a^2 and b^2 are positive integers, not in fraction.

(a) $(-4, 6)$ (b) $(-3, 8)$ (c) $(-9, 16)$ (d) $(-2, 10)$

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MCQ-8 :

What is the eccentricity of the following

$$16x^2 + 9y^2 - 64x - 90y + 145 = 0\ ?$$

a^2 and b^2 are positive integers.

(a) $\sqrt{7}/16$ (b) 7/16 (c) $\sqrt{7}/4$ (d) $^1/_4$

Solution:

Coeff.of $x^2 and\ y^2$ both are positive , so it is the equation of ellipse.

Coeff. of y^2<coeff. of x^2

$$a^2 = 16 \quad \text{and} \quad b^2 = 9$$

There is no need for eccentrity, that major axis is along x-axis or y-axis, and eccentriaty does not depend on the center of allipse

$$c^2 = a^2 - b^2$$

$$c^2 = 16 - 9$$

$$= 7$$

$$c = \sqrt{7}$$

$$e = {}^c/_a$$

$$e = {}^{\sqrt{7}}/_4$$

The answer is (c).

MCQ- 9:

What is the eccentricity of the ellipse

$$64x^2 + 9y^2 - 256x - 90y + 337 = 0\ ?$$

(a) $\sqrt{55}/8$ (b) $\sqrt{73}/8$ (c) 4/3 (d) 25/6

Solution:

Given options

(a) $\frac{\sqrt{55}}{8} < 1$ (b) $\frac{\sqrt{3}}{8} > 1$ (c) $\frac{4}{3} > 1$ (d) $\frac{25}{6} > 1$

For ellipse e < 1

The answer is (a).

MCQ- 10:

What is the length of latus rectum of the ellipse

$$4x^2 + 9y^2 - 48x - 144y + 684 = 0\ ?$$

a^2 and b^2 are positive integers.

(a) 13 (b) 9 (c) 8/9 (d) 8/3

Solution:

coeff. of x^2 < coeff. of y^2

Major axis is along x-axis.

$$a^2 = 9 \quad \text{and} \quad b^2 = 4$$

$$a = 3 \quad , \quad b = 4$$

Length of latus rectum $= 2b^2/a$

$$= \frac{2\times 4}{3}$$

$$= 8/3$$

The answer is (d).

HOW TO CONVERT INTO STANDARD FORM

Note: {Convert general equation into standard equation when it is not conform (or not given) that a^2 and b^2 are in fractions or positive integers.}

Case-1: a^2 and b^2 are not in fraction, they are positive integers.

$$4x^2 + 25y^2 - 16x + 150y + 141 = 0 \qquad \rightarrow (1)$$

Coeff. of $x^2(x - \frac{coeff.of\ x}{2.\ coeff.of\ x^2})^2$ + Coeff. of $y^2(y - \frac{coeff.of\ y}{2.\ coeff.of\ y^2})^2 =$

$$-141 + \text{Coeff. of } x^2(\frac{coeff.of\ x}{2.\ coeff.of\ x^2})^2 + \text{Coeff. of } y^2(\frac{coeff.of\ y}{2.\ coeff.of\ y^2})^2$$

First step: (For L.H.S) Incomplete equation

$$4\left(x - \frac{16}{2\times4}\right)^2 + 25(y + \frac{150}{2\times25})^2$$

Second step:

$$4(x-2)^2 + 25(y+3)^2 = -141 + 4\times2^2 + 25\times3^2$$
$$= 100$$

$$\frac{(x-2)^2}{25} + \frac{(y+3)^2}{4} = 1 \qquad \rightarrow (2)$$

is the standard equation.

By equation (2)

$$\Rightarrow a^2 = 25 \ , \quad b^2 = 4$$

According to equation (1)

coeff. of $x^2 = b^2 = 4$

coeff. of $y^2 = a^2 = 25$

Case-2: a^2 and b^2 are in fraction.

$$8x^2 + 25y^2 - 32x + 150y + 157 = 0 \qquad \rightarrow (1)$$

$$8(x-2)^2 + 25(y+3)^2 = -157 + 8\times2^2 + 25\times3^2$$
$$= 100$$

$$\frac{2(x-2)^2}{25} + \frac{(y+3)^2}{4} = 1$$

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(7) What is focus of the ellipse $8(x+2)^2+24(y+1)^2=24$.

(a) $(2,-1)$ (b) $(3,-2)$ (c) $(-2,-1)$ (d) $(-1,2)$

(8) What is a focus of the ellipse $6(x-5)^2+2(y-3)^2=12$.

(a) (5, 4) (b) (5, 2) (c) (5, 5) (d) (7, 4)

(9) e is the eccentricity of the ellipse $a^2(x-h)^2+b^2(y-k)^2=a^2b^2$.

What are the coordinates of the foci?

(a) $(h \pm ae, k)$ (b) $(h, k \pm ae)$ (c) $(0 \pm ae)$ (d) $(k, h \pm ae)$

(10) What is an equation of directrix of the ellipse $3(x-2)^2+12(y-6)^2=36$?

(a) $y=5$ (b) $x=2$ (c) $x=4$ (d) $x=6$

(11) What is an equation of directrix of the ellipse $20(x-5)^2+4(y-3)^2=80$?

(a) $y=8$ (b) $x=8$ (c) $y=10$ (d) $x=10$

(12) What is an equation of directrix of the ellipse $20(x-5)^2+4(y-3)^2=80$?

