

Chapter 3**LIMITS****INDETERMINATE FORMS**

$$\frac{0}{0}, 0 \cdot \infty, 0^0, \frac{\infty}{\infty}, \infty - \infty, \infty^\infty, 1^\infty$$

are indeterminate forms.

MCQ-1 :

Which of the following is not an indeterminate form?

- (a) $0 \cdot \infty$ (b) $\infty + \infty$ (c) $\infty - \infty$ (d) 1^∞

Solution:

since $\infty + \infty = \infty$
 $\therefore \infty + \infty$ is not a indeterminate form.

The answer is (b).

MCQ- 2:

Which of the following is an indeterminate form?

- (a) $\infty + \infty$ (b) $\infty \cdot \infty$ (c) 0^∞ (d) 1^∞

Solution:

The answer is (d).

LIMIT OF THE FUNCTION

$$\lim_{x \rightarrow a} f(x) = b$$

b is the limit of the function at $x \rightarrow a$.

DE L' HOPITAL RULE:

$f(x)$ and $g(x)$ are two functions. If the derivatives of both functions exist and $f(a) = 0 = g(a)$, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

MCQ- 3:

$$\lim_{x \rightarrow 5} \frac{(x - 5)}{x^2 - 25} = ?$$

Solution:

$$\lim_{x \rightarrow 5} \frac{(x - 5)}{x^2 - 25}, \quad (\frac{0}{0})$$

$$= \lim_{x \rightarrow 5} \frac{(x - 5)}{(x - 5)(x + 5)}$$

$$= \lim_{x \rightarrow 5} \frac{1}{x+5}$$

1

$$= \frac{1}{10}$$

The answer is (d).

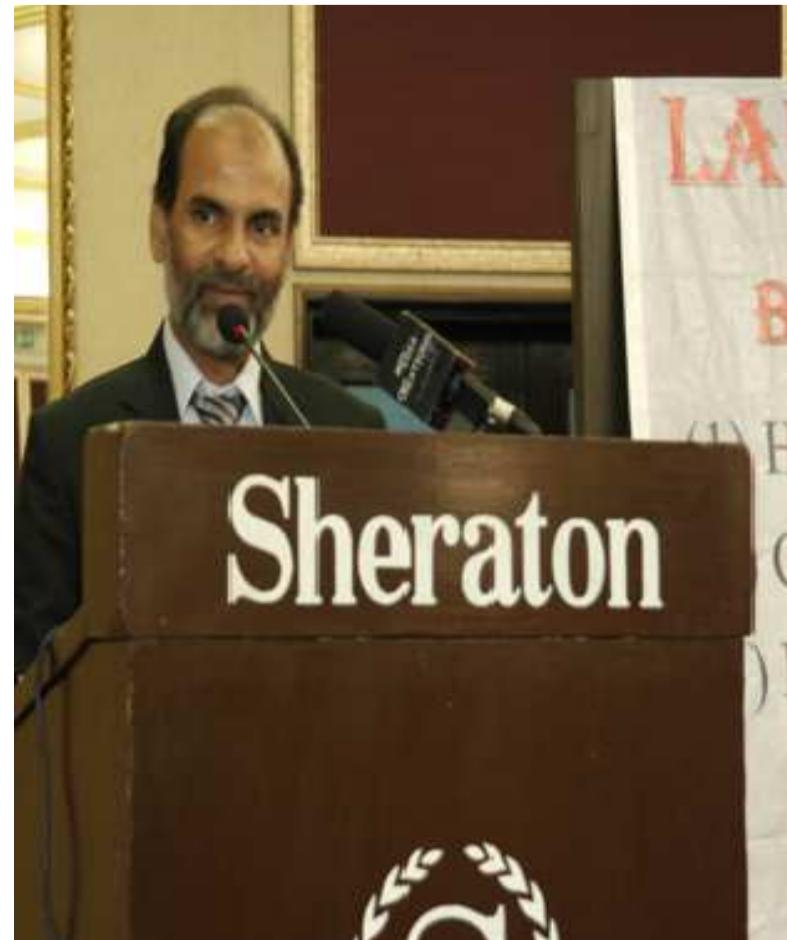
Second Method: (By De l' Hopital Rule)

$$\lim_{x \rightarrow 5} \frac{(x - 5)}{x^2 - 25}, \quad (\frac{0}{0})$$

$$= \lim_{x \rightarrow 5} \frac{1}{2x}$$

$$= \frac{1}{10}$$

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LIMIT INVOLVING TRIGONOMETRIC FUNCTIONS**Formula :**

