

Chapter 4**DERIVATIVES****Power constant and base variable:**i) Derivative of x^n w.r.t. x :base $\leftarrow x^n \rightarrow$ power

$$\frac{d}{dx}(x^n) = ?$$

Multiply power (n) to the coefficient of x^n and subtract 1 from the power (n).

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Explanation:

$$\frac{d}{dx}(5x^9 - 9x + 6) = ?$$

Derivative of each term one by one.

$$\text{i) } \frac{d}{dx}(5x^9)$$

Multiply power 9 to 5 (the coefficient of x^9) and subtract 1 from 9 (power of x)

$$\frac{d}{dx}(5x^9) = 45x^8$$

$$\text{ii) } \frac{d}{dx}(9x) = 9 \frac{d}{dx}(x) = 9 \frac{dx}{dx} = 9(1) = 9$$

$$\text{iii) } \frac{d}{dx}(6) = 0$$

So that

$$\frac{d}{dx}(5x^9 - 9x + 6) = 45x^8 - 9$$

MCQ-1 :

$$\frac{d}{dx}(5x^9 - 9x + 6) = ?$$

- (a) $45x^8 - 1$ (b) $5x^8 - 9$ (c) $45x^7 - 9x$ (d) $45x^8 - 9$

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Derivative of a function $f(x)$ of power n :base (function of x) $\leftarrow f \rightarrow$ power

$$\frac{d}{dx} f^n(x) = ?$$

Multiply power (n) to the coefficient of $f(x)$ and than subtract 1 from the power (i.e. n), and than differentiate the base (i.e. $f(x)$) w.r.t x (i.e. $f'(x)$), multiply $f'(x)$ to $nf^{n-1}(x)$

$$\frac{d}{dx} f^n(x) = nf^{n-1}(x) \cdot f'(x)$$

Case-1: $f(x)$ in terms of $\{g(x)\}^n$:

Explanation:

$$\frac{d}{dx} (6x^4 - 3)^8 = ?$$

Power : 8

Base : $6x^4 - 3$

Multiply power 8 with 1 (coefficient of the base) and subtract 1 from 8 (power) and then differentiate the base ($6x^4 - 3$).

$$\begin{aligned} \frac{d}{dx} (6x^4 - 3)^8 &= 8(6x^4 - 3)^{8-1} \cdot \frac{d}{dx} (6x^4 - 3) \\ &= 8(6x^4 - 3)^7 \cdot (24x^3) \\ &= 192x^3 (6x^4 - 3)^7 \end{aligned}$$

MCQ- 3:

$$\frac{d}{dx} (6x^4 - 3)^8 = ?$$

- | | |
|--------------------------|----------------------|
| (a) $192x^3(6x^4 - 3)^7$ | (b) $8(6x^4 - 3)^7$ |
| (c) $8x^3(6x^4 - 3)^7$ | (d) $48(6x^4 - 3)^7$ |

Solution:

$$\begin{aligned} \frac{d}{dx} (6x^4 - 3)^8 &= 8(6x^4 - 3)^{8-1} \cdot \frac{d}{dx} (6x^4 - 3) \\ &= 8(6x^4 - 3)^7 \cdot (24x^3) \\ &= 192x^3 (6x^4 - 3)^7 \end{aligned}$$

The answer is (a).

EXERCISE-2

(1) $\frac{d}{dx} \left(5\sqrt{x^2 + 1} \right) = ?$

- (a) $\frac{5x}{\sqrt{x^2+1}} + \sqrt{x^2 + 1}$ (b) $\frac{5}{2\sqrt{x^2+1}}$ (c) $\frac{5x}{\sqrt{x^2+1}}$ (d) $\frac{5\sqrt{x^2+1}}{x}$

(2) $\frac{d}{dx} \sqrt[3]{(x^2 + 1)^2} = ?$

- (a) $12x(x^2 + 1)$ (b) $\frac{2x}{3(x^2+1)^{2/3}}$ (c) $\frac{4x}{3 \cdot \sqrt[3]{x^2+1}}$ (d) $\frac{4x}{\sqrt{x^2+1}}$

