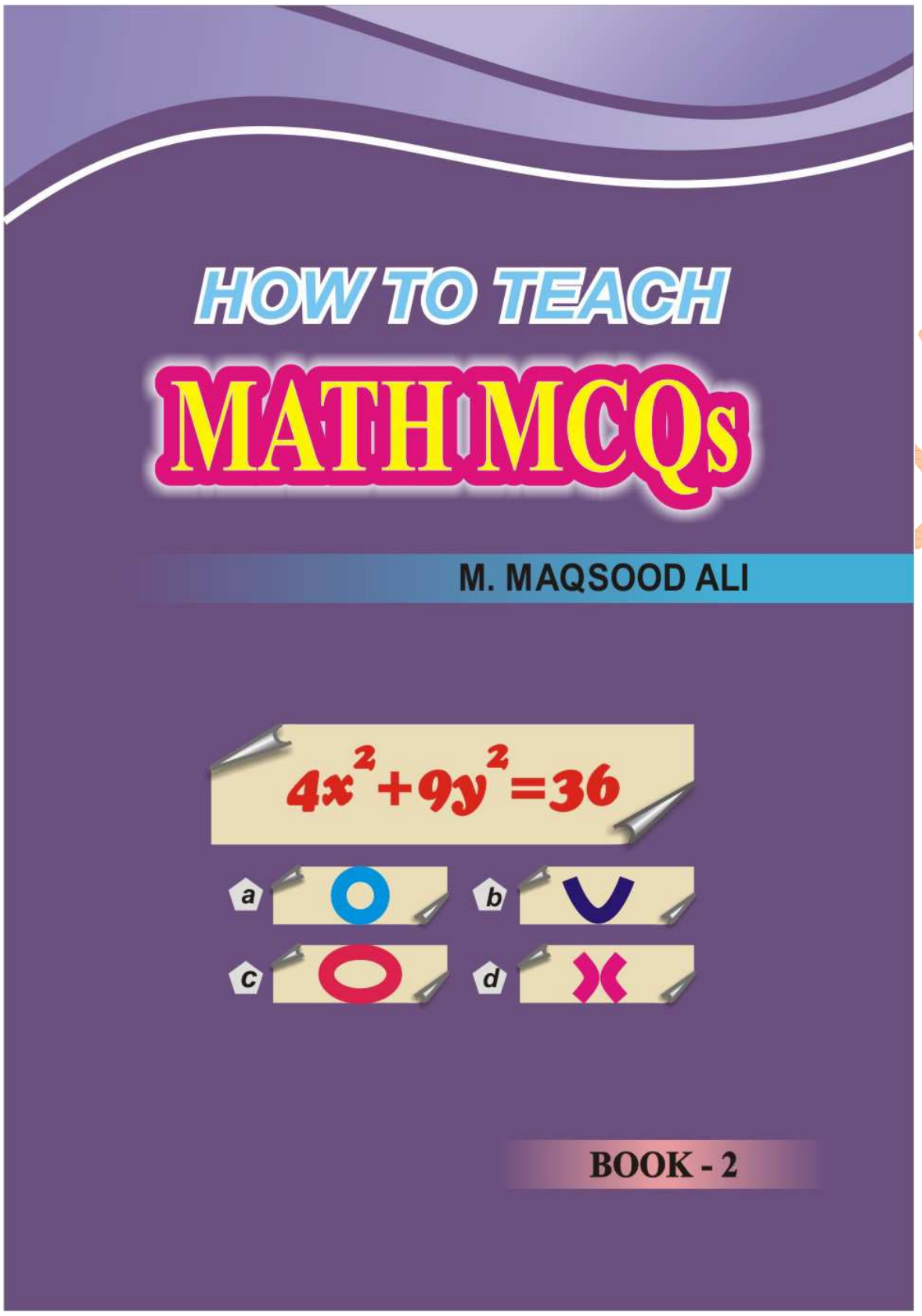
HOW TO TEACH
MATH MCQs
M. MAQSOOD ALI
$4x^2+9y^2=36$
a
b
c
d
BOOK - 2

Chapter 5

APPLICATIONS OF DERIVATIVE

SLOPE AND EQUATION OF THE TANGENT

Slope of the tangent or slope of the curve at (x_1, y_1) = $(\frac{dy}{dx})_{(x_1,y_1)}$

MCQ- 1:

What is the slope of the tangent of the curve $y^2 = 3x^2 - 5$ at the point (4 , 2)?