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

MCQ- 5:

$$\lim_{x \rightarrow 0} \frac{\cos 2x \cdot \sin 5x}{x} = ?$$

(a) 10

(b) 0

(c) 5

(d) $\frac{1}{5}$ **Solution:**

$\lim_{x \rightarrow 0} \frac{\cos 2x \cdot \sin 5x}{x}$

Multiplying and dividing by 5

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{5 \cos 2x \cdot \sin 5x}{5x} \\
 &= 5 \lim_{x \rightarrow 0} \cos 2x \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} \\
 &= 5 (1) (1) \\
 &= 5
 \end{aligned}$$

The answer is (c).

EXERCISE- 1

$$(1) \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{(x-1)^2} = ?$$

- (a) 0 (b) 1 (c) 2 (d) None

$$(2) \lim_{x \rightarrow 5} \frac{x-5}{x^2 - 25} = ?$$

- (a) 1 (b) 0 (c) ∞ (d) $\frac{1}{10}$

$$(3) \lim_{x \rightarrow 4} \left[\frac{\sqrt[3]{x^2 - 16}}{(x-4)^{1/3}} \right] = ?$$

- (a) $16^{1/3}$ (b) ∞ (c) $\sqrt[3]{4}$ (d) 2

- (4) $\lim_{x \rightarrow 2} \left[\frac{(x^3 - 8)^{3/2}}{\sqrt{x-2}} \right] = ?$
- (a) ∞ (b) $\sqrt{12}$ (c) $8\sqrt{27}$ (d) $3\sqrt{6}$
- (5) $\lim_{x \rightarrow 2} \frac{(x^2 - 4)^2}{(x-2)^2} = ?$
- (a) 64 (b) 16 (c) 4 (d) ∞
- (6) $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x - 2} = ?$
- (a) -1 (b) 5 (c) 1 (d) ∞
- (7) $\lim_{x \rightarrow 0} \frac{x^3 + 6x^2}{x} = ?$
- (a) 6 (b) ∞ (c) 0 (d) 1
- (8) $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x-3} = ?$
- (a) 1 (b) 3 (c) 5 (d) ∞
- (9) $\lim_{x \rightarrow 1} \frac{x^2 + 3x - 10}{x^2 + 2x + 1} = ?$
- (a) $-\frac{1}{6}$ (b) $-\frac{3}{2}$ (c) $\frac{2}{5}$ (d) None
- (10) $\lim_{x \rightarrow 0} \frac{\sin 5x}{x} = ?$
- (a) 0 (b) $\frac{0}{0}$ (c) ∞ (d) 5
- (11) $\lim_{x \rightarrow 0} \frac{\sin \frac{x}{5}}{x} = ?$
- (a) 5 (b) 0 (c) $\frac{1}{5}$ (d) ∞
- (12) $\lim_{x \rightarrow 0} \frac{\sin(x-3)}{x-3} = ?$
- (a) 3 (b) 1 (c) 0 (d) ∞

(13) $\lim_{x \rightarrow 0} \frac{\sin 2x}{x \cos x} = ?$

- (a) $\frac{0}{0}$ (b) $\frac{1}{2}$ (c) 0 (d) 2

(14) $\lim_{x \rightarrow \pi} \frac{1+\cos x}{\sin x} = ?$

- (a) 2 (b) $\frac{1}{2}$ (c) 0 (d) ∞

(15) $\lim_{x \rightarrow \pi} \frac{\sin x}{1-\cos x} = ?$

- (a) 2 (b) $\frac{1}{2}$ (c) 1 (d) 0

(16) $\lim_{x \rightarrow \pi} \frac{\tan 2x}{\sin 2x} = ?$

- (a) 0 (b) 1 (c) ∞ (d) 2

(17) $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 6x} = ?$

- (a) $\frac{1}{3}$ (b) 3 (c) $\frac{2}{3}$ (d) ∞

(18) $\lim_{x \rightarrow \frac{\pi}{3}} \frac{1-\cos x}{1+\cos x} = ?$

- (a) 4 (b) $\frac{1}{4}$ (c) 3 (d) $\frac{1}{3}$

(19) $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin^3 x \cos^3 x}{\tan^3 x} = ?$

