

HOW TO TEACH MATH MCQs

M. MAQSOOD ALI

$$4x^2 + 9y^2 = 36$$

- a 
- b 
- c 
- d 

BOOK - 2

Chapter 6**INTEGRATIONS****Formula:**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

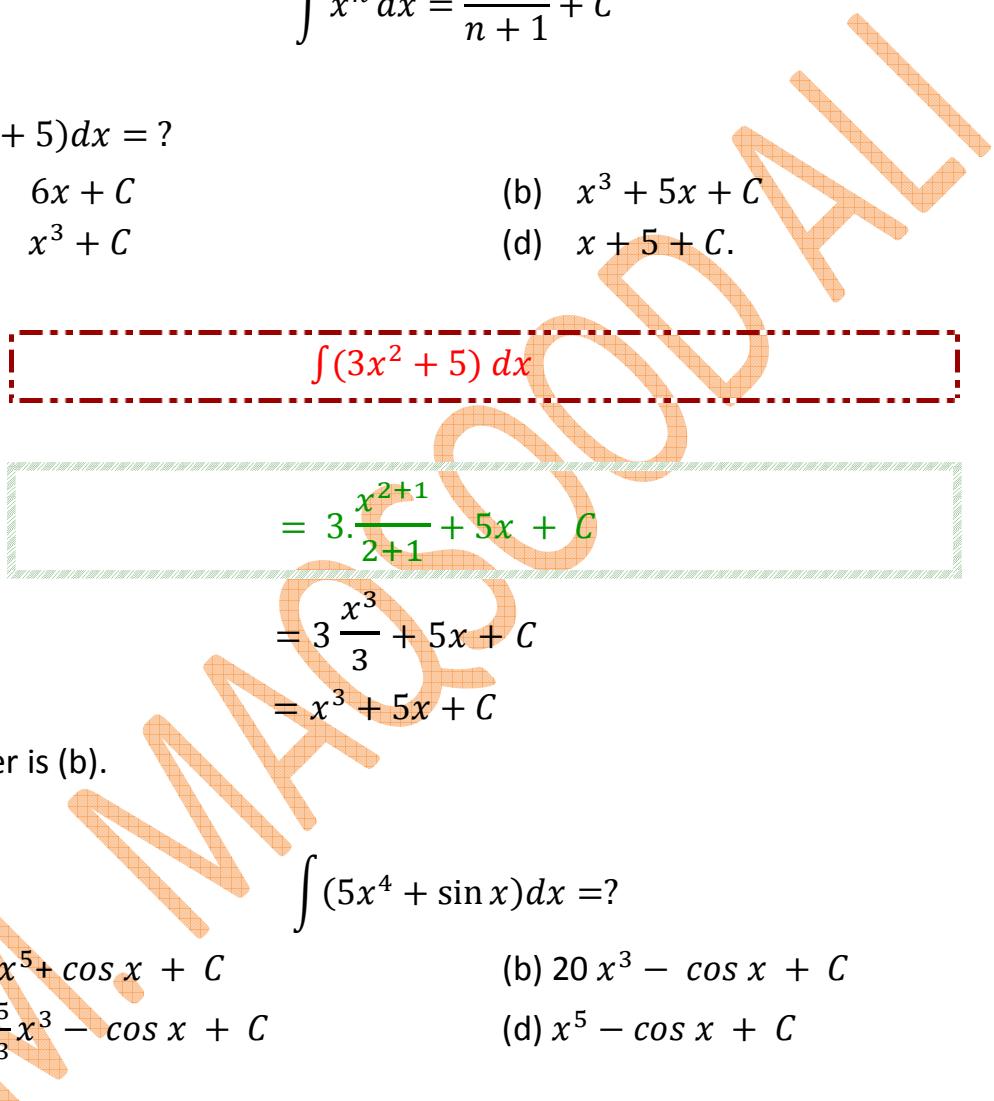
MCQ-1 :

$$\int (3x^2 + 5)dx = ?$$

- (a) $6x + C$
 (c) $x^3 + C$

- (b) $x^3 + 5x + C$
 (d) $x + 5 + C.$

Solution:



$$\begin{aligned} & \boxed{\int (3x^2 + 5) dx} \\ &= 3 \cdot \frac{x^{2+1}}{2+1} + 5x + C \\ &= 3 \frac{x^3}{3} + 5x + C \\ &= x^3 + 5x + C \end{aligned}$$

The answer is (b).

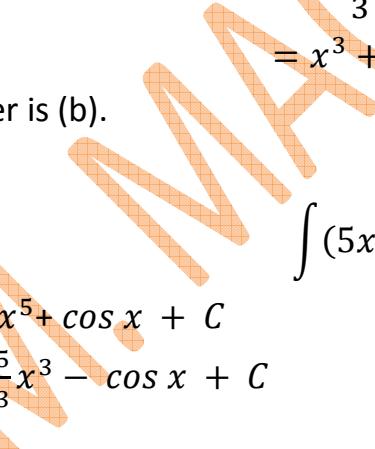
MCQ-2 :

$$\int (5x^4 + \sin x)dx = ?$$

- (a) $x^5 + \cos x + C$
 (c) $\frac{5}{3}x^3 - \cos x + C$

- (b) $20x^3 - \cos x + C$
 (d) $x^5 - \cos x + C$

Solution:



$$\begin{aligned} & \boxed{\int (5x^4 + \sin x)dx} \\ &= \frac{5x^{4+1}}{4+1} + (-\cos x) + C \end{aligned}$$

$$\begin{aligned}
 &= 5 \cdot \frac{x^5}{5} - \cos x + C \\
 &= x^5 - \cos x + C
 \end{aligned}$$

The answer is (d).