(a) 6 (b) $\frac{1}{6}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

Solution:

$$y^2 = 3x^2 - 5$$

$$2y\frac{dy}{dx} = 6x$$

$$\frac{dy}{dx} = \frac{3x}{y}$$

Slope of the tangent at (4 , 2) $= \left(\frac{dy}{dx}\right)_{(4,2)}$

$$= \frac{3\,(4)}{2}$$

$$= 6$$

The answer is (a).

MCQ- 2:

What is the point where the tangent to the curve $y = x^3 + 7$ is parallel to x-axis?

(a) $(-2, -1)$ (b) $(2, 15)$ (c) $(0, 7)$ (d) $(1, 8)$

Solution:

$$y = x^3 + 7 \rightarrow (1)$$

$$\frac{dy}{dx} = 3x^2$$

The tangent is parallel to x-axis

$$\frac{dy}{dx} = 0$$

$$3x^2 = 0$$

$$x = 0$$

Put $x = 0$ in equation (1)

$$y = 0 + 7$$

$$y = 7$$

The point is (0, 7).

The answer is (c).

EXERCISE-1

(1) What is the equation of the tangent to the curve $y = 3 - x^2$ at point $(1,2)$?

(a) $y - 2 = 2(1 - x)$ (b) $y - 2 = -2(1 - x)$

(c) $y - 2 = 2(x - 1)$ (d) $y - 2 = -2(x + 1)$

(2) What is the equation of normal to the curve $x^3 - y^2 = 0$ at point $(1, -1)$?

(a) $2(y + 1) = -3(x + 1)$ (b) $3(1 - y) = 2(x - 1)$

(c) $3(y + 1) = 2(x - 1)$ (d) $2(y + 1) = 3(1 - x)$

(3) What is the equation of the normal to the curve $y = ln\,(5 - x) + 5$ at the point where $x = 4$?

(a) $y = x + 1$ (b) $y - 5 = -1(x - 4)$

(c) $y = 5(x + 4)$ (d) $y - 5 = \frac{1}{ln(5-x)}(x - 4)$

(4) A tangent is drawn to a curve $x^2 = 8y - 16$ at a point P parallel to x −axis. What is the equation of tangent?

(a) $y = 2$ (b) $x = 2$ (c) $y = 8$ (d) $y = 4$

(5) A tangent is drawn parallel to y −axis to a curve $y^2 = 12x - 36$ at a point. What is the equation of tangent?

(a) $x = 12$ (b) $y = 3$ (c) $x = 3$ (d) $x = -3$

(6) P is a point on the curve $y^2=6x$. What is the equation of normal at P if tangent at P is parallel to y −axis?

(a) $x = 1$ (b) $y = 0$ (c) $x = 6$ (d) $x = 0$

(7) What is value of k if gradient of the curve $y^2= 6kx + 1$ at point $(a, 2)$ is 9?

(a) 2 (b) 3 (c) 6 (d) $\frac{1}{6}$

(8) $e^{kx} + 2y + kx = 0$ is the equation of a curve. What is the value of k if the gradient of the normal at $x = 0$ is $-\frac{1}{2}$?

(a) -2 (b) 2 (c) 4 (d) $\frac{1}{2}$

SPEED AND ACCELERATION

A particle covered distance s in time t, the speed and magnitude of acceleration are

(i) speed $= v = \frac{ds}{dt}$

(ii) acceleration $= a = \frac{dv}{dt}$ or $a = \frac{d^2s}{dt^2}$

MCQ- 3:

What is the acceleration of the particle if its distance s at time t is $s = 2\sqrt{t}$?