(3) $\frac{d}{dx} \left(\frac{1}{x^3 + 1} \right) = ?$

- (a) $\frac{1}{3x^2}$ (b) $-3x^2(x^3 + 1)^2$ (c) $\frac{-3x^2}{(x^3+1)^2}$ (d) $3x^2(x^3 + 1)^2$

(4) $\frac{d}{dx} \sqrt{3x + 1} = ?$

- (a) $\frac{3}{2}(3x + 1)^{1/2}$ (b) $\frac{(3x+1)^{1/2}}{3}$ (c) $\frac{3}{\sqrt{3x+1}}$ (d) $\frac{3}{2\sqrt{3x+1}}$

(5) $\frac{d}{dx} (200 + 3\sqrt{x^3}) = ?$

- (a) $1 + \frac{9}{2}\sqrt{x}$ (b) $\frac{9}{2}\sqrt{x}$ (c) $\frac{9}{2\sqrt{x}}$ (d) $\frac{9}{2}x^{\frac{3}{2}}$

(6) $\frac{d}{dx} (\sqrt[5]{x^2 + 5}) = ?$

- (a) $\frac{2}{5 \cdot \sqrt[5]{x^3}}$ (b) $\frac{1}{5x^{\frac{4}{5}}}$ (c) $2x \sqrt[5]{x}$ (d) $\frac{2}{5x^{\frac{3}{5}}} + 1$

(7) If $f(x) = (4 + x^2)^{-2/3}$, then $f'(2) = ?$

- (a) $-\frac{3}{7}$ (b) $-\frac{5}{48}$ (c) $-\frac{1}{48}$ (d) $-\frac{1}{12}$

(8) If $g(x) = \sqrt{5 - x}$, then $g'(1) = ?$

- (a) $-\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $-\frac{1}{4}$ (d) $\frac{5}{4}$

(9) If $f(x) = (5x + 3)^2$, then $f'(2) = ?$

- (a) 13 (b) 169 (c) 26 (d) 130

TRIGONOMETRIC FUNCTIONS

Formulae:

(i) $\frac{d}{dx} \sin x = \cos x$

(ii) $\frac{d}{dx} \cos x = -\sin x$

(iii) $\frac{d}{dx} \tan x = \sec^2 x$

(iv) $\frac{d}{dx} \cosec x = -\cosec x \cot x$

(v) $\frac{d}{dx} \sec x = \sec x \tan x$

(vi) $\frac{d}{dx} \cot x = -\cosec^2 x$

Case-2: $f(x)$ in terms of trigonometric functions:

(i) Power = $n = 1$ and angle x :

MCQ-4 :

$$\frac{d}{dx} \sin x = ?$$

(a) $-\cos x$

(b) $\cos x$

(c) $\sin x \cos x$

(d) $\cos^2 x$

Solution:

$$\frac{d}{dx} \sin x = \cos x$$

The answer is (b).

(ii) Power = $n \neq 1$ and angle:

MCQ-5 :

$$\frac{d}{dx} (2 \sin^5 x) = ?$$

(a) $2 \cos^5 x$

(b) $10 \sin^4 x$

(c) $10 \sin x \cos x$

(d) $5 \sin^4 x \cos x$

Solution:

Power = 5

Base (function) = $\sin x$

Multiply 5 (power) to 2 (coefficient of $\sin x$) and subtract 1 from 5 and then differentiate $\sin x$ (base) in product.

$$\frac{d}{dx} (2 \sin^5)$$

$$= 10\sin^{5-1}x \cdot \frac{d}{dx}(\sin x)$$

$$= 10\sin^4x \cos x$$

The answer is (d).

(iii) Power = $n = 1$ and angle = $u(x)$:

Function = \sin

Angle = $3x$

Derivative in two steps.

Differentiate \sin (function) and then $3x$ (angle), in product.

$$\begin{aligned} & \frac{d}{dx} \sin 3x \\ &= \cos 3x \cdot \frac{d}{dx}(3x) \\ &= \cos 3x \cdot (3) \\ &= 3\cos 3x \end{aligned}$$

The answer is (c).