- (a) $\frac{1}{32}$ (b) 8 (c) $\frac{1}{64}$ (d) 32

LIMIT $x \rightarrow +\infty$ AND $x \rightarrow -\infty$

$$\lim_{x \rightarrow +\infty} f(x)$$

and

$$\lim_{x \rightarrow -\infty} f(x)$$

Short Cut:

$$\lim_{x \rightarrow \pm\infty} \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{b_0 + b_1 x + b_2 x^2 + \cdots + b_m x^m} = \lim_{x \rightarrow \pm\infty} \frac{a_n x^n}{b_m x^m}$$

MCQ- 6:

$$\lim_{n \rightarrow \infty} \frac{5 + 3x^2 - 4x^4}{2 + x^3 + 16x^4} = ?$$

(a) $\frac{-1}{4}$

(b) $\frac{5}{2}$

(c) $\frac{1}{6}$

(d) 3

Solution:

$$\begin{aligned}
 & \lim_{x \rightarrow \infty} \frac{5 + 3x^2 - 4x^4}{2 + x^3 + 16x^4} \\
 &= \lim_{x \rightarrow \infty} \frac{x^4 \left(\frac{5}{x^4} + \frac{3}{x^2} - 4 \right)}{x^4 \left(\frac{2}{x^4} + \frac{1}{x^3} + 16 \right)} \\
 &= \frac{\frac{5}{\infty} + \frac{3}{\infty} - 4}{\frac{2}{\infty} + \frac{1}{\infty} + 16} \\
 &= \frac{0+0-4}{0+0+16} \\
 &= -\frac{4}{16} \\
 &= -\frac{1}{4}
 \end{aligned}$$

The answer is (a).

Second Method: (Shortcut)

$$\lim_{x \rightarrow \infty} \frac{5 + 3x^2 - 4x^4}{2 + x^3 + 16x^4}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow \infty} \frac{-4x^4}{16x^4} \\
 &= -\frac{4}{16} \\
 &= -\frac{1}{4}
 \end{aligned}$$

MCQ-7 :

$$\lim_{x \rightarrow \infty} \frac{3 - 5x + 9x^3}{2 + 2x^4} = ?$$

- (a) ∞ (b) 0 (c) $\frac{9}{2}$ (d) $\frac{3}{2}$

Solution:

$$\lim_{x \rightarrow \infty} \frac{3 - 5x + 9x^3}{2 + 2x^4}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow \infty} \frac{9x^3}{2x^4} \\
 &= \lim_{x \rightarrow \infty} \frac{9}{2x} \\
 &= \frac{9}{\infty} \\
 &= 0
 \end{aligned}$$

The answer is (b).

MCQ-8:

$$\lim_{x \rightarrow \infty} \frac{5 + 2x + 6x^2}{\sqrt{2x + 9x^4}} = ?$$

- (a) $\frac{5}{2}$ (b) ∞ (c) 2 (d) 0

Solution:

$$\begin{aligned}
 & \text{Lim}_{x \rightarrow \infty} \frac{5 + 2x + 6x^2}{\sqrt{2x + 9x^4}} \\
 &= \text{Lim}_{x \rightarrow \infty} \frac{6x^2}{\sqrt{9x^4}} \\
 &= \text{Lim}_{x \rightarrow \infty} \frac{6x^2}{3x^2} \\
 &= 2
 \end{aligned}$$

The answer is (c).

EXERCISE-2

(1) $\text{Lim}_{x \rightarrow \infty} \frac{5+2x+3x^2}{2+8x-9x^2} = ?$

- (a) $\frac{1}{4}$ (b) $-\frac{1}{3}$ (c) $\frac{5}{2}$ (d) $\frac{3}{8}$

(2) $\text{Lim}_{x \rightarrow \infty} \left[\frac{\sqrt{2+5x^3-9x^6}}{(5+8x^4-18x^6)^{1/2}} \right] = ?$