(a) 4t (b) $\frac{1}{2}\sqrt{t}$ (c) $\frac{4}{3}t^{\frac{3}{2}}$ (d) $\frac{1}{\sqrt{t}}$

Solution:

$$s = 2\sqrt{t}$$

$$\text{Speed} = v = \frac{ds}{dt}$$

$$= 2.\frac{1}{2\sqrt{t}} = 1/\sqrt{t}$$

The answer is (d).

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RATE OF CHANGE

i) Rate of change in y w.r.t. x $= \frac{dy}{dx}$

ii) Rate of change in y w.r.t. x at $(x = p) = (\frac{dy}{dx})_{x=p}$

MCQ- 5:

What is the rate of change in A with respect to t if $A = 6/\sqrt{t}$?

(a) $-3/t^{\frac{3}{2}}$ (b) $3\sqrt{t}$ (c) $-3\sqrt{t}$ (d) $3/2t^{\frac{3}{2}}$

Solution:

$$A = \frac{6}{\sqrt{t}}$$

$$= 6t^{-1/2}$$

$$\frac{dA}{dt} = -\frac{1}{2} \times 6 \times t^{-3/2}$$

$$= -3t^{-3/2}$$

$$= -3/t^{3/2}$$

The answer is (a).

MCQ-6 :

What is the rate of change in y with respect to x at $x = 2$, if $y = 3x^3 - 6x + 12$?

(a) 36 (b) 42 (c) 12 (d) 30

Solution:

$$y = 3x^3 - 6x + 12$$

Rate of change in y w.r.t $x = \left(\frac{dy}{dx}\right)$

$$= 9x^2 - 6$$

At $x = 2$

$$(\frac{dy}{dx})_{x=2}$$

$$= 9(2)^2 - 6$$
$$= 30$$

The answer is (d).

EXERCISE-3

(1) The area of a circle at time t second is $A = \frac{1}{16}(t^2 - 1)^2 + 9$. What is the rate of increasing in area at $t = 5$ sec?

(a) 12 (b) -39 (c) 39 (d) 30

(2) $y = 5x^3 - 3x + 9$ is the equation of a curve. What is the rate of change of y with respect to x at $x = 2$?

(a) 25 (b) 57 (c) 63 (d) 43

(3) $V = 24 - \sqrt{t}\ unit^3$ is the volume of the ice cream, at time t sec. What is the rate of decreasing in unit^3/sec at $t = 4$ sec?

(a) 0.25 (b) 0.4 (c) 22 (d) 6

(4) Some water is boiling in a vessel. The volume of the water at time t is $V = 28 - t^{3/2}\ unit^3$. What is rate of decreasing in $unit^3$/sec at t = 16 sec?

(a) -36 (b) 12 (c) 8 (d) 6

(5) An ice cream in the shape of cylinder is melting in such a way that the radius and height are always same. What is the rate of change in volume with respect to radius at $r = 2$?

(a) 6π (b) 12π (c) 8π (d) None

(6) The volume of a spherical balloon is increasing. What is the rate of change in volume with respect to radius at $r = 2$?

(a) 2π (b) π (c) 4π (d) 16π

SMALL INCREAMENT

Small increament in$y$ $(i.e.\Delta y)$, when x increases Δx

$$\Delta y = \frac{dy}{dx}\,.\Delta x$$

MCQ-7 :

$V = \frac{4}{3}\pi r^3$ is the volume of a sphere. What is the change in volume, if radius of the sphere increases from 5 to 5.01.

(a) 501π (b) $0.2\,\pi$ (c) π (d) $\frac{4}{3}\pi$

Solution:

$$V = \frac{4}{3}\pi r^3$$

$$\Delta V = \frac{dV}{dr}\,.\Delta r$$

$$\Delta V = \frac{4}{3}\pi(3r^2).\Delta r$$

$$\Delta V = 4\pi r^2.\Delta r$$

$$= 4\pi(5)^2.(0.01)$$

$$= 100\pi(0.01)$$

$$= \pi$$

The answer is (c).