EXERCISE- 1

(1) $\int (x^5 - \cos x) dx = ?$

(a) $\frac{1}{6}(x^6 + 6 \sin x) + C$

(c) $6x^6 + \sin x + C$

(2) $\int \frac{1}{5}x^4 dx = ?$

(a) $\frac{4}{5}x^3 + C$

(b) $x^5 + c$

(b) $\frac{1}{6}x^6 - \sin x + C$

(d) $\frac{x^4}{4} - \sin x + C$

(c) $\frac{1}{25}x^5 + C$

(d) None

Formula:

$$\int f^n(x) \cdot f'(x) dx = \frac{f^{n+1}(x)}{n+1} + C$$

or

$$\int y^n \cdot y' dx = \frac{y^{n+1}}{n+1} + C$$

or

$$\int y^n \cdot \frac{dy}{dx} dx = \frac{y^{n+1}}{n+1} + C$$

MCQ- 3:

$\int 3x^2(x^3+9)^8 dx = ?$

(a) $\frac{1}{9}(x^3+9)^9 + C$

(c) $24(x^3+9)^7 + C$

(b) $\frac{1}{9}x^3(x^3+9)^9 + C$

(d) $\frac{1}{3}x^2(x^3+9)^9 + C$

Solution:

$$\int 3x^2(x^3+9)^8 dx$$

$$\begin{aligned} & \left\{ \because \frac{d}{dx}(x^3+9) = 3x^2 \right. \\ &= \frac{(x^3+9)^{8+1}}{8+1} + C \\ &= \frac{(x^3+9)^9}{9} + C \end{aligned}$$

The answer is (a).

MCQ- 4:

$$\int x^4 (2x^5 + 6)^7 dx = ?$$

- (a) $\frac{x^5(2x^5+6)^8}{40} + C$
 (c) $\frac{(2x^5+6)^8}{80} + C$

- (b) $\frac{(2x^5+6)^8}{8} + C$
 (d) $\frac{x^5(2x^5+6)^8}{200} + C$

Solution:

$$\int x^4 (2x^5 + 6)^7 dx$$

$$\left\{ \because \frac{d}{dx}(2x^5+6) = 10x^4 \right.$$

Multiplying and dividing by 10

$$\begin{aligned} &= \int \frac{1}{10} \cdot 10x^4 (2x^5 + 6)^7 dx \\ &= \frac{1}{10} \int 10x^4 (2x^5 + 6)^7 dx \\ &= \frac{(2x^5+6)^8}{10 \times 8} + C \\ &= \frac{(2x^5+6)^8}{80} + C \end{aligned}$$

The answer is (c).

MCQ-5 :

$$\int 2(x^2 + 3)^2 dx = ?$$

(a) $(x^2 + 3)^3 / 3x$

(b) $\frac{2}{3}(x^2 + 3)^3 + c$

(c) $\frac{2}{5}x^5 + 4x^3 + 18x + c$

(d) $x^5 + x^3 + x + c$

Solution:

$$\int 2(x^2 + 3)^2 dx$$

$$\{\because \frac{d}{dx}(x^2 + 3) = 2x$$

Wrong method:multiply and dividing by x

$$= \int \frac{1}{x} \cdot 2x (x^2 + 3)^2 dx$$

$$\neq \frac{1}{x} \int 2x (x^2 + 3)^2 dx$$

$$= \int 2(x^4 + 6x^2 + 9) dx$$

$$= 2 \int (x^4 + 6x^2 + 9) dx$$

$$= 2 \left(\frac{x^5}{5} + 6 \frac{x^3}{3} + 9x \right) + c$$

$$= 2 \left(\frac{1}{5}x^5 + 2x^3 + 9x \right) + c$$

$$= \frac{2}{5}x^5 + 4x^3 + 18x + c$$

The answer is (c).

Formula:

$$\int \frac{f'(x)}{f^n(x)} dx = ?$$

Case-1: n=1

$$\int \frac{f'(x)}{f(x)} dx = \ln(f(x)) + C$$

Case-2: n ≠ 1

$$\begin{aligned} \int \frac{f'(x)}{f^n(x)} dx &= \int f'(x) \cdot f^{-n}(x) dx \\ &= \frac{f^{-n+1}(x)}{-n+1} + C \end{aligned}$$

MCQ- 6:

$$\int \frac{2x}{(x^2+3)} dx = ?$$

(a) $\frac{1}{(x^2+3)^2} + C$

(c) $\tan^{-1}(2x) + C$

(b) $\ln(x^2 + 3) + C$

(d) $(x^2 + 3)^2 + C$

Solution:

$$\begin{aligned} \int \frac{2x}{(x^2+3)} dx &\\ \therefore \frac{d}{dx} (x^2 + 3) &= 2x \\ \text{Power of } (x^2 + 3) \text{ is 1, so} \\ &= \ln(x^2 + 3) + C \end{aligned}$$

The answer is (b).