- (a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{6}$ (c) 4 (d) $1/2$

(3) $\text{Lim}_{x \rightarrow \infty} \left(\frac{5-3x}{x} \right)^3 = ?$

- (a) 8 (b) -27 (c) 125 (d) ∞

(4) $\text{Lim}_{x \rightarrow \infty} \frac{5+3x^3+8x^5}{2x+3x^2+2x^4} = ?$

- (a) 8 (b) 4 (c) 0 (d) ∞

(5) $\text{Lim}_{x \rightarrow -\infty} \frac{\sqrt{4x^2+1}}{x+6} = ?$

- (a) -2 (b) 2 (c) $\frac{1}{6}$ (d) $-\frac{2}{3}$

(6) $\text{Lim}_{x \rightarrow -\infty} \frac{2-3x}{\sqrt{16+x^2}} = ?$

- (a) $-\frac{3}{4}$ (b) $\frac{1}{2}$ (c) -3 (d) 3

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MCQ-10 :

Solution:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{2x}$$
$$= [\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x]^2$$
$$= (e^2)^2$$
$$= e^4$$

The answer is (d).

EXERCISE-3

$$(1) \quad \lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2} = ?$$

$$(2) \quad \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^{3x} = ?$$

- (a) e^{15} (b) e^5 (c) e^3 (d) $e^{3/5}$

$$(3) \quad \lim_{x \rightarrow 0} \frac{6^x - 1}{x} = ?$$

$$(4) \quad \lim_{x \rightarrow 0} \frac{e^{5x}-1}{x} = ?$$

$$(5) \lim_{x \rightarrow \infty} \frac{\ln(1+3^{-x})}{2+e^{-x}} = ?$$

$$(6) \lim_{x \rightarrow \infty} \frac{3+e^{-x}}{x} = ?$$

- (a) ∞ (b) 0 (c) 3 (d) $3e$

$$(7) \lim_{x \rightarrow -\infty} \frac{2+e^{-t}}{3+5^t} = ?$$

- (a) $\frac{2}{3}$ (b) $\frac{1}{5}$ (c) ∞ (d) 0

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nth TERM OF THE SEQUENCE*a* : first term*d* : common difference*a_n*: *nth* term*nth* term of the sequence

$$a_n = d \cdot n + (a - d)$$

MCQ-11 :What is the *nth* term of the sequence 3, 4, 5, ... ?

- (a) $2n + 1$ (b) $3n - 2$ (c) $3n$

- (d) $n + 2$

Solution:

3, 4, 5, ...

$$a = 3, d = 1$$

$$a_n = d \cdot n + (a - d)$$

$$a_n = 1 \cdot n + (3 - 1)$$

$$a_n = n + 2$$

The answer is (d).

MCQ-12:What is the *nth* term of the sequence 5, 8, 11, ... ?

- (a) $4n + 1$ (b) $7n - 2$ (c) $5n$

- (d) $2n + 3$

Solution:

5, 8, 11, ...

$$a = 5, d = 3$$

$$a_n = d \cdot n + (a - d)$$

$$a_n = 3 \cdot n + (5 - 3)$$

$$a_n = 3n + 2$$

The answer is (b).

LIMIT OF THE SEQUENCE

If a_n is the n th term of the sequence, then

$$\lim_{n \rightarrow \infty} a_n = l$$

where l is the limit of the sequence.

MCQ-13:

$a_n = \frac{n^2}{n^2+9}$ is the n th term of the sequence. What is the limit of the sequence?

- (a) 1 (b) 9 (c) 0 (d) ∞

Solution:

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^2}{n^2 + 9}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2}{n^2(1 + \frac{9}{n^2})}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{9}{n^2}}$$

$$= \frac{1}{1 + 0}$$

$$= 1$$

The answer is (a).

EXERCISE-4

(1) What is the nth term and limit of the sequence $\frac{4}{5}, \frac{7}{11}, \frac{10}{17}, \dots$?

(a) $\frac{4n}{5n}, \frac{4}{5}$ (b) $\frac{3n+1}{4n+1}, \frac{3}{4}$ (c) $\frac{5n-1}{6n-1}, \frac{5}{6}$ (d) $\frac{3n+1}{6n-1}, \frac{1}{2}$

(2) $a_n = \frac{2n+5}{3+7n}$ is the nth term of a sequence. What is the limit of the sequence?

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$ (c) $\frac{2}{7}$ (d) ∞

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