

## Chapter 1

## TeUTNGITHONTG

## REAL NUMBER LINE



## SETS AND INTERVALS

(i) $\{a, b\}$

It is a set and $a$ and $b$ are the elements of the set.
(ii) $[a, b]$

It is a closed interval.
If $x \in[a, b]$, then $a \leq x \leq b$.
(iii) $(a, b)$

It is an open interval.

$$
\text { If } x \in(a, b) \text {, then } a<x<b
$$

MCQ-1:
If $x \in(5,8)$, then $\qquad$ ?
(a) $x=5,8$ only
(b) $5 \leq x \leq 8$
(c) $5<x<8$
(d) $x \leq 5$ and $x \geq 8$

Solution:


All real numbers between 5 and 8 but 5 and 8 are not including, so

$$
5<x<8
$$

The answer is (c)

## 

(1) If $x \in\{3,10\}$, then which of the following is true?
(a) $x=3,10$ only
(b) $3 \leq x \leq 10$
(c) $3<x<10$
(d) $x \leq 3, x \geq 10$
(2) If $x \in[6,7]$, then which of the following is true?
(a) $x=6$,7 only
(b) $6 \leq x \leq 7$
(c) $6<x<7$
(d) $x<6, x>7$
(3) If $x \in(3,9]$, then which of the following is true?
(a) $x=3,9$ only
(b) $3 \leq x \leq 9$
(c) $3 \leq x<9$
(d) $3<x \leq 9$

## GREATEST LOWER BOUND (g.I.b)

## AND <br> LEAST UPPER BOUND (I.U.๒)

$A=\{a, b\}$ is a set.
(i) Lower bounds of $A$ : $\quad x \leq a$
(ii) Greatest lower bound of $A=a$
(iii) Upper bounds of $A$ : $\quad x \geq b$
(iv) Least upper bound of $A=b$

MCQ- 2:
If $B=\{x: 5 \leq x<7\}$, then what is the least upper bound of $B$ ?
(a) 5
(b) 6
(c) 7
(d) None

Solution:

$$
\because \text { upper bounds of } B=x \geq 7
$$

$$
\therefore \text { l.u.b of } B=7
$$

The answer is (c).

## 

(1) If $B=\{3,6,9,10\}$ is a set, then upper bounds of $B=x=$ ?
(a) $x=10$ only
(b) $x \geq 10$
(c) $x \leq 3$
(d) $x=3$ only
(2) If $B=\{5,6,7,8\}$, then least upper bound of $B=x=$ ?
(a) $x=5$
(b) $x=8$
(c) $x \leq 5$
(d) $x \geq 8$
(3) If $B=\{-3,1,5\}$, then greatest lower bound of $B=x=$ ?
(a) $x=-3$
(b) $x=5$
(c) $x \leq-3$
(d) $x \geq 5$
(4) If $B=\{-5,6,12\}$, then lower bounds of $B=x=$ ?
(a) $x=-5$ only
(b) $x=12$ only
(c) $x \leq-5$
(d) $x \geq 12$


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## FUNCTIONS

## Definition 1:

A function $f$ assigns to each element $x$ of set A an element $f(x)$ of another set B .

$$
f: A \rightarrow B, \forall x \in A \text { and } f(x) \in B
$$

## Explanation with Mapping: (Different style of mapping)

(1) $f$ is a function $A$ into $B$.

(2) $f$ is a function $A$ into $B$.

(3) $f$ is not a function $A$ into $B$.

because $f(4)=20$ and $f(4)=30$.
4 has two images 20 and 30 .
(4) $f$ is not a function $A$ into $B$.

because 5 has no image.