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(a) _____ (b) _____ (c) _____ (d) _____

(5) $\int \sin x (2 - \cos x)^6 dx = ?$

(a) $\frac{-(2-\cos x)^7}{7}$

(b) $\frac{\sin x (2-\cos x)^7}{7}$

(c) $\frac{(2-\cos x)^5}{5}$

(d) $\frac{(2-\cos x)^7}{7}$

(6) $\int \frac{\cos x}{(7+3 \sin x)^5} dx = ?$

(a) $\frac{(7+3 \sin x)^{-4}}{12}$

(b) $\frac{\cos x}{12(7+3 \sin x)^4}$

(c) $\frac{-1}{12(7+3 \sin x)^4}$

(d) $\frac{3}{4(7+3 \sin x)^4}$

(7) $\int \frac{\sec^2 x}{(3+7 \tan x)^5} dx = ?$

(a) $\frac{-1}{28(3+7 \tan x)^4}$

(b) $\frac{-7}{4(3+7 \tan x)^4}$

(c) $\frac{(3+7 \tan x)^4}{28}$

(d) None

(8) $\int \frac{(1-\tan x)^4}{\cos^2 x} dx = ?$

(a) $\frac{(1-\tan x)^5}{5}$

(b) $3(1-\tan x)^3$

(c) $\frac{(\tan x-1)^5}{5}$

(d) $\frac{(1-\tan x)^3}{3}$

(9) $\int (\cos^2 x - \sin^2 x)^6 \sin 2x dx = ?$

(a) $\frac{-(\cos^2 x - \sin^2 x)^5}{10}$

(b) $\frac{(\sin^2 x - \cos^2 x)^7}{14}$

(c) $\frac{(\cos 2x)^7}{14}$

(d) None

(10) $\int (\cos 2x)^6 \sin x \cos x dx = ?$

(a) $\frac{(\cos 2x)^7}{7}$

(b) $\frac{(\cos 2x)^5}{20}$

(c) $-5(\cos 2x)^5$

(d) $\frac{(-\cos 2x)^7}{28}$

(11) $\int (1 + \tan x)^6 (1 + \tan^2 x) dx = ?$

(a) $\frac{(1+\tan x)^5}{5}$

(b) $\frac{(1+\tan^3 x)^7}{7}$

(c) $\frac{(1+\tan x)^7}{7}$

(d) None

(12) $\int \frac{\sin 2t}{\sqrt{1-\cos 2t}} dt = ?$

(a) $\sqrt{1 - \cos 2t}$ (b) $\frac{1}{2}\sqrt{1 - \cos 2t}$ (c) $2\sqrt{1 - \cos 2t}$ (d) $\frac{1}{\sqrt{1-\cos 2t}}$

(13) $\int (2x^2 - 4x)^3 (x - 1) dx = ?$

(a) $\frac{(2x^2-4x)^4}{4}$ (b) $(x^2 - 2x)^4$ (c) $\frac{(x^2-2x)}{4}$ (d) $2(2x^2 - 4x)^2$

(14) $\int \frac{(1-\sqrt{x})^4}{\sqrt{x}} dx = ?$

(a) $\frac{2(1-\sqrt{x})^5}{5}$ (b) $\frac{2(\sqrt{x}-1)^5}{5}$ (c) $\frac{-(1-\sqrt{x})^5}{10}$ (d) $\frac{5(1-\sqrt{x})^5}{2}$

(15) $\int 2\left(1 - \frac{1}{\sqrt{x}}\right)^3 x^{-3/2} dx = ?$

(a) $\frac{\left(1-\frac{1}{\sqrt{x}}\right)^4}{4}$ (b) $\left(1 - \frac{1}{\sqrt{x}}\right)^4$ (c) $\frac{3}{2}\left(1 - \frac{1}{\sqrt{x}}\right)^4 x^{-1/3}$ (d) $\frac{(1-\sqrt{x})^4}{2}$

(16) $\int \frac{\sec^{-1} x}{x\sqrt{x^2-1}} dx = ?$

(a) $\frac{\sec^{-2} x}{2}$ (b) $\ln(\sec^{-1} x)$ (c) $\ln(x\sqrt{x^2 - 1})$ (d) $\left(\frac{\sec^{-1} x}{\sqrt{2}}\right)^2$

(17) $\int \frac{\ln x}{x} dx = ?$

(a) $\frac{(\ln x)^2}{2x^2}$ (b) $\frac{(\ln x)^2}{2}$ (c) $\ln(\ln x)$ (d) None

(18) $\int \frac{\operatorname{cosec}^2 x}{(2+5 \cot x)} dx = ?$

(a) $\frac{\ln(2+5 \cot x)}{5}$ (b) $\frac{1}{5(2+5 \cot x)^2}$
 (c) $5 \ln(2 + 5 \cot x)$ (d) $\ln(2 + \cot x)^{-1/5}$

(19) $\int \frac{\operatorname{cosec}^2 x}{\cot x} dx = ?$

(a) $\ln(\tan x)$ (b) $\frac{\cot^2 x}{2}$ (c) $\frac{1}{2\cot^2 x}$ (d) $\ln(\cot x)$

INTEGRATION FOR TRIGONOMETRIC FUNCTIONS

Formulae:

$$(i) \int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C$$

$$(ii) \int \cos(ax) dx = \frac{1}{a} \sin(ax) + C$$

Above methods can be used for other trigonometric functions.

MCQ-8 :

$$\int \sin 6x dx = ?$$

$$(a) \frac{-1}{6} \cos 6x + C$$

$$(c) -6 \cos 6x + C$$

$$(b) -\cos 6x + C$$

$$(d) \frac{-1}{12} \sin^2 6x + C$$

Solution:

$$\int \sin 6x dx$$

$$= -\frac{\cos 6x}{6} + C$$

$$= \frac{1}{6} \cos 6x + C$$

The answer is (a).