(iv) Power = $n \neq 1$, angle = $u(x)$

MCQ-6 :

$$\frac{d}{dx}(2\sin^5 3x) = ?$$

- (a) $30 \sin^4 3x \cos 3x$
 (c) $30 \sin 3x \cos 3x$

- (b) $10 \sin^4 3x \cos 3x$
 (d) $30 \sin^4 3x$

Solution:

Power = 5

Base (function) = \sin

Angle = $3x$

Derivative in three steps (Power, function, angle)

$$\begin{aligned} \frac{d}{dx}(2\sin^5 3x) &= 2 \times 5\sin^{5-1} \frac{d}{dx}(\sin 3x) \\ &= 10\sin^4 3x \cdot \cos 3x \frac{d}{dx}(3x) \\ &= 10\sin^4 3x \cdot \cos 3x \cdot (3) \\ &= 30\sin^4 3x \cos 3x \end{aligned}$$

The answer is (a).

EXERCISE-3

- (1) $\frac{d}{dx} (\cos x + \sin x) = ?$
 (a) $\sin x - \cos x$ (b) $\cos x - \sin x$ (c) $\sin x + \cos x$ (d) $\cos x + \sin x$
- (2) $\frac{d}{dx} \sin x^4 = ?$
 (a) $4x^3 \sin x^4$ (b) $4x^3 \sin x^4 \cos x$ (c) $4x^3 \cos x^4$ (d) $\sin x^4 \cos x^4$
- (3) $\frac{d}{dx} (10 + \tan x^2) = ?$
 (a) $1 + \sec^2 x^2$ (b) $2x \sec^2 x^2$ (c) $2x \sec^2 x$ (d) $1 + 2x \sec^2 x^2$
- (4) $\frac{d}{dx} (\cos^3 x^2) = ?$
 (a) $-6x \sin x^2 \cos^2 x^2$ (b) $-6x \cos^2 x \sin x$
 (c) $6x \cos^2 x^2$ (d) $-6 \sin^2 x \cos x$
- (5) $\frac{d}{dx} \sqrt{\tan x} = ?$
 (a) $\frac{\sec x}{2\sqrt{\tan x}}$ (b) $\frac{1}{2\sqrt{\tan x} \cdot \sec x}$ (c) $\frac{\sec^2 x}{2\sqrt{\tan x}}$ (d) $\frac{1}{2\sqrt{\tan x} \sec x}$
- (6) $\frac{d}{dx} \cot x^3 = ?$
 (a) $3 \cot^2 x$ (b) $3 \cot x^2$ (c) $-3x^2 \operatorname{cosec}^2 x$ (d) $-3x^2 \operatorname{cosec}^2 x^3$
- (7) $\frac{d}{dx} \operatorname{cosec}^2 x = ?$
 (a) $\frac{-2 \cos x}{\sin^3 x}$ (b) $2 \operatorname{cosec}^2 x \cot x$
 (c) $-2 \operatorname{cosec} x \cot x$ (d) $\frac{-\operatorname{cosec} x}{\cot x}$
- (8) $\frac{d}{dx} \sec(2x^2) = ?$
 (a) $4x \sec x \tan x$ (b) $\sec(2x^2) \tan(2x^2)$
 (c) $4x \sec(2x^2) \tan(2x^2)$ (d) $4x \sec^2(2x^2)$

INVERSE TRIGONOMETRIC FUNCTIONS

Formulae:

$$\begin{array}{ll} \text{(i)} \quad \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} & \text{(ii)} \quad \frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \\ \text{(iii)} \quad \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} & \text{(iv)} \quad \frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2} \\ \text{(v)} \quad \frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}} & \text{(vi)} \quad \frac{d}{dx} \cosec^{-1} x = \frac{-1}{x\sqrt{x^2-1}} \end{array}$$

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NATURAL LOG ($\ln x$)**Formula:**

(i) $\frac{d}{dx} \ln x = \frac{1}{x}$

where $u = u(x)$

(i) $\frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{d}{dx} u$

Properties:

(i) $\log_e x = \ln x$

(iii) $\ln(a \cdot b) = \ln a + \ln b$

(v) $\ln a^x = x \ln a$

(vii) $\ln e^x = x$

(ii) $\log_{10} x = \log x$

(iv) $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

(vi) $e^{\ln x} = x$

MCQ- 7:

$$\frac{d}{dx} \ln x^3 = ?$$

(a) $1/x^3$

(b) $3/x^3$

(c) $3/x$

(d) $3x^5$

Solution:

Derivative in two steps.

$$\frac{d}{dx} \ln x^3 = \frac{1}{x^3} \cdot \frac{d}{dx} (x^3) \quad \{ \because \frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{d}{dx} u \}$$

$$= \frac{1}{x^3} \cdot (3x^2)$$

$$= 3/x$$

The answer is (c).

MCQ-8:

$$\frac{d}{dx} \{\ln(x^2 + 3)\}^5 = ?$$

(a) $\frac{5}{x^2+3}$

(c) $10x \{\ln(x^2 + 3)\}^4 / (x^2 + 3)$

(b) $\frac{5\{\ln(x^2+3)\}^4}{x^2+3}$

(d) $10 \{\ln(x^2 + 3)\}^4 / (x^2 + 3)$

Solution:

Derivative in three steps :

$$\begin{aligned}
 \frac{d}{dx} \{\ln(x^2 + 3)\}^5 &= 5 \{\ln(x^2 + 3)\}^4 \cdot \frac{d}{dx} \ln(x^2 + 3) \\
 &= 5 \{\ln(x^2 + 3)\}^4 \cdot \frac{1}{x^2+3} \cdot \frac{d}{dx}(x^2 + 3) \\
 &= 5 \{\ln(x^2 + 3)\}^4 \cdot \frac{1}{x^2+3} \cdot (2x) \\
 &= \frac{10x \{\ln(x^2 + 3)\}^4}{x^2+3}
 \end{aligned}$$

The answer is (c).

MCQ-9 :

$$\frac{d}{dx} (\ln x)^8 = ?$$

(a) $1/x^8$ (b) $8(\ln x)^7/x$ (c) $8(\ln x)^7$ (d) $1/x$ **Solution:**

Derivative in two steps.

power = 8

base = $\ln x$

$$\begin{aligned}
 \frac{d}{dx} (\ln x)^8 &= 8 (\ln x)^7 \cdot \frac{d}{dx} (\ln x) \\
 &= 8 (\ln x)^7 \cdot \frac{1}{x} \\
 &= 8 (\ln x)^7 / x
 \end{aligned}$$

The answer is (b).

EXERCISE-5

$$(1) \frac{d}{dx} \ln(x^3 + 1) = ?$$

(a) $\frac{3x}{1+x^3}$ (b) $\frac{1}{x^3+1}$ (c) $\frac{1}{x^3+1} \ln(x^3 + 1)$ (d) $\frac{3x^2}{1+x^3}$

$$(2) \frac{d}{dx} \ln \sqrt{x+1} = ?$$

(a) $\frac{1}{2} \sqrt{x+1}$ (b) $\frac{1}{\sqrt{x+1}}$ (c) $\frac{1}{2(x+1)}$ (d) $\frac{1}{2\sqrt{x+1}}$