## RELATIONS

A relation is a set of ordered pairs.
or
Any subset of the set of Cartesian product $A \times B$ is called relation.
Examples:
If $A=\{1,3,5\}$ and $B=\{2,4,6\}$ then the following subsets are relation between $A$ and $B$.
(i) $\{(1,2),(1,4),(2,6)\}$
(ii) $\{(3,4),(1,6),(3,4),(5,6)\}$

## FUNCTIONS

## Definition 2:

A relation $f$ of between $A$ and $B$ is called a function if every $x \in A$ there is exactly one $y \in B$.

The element $y$ is denoted by $f(x)$.
Examples:
If $A=\{1,3,5\}$ and $B=\{2,4,6\}$ then the following subsets are relations are the functions $A$ into $B$.

Examples:
(i) $f=\{(1,2),(3,4),(5,6)\}$
(ii) $f=\{(1,4),(3,6),(5,4)\}$
(iii) $f=\{(1,2),(3,2),(5,2)\}$
(iv) $f=\{(1,2),(3,4),(5,6)\}$

Following are not functions from $A$ into $B$.
(i) $f=\{(1,2),(3,4),(3,6),(5,6)\}$ because 3 has two images 4 and 6 .
(ii) $f=\{(1,2),(3,6)\}$
because image of 5 does not exist.

MCQ- 3:
$f: A \rightarrow B$ is a function such that $A=\{2,5\}$ and $B=\{3,8,9\}$.
Which of the following is correct?
(a) $f=\{(2,3),(5,9)\}$
(b) $f=\{(2,3),(2,5)\}$
(c) $f=\{(2,3),(5,8),(5,9)\}$
(d) $f=\{(5,8)\}$

Solution:
Option (a) : Each element of set $A$ has an element in set $B$.

The answer is (a).

## Explanation:

Options (b), (c) and (d) are not correct because
Option (b): $\quad\{(2,3),(2,5)\}$
$2 \epsilon A$ has two images in set $B$,
that are 3 and 5
option (c): $\{(2,3),(5,8),(5,9)\}$
$5 \in A$ has two images in set $B$,
that are 8 and 9 .
option (d) : \{(5, 8), \}
$(2, ?)$
$2 \in A$ has no image in $B$, such that $(2, ?)$.

## MCQ- 4:

If $f: \mathbb{N} \rightarrow \mathbb{N}$, then which of the following is correct?
(a) $f(x)=\frac{1}{2} x^{2}$
(b) $f(x)=\sqrt{x}+3$
(c) $f(x)=x^{3}+3$
(d) $f(x)=x^{2}-1$

## Solution:



$$
\therefore f(x) \in \mathbb{N}=\{1,2,3, \ldots\}
$$

Option(c):

$$
\begin{array}{cl}
x^{3} \in \mathbb{N} & , \forall x \in \mathbb{N} \\
\text { and } \quad\left(x^{3}+3\right) \in \mathbb{N} & , \quad \forall x \in \mathbb{N}
\end{array}
$$

The answer is (c).

## Explanation:

Option(a) : check at 3

$$
f(3)=\frac{1}{2} \times 3^{2}=\frac{9}{2} \notin \mathbb{N}
$$

Option b: $f(5)=\sqrt{5}+3 \notin \mathbb{N}$
Option d: $f(1)=1^{2}-1=0 \notin \mathbb{N}$

## 

(1) $\quad A=\{1,3,6\}, B=\{5,8\}$ and $f: A \rightarrow B$.

Which of the following is a function?
(a) $f=\{(1,5),(3,8),(3,5)\}$
(b) $f=\{(1,8),(3,5)\}$
(c ) $f=\{(6,5),(1,8),(3,3)\}$
(d) $f=\{(1,5),(3,5),(6,5)\}$
(2) If $f: \mathbb{N} \rightarrow \mathbb{Z}$, then which of the following is true?
(a) $f(x)=\frac{1}{2}(2 x+1)$
(b) $f(x)=2 x^{3}-15 x$
(c) $f(x)=\sqrt{x+2}$
(d) $f(x)=x+\sqrt{x}$