Formulae: Product into Sum:

$$(i) \sin \alpha \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

$$(ii) \cos \alpha \sin \beta = \frac{1}{2} \{ \sin(\alpha + \beta) - \sin(\alpha - \beta) \}$$

$$(iii) \cos \alpha \cos \beta = \frac{1}{2} \{ \cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

$$(iv) \sin \alpha \sin \beta = -\frac{1}{2} \{ \cos(\alpha + \beta) - \cos(\alpha - \beta) \}$$

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MCQ- 10: $\int \sqrt{1 - \cos x} dx = ?$

(a) $\frac{1}{2\sqrt{1 - \cos x}}$

(b) $\frac{1}{2} \sin \frac{x}{2}$

(c) $-2\sqrt{2} \cos \frac{x}{2} + C$

(d) $\sqrt{2} \cos x$

Solution:

$$\begin{aligned} & \boxed{\int \sqrt{1 - \cos x} dx} \\ &= \int \sqrt{2 \sin^2 \frac{x}{2}} dx \\ &= \sqrt{2} \int \sin \frac{x}{2} dx \\ &= \frac{\sqrt{2}}{\frac{1}{2}} (-\cos \frac{x}{2}) + C \\ &= -2\sqrt{2} \cos \frac{x}{2} + C \end{aligned}$$

The answer is (c).

MCQ- 11:

$\int \sqrt{1 + \cos x} dx = ?$

(a) $2\sqrt{2} \sin \frac{x}{2} + C$

(b) $-\frac{1}{2\sqrt{1 + \cos x}}$

(c) $\frac{1}{2} \sin \frac{x}{2}$

(d) $-2\sqrt{2} \cos \frac{x}{2} + C$

Solution:

$$\begin{aligned} & \boxed{\int \sqrt{1 + \cos x} dx} \\ &= \int \sqrt{2 \cos^2 \frac{x}{2}} dx = \sqrt{2} \int \cos \frac{x}{2} dx \\ &= \frac{\sqrt{2}}{\frac{1}{2}} (\sin \frac{x}{2}) + C = 2\sqrt{2} \sin \frac{x}{2} + C \end{aligned}$$

The answer is (a).

Formulae:**For even power of $\sin x$ and $\cos x$:**

(1) $\sin^2 x = \frac{1-\cos 2x}{2}$

(2) $\cos^2 x = \frac{1+\cos 2x}{2}$

These formulae are used for

$\int \sin^2 x \, dx, \int \cos^2 x \, dx, \int \sin^4 x \, dx, \int \cos^4 x \, dx, \int \sin^2 x \cos^2 x \, dx$

MCQ-12 :

$\int \cos^2 x \, dx = ?$

(a) $\frac{\cos^2 x}{3} + C$

(b) $\frac{\cos^2 x \sin x}{3} + C$

(c) $(x + \sin 2x) + C$

(d) $\frac{1}{4}(2x + \sin 2x) + C$

Solution:

$\int \cos^2 x \, dx$

$= \int \frac{1+\cos 2x}{2} \, dx$

$= \frac{1}{2} \int (1 + \cos 2x) \, dx$

$= \frac{1}{2} \left\{ \int dx + \int \cos 2x \, dx \right\}$

$= \frac{1}{2} \left\{ x + \frac{\sin 2x}{2} \right\} + C$

$= \frac{1}{2} \left\{ \frac{2x + \sin 2x}{2} \right\} + C$

$= \frac{1}{4} (2x + \sin 2x) + C$

The answer is (d).

Formulae:**For odd power of $\sin x$ or $\cos x$:**{The power of one function $\sin x$ or $\cos x$ is odd and the power of other function may be odd or even.}

(i) $\sin^2 x + \cos^2 x = 1$

(ii) $\frac{d}{dx} \sin x = \cos x$

(iii) $\frac{d}{dx} \cos x = -\sin x$

These formulae are used for

$\int \sin^3 x dx, \int \cos^3 x dx, \int \sin^5 x dx, \int \cos^5 x dx$

$\int \sin^3 x \cos^2 x dx, \int \sin^4 x \cos^3 x dx, \int \sin^3 x \cos^5 x dx$

MCQ- 13:

$\int \sin^4 x \cos^3 x dx = ?$

(a) $\frac{1}{5} \sin^5 x + C$

(b) $\sin^5 x - \sin^7 x + C$

(c) $\frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$

(d) $\frac{\sin^5 x}{5} + \frac{\cos^4 x}{4} + C$

Solution:

$$\int \sin^4 x \cos^3 x dx$$

$$= \int \sin^4 x \cos^2 x \cos x dx$$

$$= \int \sin^4 x (1 - \sin^2 x) \cos x dx$$

$$= \int (\sin^4 x - \sin^6 x) \cos x dx$$

$$= \int (\sin^4 x \cos x - \sin^6 x \cos x) dx$$

$$= \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C$$

The answer is (c).

For integration of $\tan x$ any power and $\sec x$ with even power:

$$(i) 1 + \tan^2 x = \sec^2 x$$

$$(ii) \frac{d}{dx} \tan x = \sec^2 x$$

These formulae are used for

$$\int \tan^2 x \, dx, \int \tan^4 x \, dx, \int \tan^3 x \, dx, \int \tan^5 x \, dx,$$

$$\int \sec^4 x \, dx, \int \tan^6 x \sec^2 x \, dx, \int \tan^5 x \sec^4 x \, dx$$

MCQ-14 :

$$\int \tan^3 x \sec^2 x \, dx = ?$$

$$(a) \frac{1}{4} \tan^4 x + \frac{1}{3} \sec^3 x + C$$

$$(c) \frac{1}{5} \sec^5 x + C$$

$$(b) \frac{1}{4} \tan^4 x + C$$

$$(d) \frac{1}{12} \tan^4 x \sec^3 x + C$$

Solution:

$$\int \tan^3 x \sec^2 x \, dx$$

$$\left\{ \because \frac{d}{dx} \tan x = \sec^2 x \right.$$

$$= \frac{\tan^4 x}{4} + C$$

$$= \frac{1}{4} \tan^4 x + C$$

The answer is (b).