(3) $\frac{d}{dx} \ln \sqrt{\sin x} = ?$

(a) $\frac{1}{2\sqrt{\sin x}}$

(b) $\frac{\cos x}{2\sqrt{\sin x}}$

(c) $\frac{1}{2} \tan x$

(d) $\frac{1}{2} \cot x$

(4) $\frac{d}{dx} \sqrt{\sin^{-1} x} = ?$

(a) $\frac{\sin x}{2\sqrt{1-x^2}}$

(b) $\frac{1}{2\sqrt{1-x^2} \sin^{-1} x}$

(c) $\frac{\cos^{-1} x}{2\sqrt{\sin^{-1} x}}$

(d) $\frac{\sqrt{\sin^{-1} x}}{2\sqrt{1-x^2}}$

(5) $\frac{d}{dx} \cot^{-1}(\ln e^x) = ?$

(a) $\frac{-1}{1+x^2}$

(b) $\frac{1}{\cot^{-1} e^x}$

(c) $\frac{\cot^{-1} x}{e^x}$

(d) $\frac{-1}{1+(\ln e^x)^2}$

(6) $\frac{d}{dx} 10^{\log \sin x} = ?$

(a) $\sin x$

(b) $\cot x 10^{\log \sin x}$

(c) $\cos x$

(d) $\log \cos x$

(7) $\frac{d}{dx} \ln e^{\sin(\sin^{-1} x)} = ?$

(a) $\frac{1}{e^{\sin(\sin^{-1} x)}}$

(b) $\frac{\sin^{-1} x}{x}$

(c) $\frac{1}{e^x}$

(d) 1

(8) $\frac{d}{dx} \ln(e^{2x^3+9}) = ?$

(a) $6x^2$

(b) $\frac{6x^2}{2x^3+9}$

(c) $\ln(2x^3 + 9)$

(d) $6x^2 e^{2x^3+9}$

(9) $\frac{d}{dx} \ln(e^x \cdot 10^{x^2}) = ?$

(a) $\frac{1}{e^x \cdot 10^{x^2}}$

(b)

(c) $e^x(10^{x^2} + 2x)$

(d) $\frac{2x}{e^x + 10^{x^2}}$

(10) $\frac{d}{dx} \ln\left(\frac{e^{x^3}}{5^x}\right) = ?$

(a) $3x^2 - \ln 5$

(b) $\frac{3x^2 e^{x^3}}{5^x \ln 5}$

(c) $\frac{3x^2}{5^x \ln 5}$

(d) $\frac{5^x \ln 5}{3x^2 e^{x^3}}$

$$(11) \frac{d}{dx} \ln(\sin x \cdot \cos x) = ?$$

(a) $\frac{1}{\sin x \cos x}$

(b) $\frac{\sin x + \cos x}{\sin x \cos x}$

(c) $\cot x - \tan x$

(d) $\cos x - \sin x$

$$(12) \frac{d}{dx} e^{\ln x^3} = ?$$

(a) $3x^2 e^{\ln x^3}$

(b) $3x^2$

(c) $e^{\ln x^3}$

(d) $\frac{e^{\ln x^3}}{x^3}$

$$(13) \frac{d}{dx} (\sec 3x + \ln(5x)^3) = ?$$

(a) $\sec 3x \tan 3x + 3\ln(5x)^2$

(b) $3 \sec x \tan x + \frac{5}{(5x)^3}$

(c) $3(\sec 3x \tan 3x + x)$

(d) $3 \frac{(x \sec 3x \tan 3x + 1)}{x}$

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EXPONENTIAL X**Formula:**

(i) $\frac{d}{dx} e^x = e^x$

(ii) $\frac{d}{dx} e^u = e^u \cdot \frac{du}{dx}$ {where $u = u(x)$ }

Explanation:

$$\begin{aligned} y &= e^x \\ \ln y &= \ln e^x \\ &= x \ln e \\ &= x(1) \\ &= x \end{aligned}$$

Differentiate w.r.t. x

$$\begin{aligned} \frac{d}{dx} (\ln y) &= \frac{d}{dx} (x) \\ \frac{1}{y} \cdot \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= y \\ \frac{dy}{dx} &= e^x \end{aligned}$$

MCQ-10 :

$$\frac{d}{dx} e^{5x^3+2} = ?$$

(a) $(5x^3 + 2)e^{5x^3+2}$

(b) e^{5x^3+2}

(c) e^{15x^2}

(d) $15x^2 e^{5x^3+2}$

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MCQ-12 :

$$\frac{d}{dx} 5^{(x^3+1)} = ?$$

- (a) $5^{(x^3+1)} \cdot \ln 15x^2$ (b) $3x^2 \cdot 5^{(x^3+1)} \cdot \ln 5$
 (c) $5^{(x^3+1)} \cdot \ln 5$ (d) $(x^3 + 1)5^{x^3}$

Solution:

$$\begin{aligned}\frac{d}{dx} 5^{(x^3+1)} &= 5^{(x^3+1)} \cdot \ln 5 \cdot \frac{d}{dx} (x^3 + 1) \\&= 5^{(x^3+1)} \cdot \ln 5 (3x^2) \\&= 3x^2 5^{(x^3+1)} \cdot \ln 5\end{aligned}$$

The answer is (b).