## VALUE OF THE FUNCTION

If $f$ is a function $f: A \rightarrow B$
(i) $f(a)$ is the value of the function at $x=a$,
if $a \in A$ and the value of the function exist if $f(a) \in B$.
(ii) $f(a)$ is not the value of the function at $x=a$,
if $a \notin A$.
MCQ-5:
If $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by

$$
f(x)=\sqrt{x}
$$

then $f(16)=$ ?
(a) 16
(b) 2
(c) 4
(d) 8

## Solution:



The answer is (c).
MCQ-6:
If $f: \mathbb{Z} \rightarrow \mathbb{R}$ is defined by

$$
f(x)= \begin{cases}2+x^{2} & \text { when } \\ 5-x & \text { when } \\ 5 \in \mathbb{Z}^{+} \\ \hline\end{cases}
$$

then $f(-6)=$ ?
(a) 49
(b) 38
(c) 0
(d) 11

Solution:

$$
\mathbb{Z} \text { : set of integers }
$$

$\mathbb{Z}^{-}$: set of negative integers
$\mathbb{Z}^{+}$: set of positive integers


$$
\begin{aligned}
& \therefore f(x)=5-x \\
& f(-6)=5-(-6) \\
& \quad=11
\end{aligned}
$$

The answer is (d).
MCQ- 7:

$$
\begin{aligned}
& \text { If } \quad f: \mathbb{N} \rightarrow \mathbb{R} \text { is defined by } \\
& f(x)=8 x^{2}+3
\end{aligned}
$$

then $f(-2)=$ ?
(a) -29
(b) 35
(c) 0
(d) None

Solution:


The answer is (d).
MCQ ${ }^{8:}$
If $f:[-3,8] \rightarrow \mathbb{R}$ is defined by

$$
f(x)=2 \sqrt{x}+3
$$

then $f(9)=$ ?
(a) 3
(b) 21
(c) 9
(d) None

Solution:


The answer is (d).

## 

(1) A function $f:[2,8] \rightarrow \mathbb{R}$ is defined by $f(x)=2 x^{3}-5$.

What is the value of $f(1)$ ?
(a) 12
(b) 6
(c) -3
(d) None
(2) A function $f:[6,12] \rightarrow \mathbb{N}$ is defined by $f(x)=2 x^{2}+6 x$.

What is the value $x$ when $f(x)=7 / 2$ ?
(a) $-1 / 2$
(b) $5 / 2$
(c) $3 / 2$
(d) $-7 / 2$
(3)

$$
f(x)=\left\{\begin{array}{lr}
2 x^{2}-1, & 2 \leq x<5 \\
x^{2}+2, & 5 \leq x \leq 10
\end{array}\right.
$$

$f(5)=$ ?
(a) 27
(b) 49
(c) 76
(d) None


## ONE TO ONE FUNCTIONS

A function $f: A \rightarrow B$ is said to be one to one if $x_{1} \neq x_{2}$ implies that $f\left(x_{1}\right) \neq f\left(x_{2}\right)$ such that $x_{1}, x_{2} \in A$ and $f\left(x_{1}\right), f\left(x_{2}\right) \in B$.
Examples:
(a) Following functions are one to one functions.

(2)


(4) If $\mathrm{A}=\{1,2,4\}$ and $\mathrm{B}=\{1,10,20\}$

$$
\text { and } f=\{(1,1),(2,10),(4,20)\}
$$

(5) $\mathrm{f}(\mathrm{x})=5 \mathrm{x}, \forall \mathrm{x} \in \mathbb{R}$
(6) $\quad \mathrm{f}(\mathrm{x})=\mathrm{x}^{2}+3 \mathrm{x}, \forall \mathrm{x} \in \mathbb{R}$
(b) Following functions, are not one - one functions.
(1) $\mathrm{f}(\mathrm{x})=|\mathrm{x}|$
(2) $f(x)=6 \quad$ (or any constant function)
(3)


MCQ- 9:
If $f: A \rightarrow B$ such that $A=\{2,6,8\}$ and $B=\{1,3,5,7,9\}$ then which of the following is one to one function?
(a) $f=\{(2,1),(8,3),(6,1)\}$
(b) $f=\{(2,3),(8,7),(6,5),(2,9)\}$
(c) $f=\{(2,1),(8,9)\}$
(d) $f=\{(2,1),(8,9),(6,5)\}$

Solution:
$f$ is a function and the images of 2,8 and 6 are
different
$\therefore f$ is one to one.