Power of $\tan x$ is odd and $\sec x$ with any power:

$$(i) \quad 1 + \tan^2 x = \sec^2 x$$

$$(ii) \quad \frac{d}{dx} \sec x = \sec x \tan x$$

These formulae are used for

$$\int \tan^3 x \sec^3 x dx, \int \tan^5 x \sec^5 x dx, \int \tan x \sec^8 x dx$$

MCQ-15 :

$$\int \tan x \sec^6 x dx = ?$$

$$(a) \quad \frac{1}{6} \sec^6 x + C$$

$$(c) \quad \frac{1}{6} \tan^6 x + C$$

$$(b) \quad \frac{\tan^2}{2} + \frac{\sec^7 x}{7} + C$$

$$(d) \quad \frac{\sec^7 x}{7} + C$$

Solution:

$$\begin{aligned} & \boxed{\int \tan x \sec^6 x dx} \\ &= \int \sec^5 x (\sec x \tan x) dx \\ & \quad \left\{ \because \frac{d}{dx} \sec x = \sec x \tan x \right. \\ &= \frac{1}{6} \sec^6 x + C \end{aligned}$$

The answer is (a).

For integration of $\cot x$ any power and $\cosec x$ with even power:

$$(i) \quad 1 + \cot^2 x = \cosec^2 x$$

$$(ii) \quad \frac{d}{dx} \cot x = -\cosec^2 x$$

These formulae are used for

$$\int \cot^2 x \, dx, \int \cot^4 x \, dx, \int \cot^3 x \, dx, \int \cot^5 x \, dx,$$

$$\int \cosec^4 x \, dx, \int \cot x \cosec^2 x \, dx, \int \cot^5 x \cosec^4 x \, dx$$

MCQ-16 :

$$\int \sqrt{\cosec^4 x} \cot x \, dx = ?$$

(a) $\frac{1}{3} \cosec^3 x + \cot^2 x + C$

(c) $-\frac{1}{2} \cot^2 x + C$

(b) $\cosec^2 x + \cot x + C$

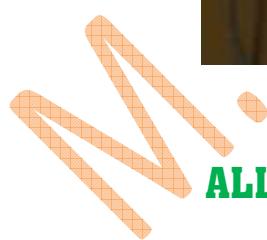
(d) $\frac{1}{3} \cosec^3 x + C$

Solution:

$$\int \sqrt{\cosec^4 x} \cot x \, dx$$

$$= \int \cot x \cosec^2 x \, dx$$

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EXERCISE-3

(1) $\int 2 \cos^2 x \, dx = ?$

(a) $\frac{2}{3} \cos^3 x + C$

(b) $x + \frac{1}{2} \sin 2x + C$

(c) $\frac{1}{2} x^2 + \sin 2x + C$

(d) $x + \cos 2x + C$

(2) $\int 2 \sin^2 x \, dx = ?$

(a) $\frac{2}{3} \sin^3 x + C$

(b) $x - \frac{1}{2} \sin 2x + C$

(c) $4 \sin^3 x \cos x + C$

(d) $2x + \frac{1}{2} \cos x + C$

(3) $\int \cos^3 x \, dx = ?$

(a) $-3 \cos^2 x \sin x + C$

(b) $\frac{1}{4} \sin x \cos^4 x + C$

(c) $\sin x + \sin^3 x + C$

(d) $\sin x - \frac{1}{3} \sin^3 x + C$

(4) $\int 2 \cos 4x \cos 2x \, dx = ?$

(a) $\frac{1}{6} \sin 6x + C$

(b) $\frac{4}{3} \cos 3x + 2 \sin x + C$

(c) $\frac{1}{6} \sin 6x + \frac{1}{2} \sin 2x + C$

(d) $\frac{1}{4} \sin 4x \sin 2x + C$

(5) $\int \sqrt{1 + \cos x} \, dx = ?$

(a) $2\sqrt{2} \sin \frac{x}{2} + C$

(b) $\frac{1}{2} \sqrt{x - \sin x} + C$

(c) $2\sqrt{x + \sin x} + C$

(d) $\sqrt{2} \cos \frac{x}{2} + C$

(6) $\int \tan^8 x \sec^4 x \, dx = ?$

(a) $\frac{1}{40} \tan^9 x \sec^5 x + C$

(b) $\frac{1}{8} \tan^8 x + \frac{1}{12} \tan^{12} x + C$

(c) $\frac{1}{9} \tan^9 x + \tan x + C$

(d) $\frac{1}{9} \tan^9 x + \frac{1}{11} \tan^{11} x + C$

INVERSE TRIGONOMETRIC FUNCTIONS

(i) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + C$

(ii) $\int \frac{1}{1+x^2} dx = \tan^{-1}x + C$

(iii) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}x + C$

and

(i) $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a} + C$

(ii) $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

(iii) $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1} \frac{x}{a} + C$

EXERCISE-4

(1) $\int \frac{1}{\sqrt{9-x^2}} dx = ?$

(a) $\sin^{-1} \frac{x}{3} + C$

(c) $-\frac{1}{2} \sqrt{9-x^2} + C$

(b) $\frac{1}{3} \sin^{-1} \frac{x}{3} + C$

(d) $-\frac{1}{3} \cos^{-1} x + C$

(2) $\int \frac{1}{4+x^2} dx = ?$

(a) $\tan^{-1} \frac{x}{2} + C$

(c) $\frac{1}{2} \tan^{-1} \frac{x}{2} + C$

(b) $\cos^{-1} \frac{x}{2} + C$

(d) $\frac{1}{2} \tan^{-1} x + C$

Method:

$$\int \frac{x^n}{x^n + a} dx$$

Power of x are same in numerator and denominator

$$\int \frac{x^n}{x^n + a} dx$$

Adding and subtracting a in numerator.