EXERCISE-6

$$(1) \frac{d}{dx} (e^{5x^2} \cdot 4) = ?$$

- (a) $e^{5x^2} + 40xe^{5x^2}$ (b) $20x \ln (5x^2)$ (c) $40e^{5x^2}$ (d) $40x e^{5x^2}$

$$(2) \frac{d}{dx} (\sin 2x + e^{3x^2}) = ?$$

- (a) $2 \cos 2x + 6x e^{3x^2}$ (b) $2 \sin 2x \cos 2x + 6x e^{3x^2}$
 (c) $2(\cos x + 3x e^{6x})$ (d) *None*

$$(3) \frac{d}{dx} e^{x^2+9} = ?$$

- (a) e^{2x} (b) $2x e^{x^2+9}$ (c) $(2x+1)e^{x^2+9}$ (d) $2x \ln (x^2 + 9)$

$$(4) \frac{d}{dx} (e^{x^2} \cdot e^{\tan x}) = ?$$

- (a) $e^{2x+\sec^2 x}$ (b) $(2x + \sec^2 x) e^{x^2+\tan x}$
 (c) $2x + \sec^2 x$ (d) $2x \sec^2 x e^{x^2} e^{\tan x}$

$$(5) \frac{d}{dx} e^{\cos x} + 5 = ?$$

- (a) $-\sin x e^{\cos x} + 5$ (b) $e^{-\sin x}$ (c) $-\sin x e^{\cos x}$ (d) $5 + \frac{1}{e^{\sin x}}$

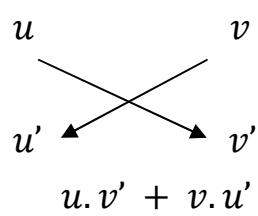
PRODUCT RULE

if u and v are derivable function of x , then

$$\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

or

$$(u \cdot v)' = u \cdot v' + v \cdot u'$$

Shortcut:**MCQ- 13:**

$$\frac{dy}{dx} = ? , \text{ if } y = x^2 \tan x$$

- (a) $2x \sec^2 x$
- (c) $x^2 \sec 2x \tan x$

- (b) $x^2 \sec^2 x$
- (d) $x^2 \sec^2 x + 2x \tan x$

Solution:

x^2 $\tan x$
 $2x$ $\sec^2 x$

$$\frac{dy}{dx} = x^2 \sec^2 x + 2x \tan x$$

The answer is (d).

MCQ-14 :

$$\frac{dy}{dx} = ? , \text{ if } y = e^{2x} \tan^{-1} x$$

(a) $\frac{e^{2x}}{1+x^2} + 2e^{2x}\tan^{-1}x$

(b) $e^{2x}\sec^{-1}x + 2e^{2x}\tan^{-1}x$

(c) $\frac{e^{2x}}{\sqrt{1-x^2}} + e^{2x}\tan^{-1}x$

(d) $e^{2x}(1+x^2) + e^{2x}\tan^{-1}x$

Solution:

$$\begin{array}{ccc} e^{2x} & & \tan^{-1}x \\ & \swarrow \times \searrow & \\ 2e^{2x} & & \frac{1}{1+x^2} \end{array}$$

$$\frac{dy}{dx} = \frac{e^{2x}}{1+x^2} + 2e^{2x}\tan^{-1}x$$

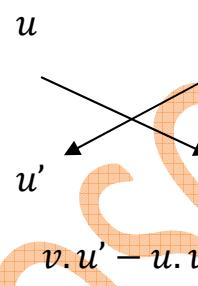
The answer is (a).