The answer is (d).
Explanation:
Option (a) : $\{(2,1),(8,3),(6,1)\}$
$f$ is not one to one, the images of 2 and 6 are same.

Option (b) : $\{(2,3),(8,7),(6,5),(2,9)\}$
$f$ is not a function.
$\because 2$ is repeated in the domain.

Option (c) : \{(2, 1), (8, 9) \} (6, ?)
$\Rightarrow f(6)$ does not exist.
$\therefore f$ is not a function.

## ONTO FUNCTIONS

A function $f: A \rightarrow B$ is said to be onto if for every $y \in B$ there is some $x \in A$ such that $y=f(x)$.
Examples:
(A) Following functions are onto functions.
(1)

because each element of $B$ is the image of some element of $A$, under $f$.
(2)

because each element of $B$ is the image of an element of $A$ under $f$.
(B) Following functions are not onto functions.
(1)

because $8 \in B$ is not the image of any element of $A$ under $f$.

MCQ- 10 :
If $f: A \rightarrow B$ such that $A=\{5,6,7\}$ and $B=\{2,4\}$ then which of the following is onto function?
(a) $f=\{(5,2),(6,4)\}$
(b) $f=\{(5,2),(6,2),(7,4)\}$
(c) $f=\{(5,2),(6,4),(6,2)\}$
(d) $f=\{(5,2),(6,2),(7,2)\}$

Solution:
Option (b) f is a function and onto.
Because all the elements in B are the images of the elements in A .
The answer is (b).

## Explanation:

Option (a) and option (c) are not functions.
Option (d) is a function but not onto because for $4 \epsilon B$ is not the image of any element in A .

## 



## COMPOSITE FUNCTIONS

Suppose that $\mathrm{A}, \mathrm{B}$ and C are three sets. Let f be function A into B

$$
y=f(x), \forall x \in A \text { where } y \in B
$$

Geometrically it can be represented as


Let g be a function B into C , then
$z=g(y), \forall y \in B$ where $z \in C$

$h$ is a new function $A$ into $C$ defined as

$$
h(x)=g[f(x)]
$$

is called composite function.
Geometrically it can be represented as
h


The composite function $g[f(x)]$ is written as

$$
\left(g_{o} f\right)(x)
$$

read " $g$ composed with $f$ ".

## Shortcut"

(i) $f_{o} g=f(g)$
(ii) $g_{o} f=g(f)$
(iii) $f_{o} f=f(f)$

MCQ- 11:
If $f(x)=5 x-3$ and $g(x)=2 x^{2}$, then $f o g=$ ?
(a) $10 x^{2}-3$
(b) $10 x^{3}-6 x^{2}$
(c) $2(5 x-3)^{2}$
(d) $2 x^{2}+5 x-3$

Solution:

| $f o g=f(g)$ |  |
| :--- | ---: |
|  | $=5 g-3$ |
| $=5\left(2 x^{2}\right)-3$ | \{Replace $x$ with $g$ |
| $=10 x^{2}-3$ | \{Put the value of $g(x)$ |

The answer is (a).

## 

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## POLYNOMIAL FUNCTIONS

A function is of the form

$$
P(x)=a_{n} x^{n}+a_{n-1} x^{n-1}+\cdots+a_{1} x+a_{0}
$$

is called polynomial function in x of degree n , where n is non-negative integer and the highest power of $x$, where $a_{0}, a_{n-1}, \ldots, a_{1}, a_{0}$ are real numbers.