{The constant same as in denominator}

$$\begin{aligned} &= \int \frac{x^n + a - a}{x^n + a} dx \\ &= \int \left(\frac{x^n + a}{x^n + a} - \frac{a}{x^n + a} \right) dx \\ &= \int \left(1 - \frac{a}{x^n + a} \right) dx \\ &= \int dx - a \int \frac{1}{x^n + a} dx \end{aligned}$$

MCQ-18: $\int \frac{x}{x-8} dx = ?$

(a) $\ln(x-8) + C$

(c) $\frac{(x-8)^2}{2} + C$

(b) $x + 8\ln(x-8) + C$

(d) $\frac{1}{2}x^2 + \ln(x-8) + C$

Solution:

$$\begin{aligned} &\int \frac{x}{x-8} dx \\ &= \int \frac{x-8+8}{x-8} dx \\ &= \int \left(\frac{x-8}{x-8} + \frac{8}{x-8} \right) dx \\ &= \int \left(1 + \frac{8}{x-8} \right) dx \\ &= \int dx + 8 \int \frac{1}{x-8} dx \\ &= x + 8\ln(x-8) + C \end{aligned}$$

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Formulae:

(i) $\int e^x dx = e^x + C$

(ii) $\int e^{nx} dx = \frac{e^{nx}}{n} + C$

MCQ- 20:

$\int (e^{6x} + 8x)dx = ?$

(a) $6e^{6x} + 8 + C$

(c) $\frac{1}{6}e^{6x} + 4x^2 + C$

(b) $\frac{1}{6}e^{6x} + 8 + C$

(d) $6e^{6x} + 4x^2 + C$

Solution:

$$\boxed{\int (e^{6x} + 8x)dx}$$

$$\begin{aligned}
 &= \frac{e^{6x}}{6} + \frac{8x^2}{2} + C \\
 &= \frac{1}{6}e^{6x} + 4x^2 + C + C
 \end{aligned}$$

The answer is (c).

MCQ- 21:

$\int_0^3 4e^{2x} dx = ?$

(a) $2(e^6 - 1) + C$

(b) $2e^6 + C$

(c) $\frac{e^6 + 1}{6} + C$

(d) $\frac{3e^6}{9} + C$

Solution:

$$\boxed{\int_0^3 4e^{2x} dx}$$

$$\begin{aligned}
 &= \left[\frac{4}{2} e^{2x} \right]_0^3 \\
 &= 2 [e^{2x}]_0^3 \\
 &= 2 (e^6 - e^0) \\
 &= 2 (e^6 - 1)
 \end{aligned}$$

The answer is (a).

EXERCISE-6

(1) $\int e^{5x+9} dx = ?$

- (a) $\frac{1}{5} e^{5x+9}$ (b) $5e^{5x+9}$ (c) e^{5x+9} (d) $\frac{e^{5x}}{5}$

(2) $3 \int x^2 e^{x^3} dx = ?$

- (a) $\frac{e^{x^3}}{3}$ (b) $\frac{x^3}{3} + e^{x^3}$ (c) $x e^{x^3}$ (d) e^{x^3}

(3) $\int \frac{e^x}{1+e^x} dx = ?$

- (a) $\frac{(1+e^x)^{-2}}{-2}$ (b) $\frac{1}{(1+e^x)^2}$ (c) $\ln(1+e^x)$ (d) $x + e^x$

(4) $\int e^x \sec^2(e^x) dx = ?$

- (a) $\tan x$ (b) $\frac{\sec^3(e^x)}{3}$ (c) $\frac{\tan^3(e^x)}{3}$ (d) $\tan(e^x)$

(5) $\int \ln e^x dx = ?$

- (a) $\frac{x^2}{2}$ (b) $\frac{(\ln e^x)^2}{2}$ (c) $\frac{1}{e^x}$ (d) $\ln e^x + 1$

(6) $\int \frac{1}{x^2} e^{x+2 \ln x} dx = ?$

- (a) $\frac{1}{2} e^{x+2 \ln x}$ (b) e^x (c) $e^{x+2 \ln x}$ (d) None

(7) $\int \frac{2}{x} \ln \left(\frac{x}{e^x} \right) dx = ?$

- (a) $\ln \left(\frac{x}{e^x} \right)$ (b) $\ln x - e^x$ (c) $(\ln x)^2 - 2x$ (d) $\ln x - x^2/2$

(8) $\int \ln(e^x e^{\sin x}) dx = ?$

- (a) $\frac{1}{e^{x+\sin x}}$ (b) $\frac{1}{2}(x^2 - 2 \cos x)$ (c) $e^{x^2-\cos x}$ (d) $\ln(\sin x)$

(9) $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} dx = ?$

- (a) $e^{\cos^{-1}x}$ (b) $\frac{1}{2}\sqrt{1-x^2}e^{\sin^{-1}x}$ (c) $x\sqrt{1-x^2}e^{\cos^{-1}x}$ (d) $e^{\sin^{-1}x}$