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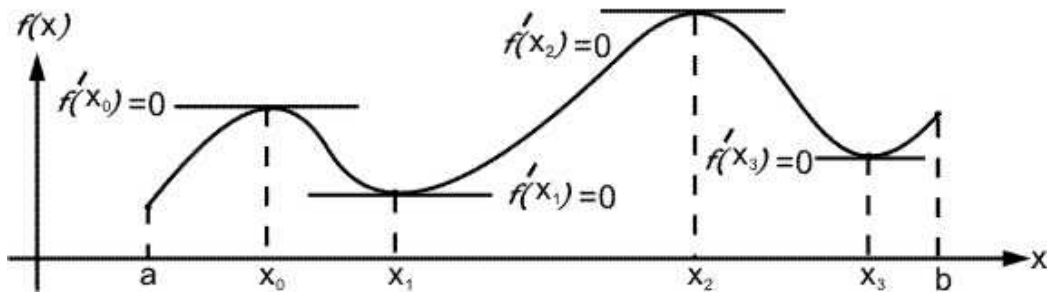
MAXIMA AND MINIMA



$y = f(x)$ is a curve, shown in the above diagram. The points on the curve where tangent is parallel to x-axis or $f'(x) = 0$ are called stationary points.

In above diagram the stationary points are at $x_o, x_1, x_2 \; and \; x_3$.

The value of f is relative minimum or relative maximum at $x = x_o, x_1, x_2, x_3$.

Second Derivative Test:

At $x = p$

i) If $f''(p) < 0$, then

f is relative maximum at $x = p$.

ii) If $f''(p) > 0$, then

f is relative minimum at $x = p$.

MCQ- 8:

What is the abscissa of stationary point on the curve $f(x) = 3x^2 + 12x$?

(a) -12 (b) 3 (c) 6 (d) -2

Solution:

$$f(x) = 3x^2 + 12x$$

$$f'(x) = 6x + 12$$

For stationary point

$$f'(x) = 0$$

$$6x + 12 = 0$$

$$x = -2$$

The answer is (d).

MCQ- 9:

What is the relative minimum value of f if $f(x) = 5x^2 - 2$?

(a) 3 (b) -2 (c) 0 (d) 10

Solution:

$$f(x) = 5x^2 - 2$$

$$f'(x) = 10x$$

For stationary point

$$f'(x) = 0$$

$$10x = 0$$

$$x = 0$$

Note: Do not use second derivative test, because there is only one value of x that is zero.

Do not need of the process.

$$f''(x) = 10$$

$$f''(0) = 10 > 0$$

$\therefore$ f is relative minimum $at\ x = 0$

f is relative minimum $at\ x = 0$

At $x = 0$, the value of f is

$$f(0) = 0 - 2 = -2$$

The answer is (b).

MCQ-10 :

What is the relative maximum value of f if $f(x) = 5x^2 + 10x$?

(a) 10 (b) – 15 (c) – 5 (d) None

Solution:

$$f(x) = 5x^2 + 10x$$

$$f'(x) = 10x + 10$$

For stationary points

$$f'(x) = 0$$

$$10x + 10 = 0$$

$$x = -1$$

Note: There is only one value of x (i.e. -1), but here is an option (d) None, so use "second derivative test" to check the value of f is relative minimum or maximum at $x = -1$.

$$f''(x) = 10$$

$$f''(-1) = 10 > 0$$

EXERCISE-5

(1) What is the x – coordinate of the stationary point on the curve

$$y = \frac{1}{3}x^3 - x^2 + x?$$

(a) 1 (b) $\frac{1}{2}$ (c) 7 (d) 2.5

(2) What is the x – coordinate of minimum point on the curve

$$y = x^3 - x^2 - 5x?$$

(a) $\frac{1}{2}$ (b) 3.6 (c) $\frac{5}{3}$ (d) 7

(3) What is the relative maximum point on the curve $y = \frac{1}{3}x^3 - x^2$?

(a) $\left(2, -\frac{4}{3}\right)$ (b) (3,0) (c) (0,0) (d) $(-3, -18)$

(4) k is abscissa of relative minimum point on the curve $y = x^3 - x^2 + k$. What is the value of k?

(a) 7/3 (b) $-\frac{2}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

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