QUOTIENT RULE

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

or

$$\left(\frac{u}{v} \right)' = \frac{vu' - uv'}{v^2}$$

Shortcut:**MCQ- 15:**

$$\frac{dy}{dx} = ?, \text{ if } y = x^3 / \cos x$$

- (a) $(-x^3 \sin x - 3x^2 \cos x) / \cos^2 x$ (b) $-3x^2 / \sin x$
 (c) $(3x^2 \cos x + x^3 \sin x) / \cos^2 x$ (d) $(3x^2 \cos x - x^3 \sin x) / \cos^2 x$

Solution:

x^3
 $\cos x$
 $3x^2$
 $-\sin x$

$$\frac{dy}{dx} = (3x^2 \cos x + x^3 \sin x) / \cos^2 x$$

The answer is (c).

EXERCISE-7

(1) $\frac{d}{dx}(x^2 \cdot \sin x) = ?$

- (a) $x^2 \sin x + 2x \cos x$ (b) $2x \cos x$ (c) $x^2 \cos x + 2x \sin x$ (d) $x^2 \cos x$

(2) $\frac{d}{dx}(x^3 \cos^{-1} x) = ?$

- (a) $-3x^2 / \sqrt{1-x^2}$ (b) $-3x^2 \cos^{-1} x - x^3 \sin^{-1} x$
 (c) $x^3 \sqrt{1-x^2} + 3x^2 \cos^{-1} x$ (d) $3x^2 \cos^{-1} x - \frac{x^3}{\sqrt{1-x^2}}$

(3) $\frac{d}{dx}\left(\frac{\ln x}{x}\right) = ?$

- (a) $\frac{(1-\ln x)}{x^2}$ (b) $\frac{(x-\ln x)}{x^2}$ (c) 0 (d) $\frac{1}{x^2}$

(4) $\frac{d}{dx}\left(\frac{x}{e^x}\right) = ?$

- (a) $1/e^x$ (b) $(x - e^x)/e^{2x}$ (c) $(xe^x - 1)/e^{2x}$ (d) $(1 - x)/e^x$

IMPLICIT EQUATIONS
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PARAMETRIC EQUATIONS

$$x = f(t) , \quad y = g(t)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

MCQ-17 :

$\frac{dy}{dx} = ?$, if $x = 6t^3$ and $y = 8t^2$?

(a) $\frac{9t}{8}$

(b) $\frac{8}{9t}$

(c) $48t^5$

(d) $\frac{16}{t}$

Solution:

$x = 6t^3 , \quad y = 8t^2$

$\frac{dx}{dt} = 18t^2 , \quad \frac{dy}{dt} = 16t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{16t}{18t^2} \\ = \frac{8}{9t}$$

The answer is (b).

EXERCISE-9

(1) $x = t^2 + 3, y = 3t^3$. What is the value of $\frac{dy}{dx}$ at $t = 4$?

(a) $\frac{1}{18}$

(b) 1.8

(c) 18

(d) 36

(2) Given that $x = \sin^2 t, y = \cos^2 t$. What is the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{6}$?

(a) -1

(b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{1}{\sqrt{2}}$

HIGHER DERIVATIVES**MCQ-18 :**

$$f''(2) = ?, \text{ if } f(x) = 3x^4 - 16x$$

- (a) 144 (b) 72 (c) 92 (d) 108

Solution:

$$\begin{aligned} f(x) &= 3x^4 - 16x \\ f'(x) &= 12x^3 - 16 \\ f''(x) &= 36x^2 \\ f''(2) &= 36(2)^2 \\ f''(2) &= 144 \end{aligned}$$

The answer is (a).

EXERCISE-10

$$(1) \text{ If } f(x) = \sin x, \text{ then } f''\left(\frac{\pi}{6}\right) = ?$$

- (a) $\frac{-\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

$$(2) \text{ If } f(x) = x^4 - x^2, \text{ then } f'''(2) = ?$$

- (a) 48 (b) 24 (c) 46 (d) 22

$$(3) \text{ If } y = e^{3x}, \text{ then the value of } y'' \text{ at } x = 0 \text{ is } \underline{\hspace{2cm}}.$$

- (a) 0 (b) 9 (c) 1 (d) $9e^3$

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