(10) $\int \frac{e^{\tan x}}{\cos^2 x} dx = ?$

- (a) $\tan x e^{\sec^2 x}$ (b) $\ln(\tan x)$ (c) $e^{\tan x}$

(d) $e^{\sec^2 x}$

(11) $\int e^{\sec x} \sec x \tan x dx = ?$

- (a) $\ln(\sec x \tan x)$ (b) $e^{\sec x}$

(c) $\sec x$

(d) $\frac{e^{\sec^2 x}}{2}$

(12) $\int \ln\left(\frac{1}{e^{\cosec^2 x}}\right) dx = ?$

- (a) $\ln(\cot x)$ (b) $e^{\cot x}$

(c) $\cot x$

(d) None

(13) $\int \ln\left(\frac{1}{e^{\sin x}}\right) dx = ?$

- (a) $e^{\sin x}$

(b) $e^{\cos x}$

(c) $\cos x$

(d) $\ln(e^{\cos x})$

M. MAQSOOD

INTIGRATION BY PARTS

Formula:

$$\int u \cdot v dx = u \int v dx - \int \left(\frac{du}{dx} \cdot \int v dx \right) dx$$

Shortcut:



MCQ-22 :

$$\int x \sin x dx = ?$$

- (a) $-x \cos x + \cos x + C$
- (b) $-x \cos x + \sin x + C$
- (c) $\frac{1}{2} x^2 \cos x + C$
- (d) $\sin x + \frac{1}{2} x^2 \cos x + C$

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APPLICATIONS OF INTEGRATION

AREA UNDER THE CURVE ABOVE X-AXIS

Area under the curve above x-axis and between the lines $x = a$ and $x = b$ is

$$\text{Area} = \int_a^b y dx$$

MCQ-23 :

What is the area under the curve $y = \cos x$ above x-axis between the lines $x = 0$ and $x = \pi/2$?

(a) 1

(b) $\frac{1}{2}$

(c) 2

(d) $\frac{1}{4}$

Solution:



$$\text{Area} = \int_a^b y dx$$

$$\text{Area} = \int_0^{\frac{\pi}{2}} \cos x dx$$

$$= [\sin x]_0^{\frac{\pi}{2}}$$

$$= \sin \frac{\pi}{2} - \sin 0$$

$$= 1 - 0$$

$$= 1$$

The answer is (a).

MCQ- 24:

What is area under the curve $y = 3x^2$ above x-axis between $x = 0$ and $x = 2$?

- (a) 6 (b) 8 (c) 10 (d) 4

Solution:

$$\text{Area} = \int_a^b y \, dx$$

$$y = 3x^2$$

$$\text{Area} = 3 \int_0^2 x^2 \, dx$$

$$= 3 \left[\frac{x^3}{3} \right]_0^2$$

$$= [x^3]_0^2$$

$$= 8 - 0$$

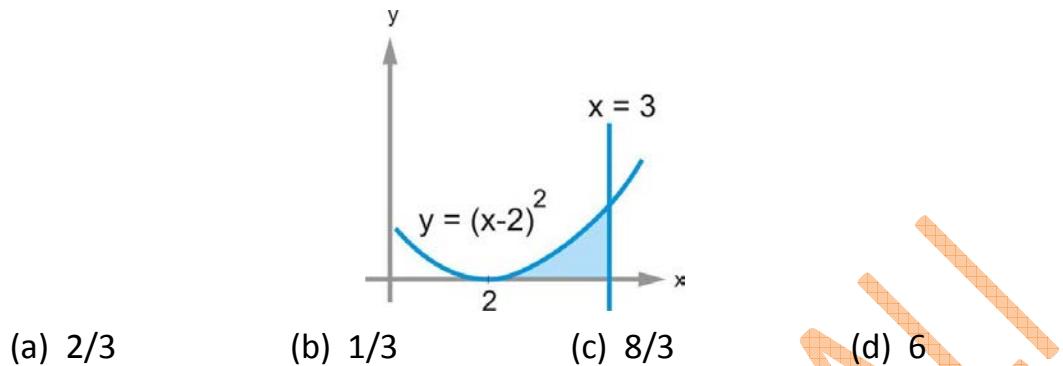
$$= 8$$

The answer is (b).

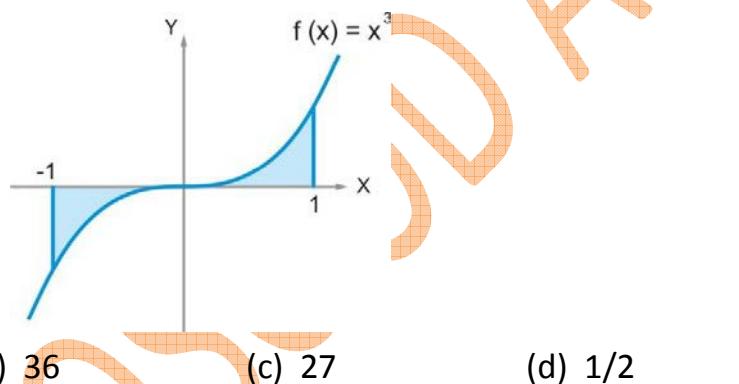
EXERCISE-8

- (1) What is the area above x-axis under the curve $y = 5x^4$ between $x = 0$ and $x = 2$?
 (a) 32 (b) 160 (c) 150 (d) 64
- (2) What is the area above x-axis under the curve $y^2 = 36x$ between $x = 4$ and $x = 9$?
 (a) 104 (b) 24 (c) 76 (d) 80