(a) $x=-2$ (b) $x=0$ (c) $y=-2$ (d) $y=6$

(13) $e=\frac{c}{a}$ is the eccentricity of the ellipse $b^2(x-h)^2+a^2(y-k)^2=a^2b^2$.

Which are the equation of the directrices?

(a) $x=k\pm\frac{a^2}{c}$ (b) $y=k\pm\frac{a^2}{c}$ (c) $y=h\pm\frac{a^2}{c}$ (d) $x=h\pm\frac{a^2}{c}$

(14) What is the eccentricity of the ellipse $25(x-9)^2+9(y-7)^2=225$?

(a) $\frac{4}{5}$ (b) $\frac{5}{2}$ (c) $\frac{11}{14}$ (d) $\frac{3}{5}$

(15) What is the eccentricity of the ellipse $b^2(x-h)^2+a^2(y-k)^2=a^2b^2$?

(a) $\frac{c-k}{a-h}$ (b) ac (c) $\frac{a}{c}$ (d) $\frac{c}{a}$

(16) What is the length of latus rectum of the ellipse

$64(x-3)^2+16(y+7)^2=1024$?

(a) 1.8 (b) $\frac{3}{2}$ (c) 0.5 (d) 4

EQUATION OF TANGENT TO THE ELLIPSE

Equation of tangent to the ellipse at point (x_1, y_1).

S.No.	EQUATION OF ELLIPSE	EQUATION OF TANGENT
1	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$
2	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	$\frac{xx_1}{b^2}+\frac{yy_1}{a^2}=1$

OR

1	$b^2x^2+a^2y^2=a^2b^2$	$b^2xx_1+yy_1a^2=a^2b^2$
2	$a^2x^2+b^2y^2=a^2b^2$	$a^2xx_1+b^2yy_1=a^2b^2$

Note: The tangents at the vertices and end points of minor axis is parallel to x-axis or y-axis.

Case-1: Major axis is along x-axis:

i) Equation of tangent at $(a, 0)$ and $(-a, 0)$:

$x = a$ and $x = -a$ respectively.

ii) Equation of tangent at $(0, b)$ and $(0, -b)$:

$y = b$ and $y = -b$ respectively.

Case-2: Major axis is along y-axis:

i) Equation of tangent at $(0, a)$ and $(0, -a)$:

$y = a$ and $y = -a$ respectively.

ii) Equation of tangent at $(b, 0)$ and $(-b, 0)$:

$x = b$ and $x = -b$ respectively.

MCQ-11 :

What is the equation of tangent to the ellipse $16x^2 + 25y^2 = 400$ at point $\left(3, \frac{8}{5}\right)$?

(a) $6x + 5y - 50 = 0$ (b) $6x + 5y - 16 = 0$

(c) $15x + 5y + 53 = 0$ (d) $2x + 8y + 13 = 0$

Solution:

$$16x^2 + 25y^2 = 400$$

Point $\left(3, \frac{8}{5}\right)$

$$x_1 = 3 \quad , \quad y_1 = {}^{8}/_{5}$$

Equation of tangent

$$16xx_1 + 25yy_1 = 400$$

$$16(3)x + 25\left(\frac{8}{5}\right)y = 400$$

$$48x + 40y = 400$$

$$6x + 5y = 50$$

The answer is (a).

EXERCISE-4

(1) What is the equation of tangent to the ellipse $9x^2 + 4y^2 = 36$ at (2, -3) ?

(a) $6x - 8y = 36$ (b) $2x - 3y = 6$

(c) $18x - 12y = 36$ (d) $9x + 4y = 36$

(2) What is the equation of the tangent drawn at a vertex of the ellipse $16x^2 + 9y^2 = 144$?

(a) $y = 4$ (b) $x = 4$ (c) $y = 2$ (d) $x = -2$

(3) What is the equation of the tangent drawn to a vertex of the ellipse $4(x-2)^2 + 9(y-1)^2 = 36$?

(a) $x = -5$ (b) $y = -5$ (c) $x = 5$ (d) $y = 5$

CONDITION OF TANGENCY

S.No	EQ. OF ELLIPSE	EQ. OF ST. LINE	CONDITION OF TANGENCY
1	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $b^2x^2+a^2y^2=a^2b^2$	$y = mx + c$	$c^2 = a^2m^2 + b^2$
2	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ or $a^2x^2+b^2y^2=a^2b^2$	$y = mx + c$	$c^2 = b^2m^2 + a^2$

MCQ- 12:

a^2 =?, if the line $y = \sqrt{5}\,x + 7$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1$

(a) 25 (b) 9 (c) 49 (d) 16

Solution:

$$y = \sqrt{5}\,x + 7$$

$$m = \sqrt{5} \quad , \quad c = 7$$

$$\frac{x^2}{a^2}+\frac{y^2}{4}=1$$

$$b^2 = 4$$

Condition of tangency

$$c^2 = a^2m^2 + b^2$$

$$49 = 5a^2 + 4$$

$$a^2 = 9$$

The answer is (b).

EXERCISE-5

(1) $b^2 =$? if the line $y = 3x + 10$ touches the ellipse $\frac{x^2}{6} + \frac{y^2}{b^2} = 1$?

(a) 46 (b) 64 (c) 32 (d) 82