## Particular Polynomial Functions:

(i) If $n=0 \Rightarrow P(x)=a_{0}$ (constant function, degree 0)
(ii) If $n=1 \Rightarrow P(x)=a_{1} x+a_{0} \quad$ (linear function, degree 1)
(iii) If $n=2 \Rightarrow P(x)=a_{2} x^{2}+a_{1} x+a_{0} \quad$ (quadratic function, degree 2)

MCQ- 12:
Which of the following is a polynomial function?
(a) $f(x)=2 x+\sqrt{x}$
(b) $f(x)=\frac{3}{2} x^{2}+2 x$
(c) $f(x)=5 x^{\frac{2}{3}}+2$
(d) $f(x)=2 x^{-3}+6 x^{2}$

Solution:
Option(b): the powers of $x$ are non-negative integers.
The answer is (b).
MCQ-13 :
What is the degree of the polynomial function $f(x)=5 x^{4}+3 x^{2}+9 x$ ?
(a) 5
(b) 9
(c) 4
(d) 1

Solution:
The highest power of $x$ is 4 .
The degree of the polynomial function is 4 .
The answer is (c).

## 

(1) Which of the following is a polynomial function?
(a) $f(x)=5$
(b) $f(x)=2 \sqrt{x}+3$
(c) $f(x)=\frac{2}{x^{2}}+7 x$
(d) $f(x)=5 x^{-6}+6$

## EVEN FUNCTIONS

A function is said to be even function, if

$$
f(-x)=f(x)
$$

## ODD FUNCTIONS

A function is said to be odd function, if

$$
f(-x)=-f(x)
$$

## Note:

(1) $\quad f(x)=g(x)+h(x)+t(x)+\cdots$
(i) $f$ is even function if $g, h, t \ldots$ are even functions.
(ii) $f$ is odd function if $g, h, t \ldots$ are odd functions.
(2) $\quad f(x)=g(x) \cdot f(x)$
(i) $f$ is even function if $g$ and $h$ both are even or both are odd functions.
(ii) $f$ is odd function if one of them $g$ or $h$ are odd function and other is even function.
(3) $\quad f(x)=\frac{g(x)}{f(x)}$
(i) $f$ is even function if $g$ and $h$ both are even or both are odd functions.
(ii) $f$ is odd function if one of them $g$ or $h$ are odd function and other is even function.
(4) A constant function is an even function.

MCQ-14:
Which of the function is an odd function?
(a) $f(x)=x^{3}+5$
(b) $f(x)=2 x^{2}$
(c) $f(x)=3 x^{3}+\sin x$
(d) $f(x)=x^{5}+\cos x$

## Solution:

$$
\begin{aligned}
& f(-x)=3(-x)^{3}+\sin (-x) \\
& \quad=-3 x^{3}-\sin x \\
& =-\left(3 x^{3}+\sin x\right) \\
& =-f(x)
\end{aligned}
$$

The answer is (c).

## Shortcut:

> Since $x^{3}$ and $\sin x$ are odd functions.
> So that $f(x)$ is an odd function.

The answer is (c).

## Explanation:

Option (a) : There is a constant term 5 , so it can not be an odd function.
Option (b) : The power of $x$ is an even number, which is 2 , so it is not an odd function.
option (d) : $\cos (-x)=\cos x \neq-\cos x$
$\therefore$ it is not an odd function.
Option (c) : the power of $x$ is odd number, that is 3 so $(-x)^{3}=-x^{3}$ and $\sin (-x)=-\sin x$
So $f$ is an odd function.

## 

(1) Which of the following is neither even nor odd function?
(a) $f(x)=3 x^{3}+2 x$
(b) $f(x)=2 x^{2}+9$
(c) $f(x)=5 x^{3}+6$
(d) $f(x)=x^{2}+\cos x$

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