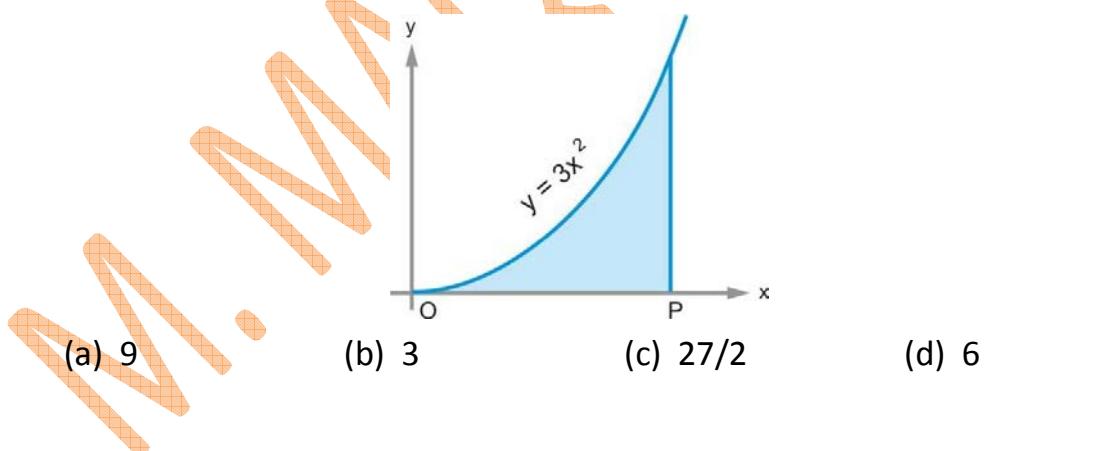
(3) What is the area of shaded region?



(4) What is the area of shaded region?



(5) What is the value of P , if the area of the shaded region is 27 ?



SOLUTIONS OF DIFFERENTIAL EQUATIONS

MCQ- 25:

What is the solution of differential equation $\frac{dy}{dx} = 6$?

- (a) $y = x^2 + C$
- (b) $y = 6x + C$
- (c) $y = C$
- (d) $y = 3x^2 + C$

Solution:

$$\frac{dy}{dx} = 6$$

Separate the variables and integrate.

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Separate the variables.

$$\int 2y \, dy = \int dx$$

$$2 \frac{y^2}{2} = x + C$$

$$y^2 = x + C \rightarrow (1)$$

Putting $x = 0, y = 2$

$$2^2 = 0 + C$$

$$C = 4$$

Putting $C = 4$ in equation (1)

$$y^2 = x + 4$$

The answer is (a).

EXERCISE-9

- (1) What is the solution of the differential equation $\frac{d}{dx}(\sin x) = 2?$
 - (a) $\cos x = 0$
 - (b) $\cos x = 2x + C$
 - (c) $2x - \sin x = C$
 - (d) $\sin x = C$
- (2) What is the solution of the differential equation $\frac{d}{dx}(x^2 + 2) = 6x?$
 - (a) $2x = 6$
 - (b) $x^2 + 2 = 6$
 - (c) $x^2 = 4 + C$
 - (d) $2 - 2x^2 = C$
- (3) What is the solution of the differential equation $\frac{dy}{dx} = 0?$
 - (a) $y = 0$
 - (b) $y + C = 0$
 - (c) $y^2/2 = C$
 - (d) $y = x + C$
- (4) Solve the following differential equation for y as a function of x

$$y' = y + 1$$
 - (a) $y = 1 + C$
 - (b) $\ln(y + 1) = C$
 - (c) $y = e^{x+C} - 1$
 - (d) $\ln y + 1 = x + C$
- (5) What is the solution of the following differential equation
$$y' = 2xy^2 + 2x?$$
 - (a) $\ln(y^2 + 1) = x^2 + C$
 - (b) $y = \tan(x^2 + C)$
 - (c) $\tan^{-1} y = 2x + C$
 - (d) $e^{y^2+1} = x^2 + C$

- (6) What is the solution of the following differential equation? The value of integral constant is zero.

$$x \frac{dy}{dx} = y + 1$$

- (a) $\ln(y + 1) = x^2/2$ (b) $y = \ln x - 1$ (c) $y + 1 = e^x$ (d) $y = x - 1$

- (7) What is the solution of the following differential equation? Integral constant is zero.

$$\frac{1}{x^2}y' = 3e^{-y}$$

- (a) $y = 3 \ln x$ (b) $\ln y = x^3$ (c) $y = e^{x^3}$ (d) $x^3 = \ln y + 2$

- (8) Solve the following differential equation, where integral constant is zero.

$$x^{-2} \frac{dy}{dx} = 3\sqrt{1 - y^2}$$

- (a) $\frac{1}{2}\sqrt{1 - y^2} = x^3$ (b) $\frac{1}{2\sqrt{1-y^2}} = x^3$ (c) $y = \sin x^3$ (d) $\sin^{-1} y = 9x$

- (9) What is the solution of the differential equation

$$\frac{dy}{dx} (2x - 1)^2 - 4 = 0, \quad y = 3 \text{ when } x = 0.$$

- (a) $y = -1 - 4/(2x - 1)$ (b) $y = 1 - 2/(2x - 1)$
 (c) $\frac{2}{(2x-1)^3}$ (d) $8(2x - 1)^3 + 1$

- (10) What is the solution of the differential equation

$$x^3 \frac{dy}{dx} = 6, \quad y(1) = 5$$

- (a) $x^2 y = 8$ (b) $y = 2 - \frac{3}{x^2}$ (c) $y = 23 - \frac{18}{x^4}$ (d) $y = 8 - \frac{3}{x^2}$

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