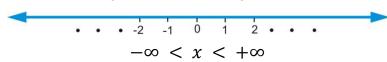


Chapter 1

FUNCTIONS

REAL NUMBER LINE



SETS AND INTERVALS

(i) $\{a, b\}$

It is a set and a and b are the elements of the set.

(ii) [a,b]

It is a closed interval.

If $x \in [a, b]$, then $a \le x \le b$.

(iii) (a,b)

It is an open interval.

If $x \in (a, b)$, then a < x < b.

MCQ-1:

If $x \in (5,8)$, then____?

(a)
$$x = 5.8$$
 only

(b)
$$5 \le x \le 8$$

(c)
$$5 < x < 8$$

(d)
$$x \le 5$$
 and $x \ge 8$

Solution:



All real numbers between 5 and 8 but 5 and 8 are not including, so

The answer is (c)

EXERCISE-1

- (1) If $x \in \{3,10\}$, then which of the following is true?
 - (a) x = 3, 10 only
- (b) $3 \le x \le 10$

(c) 3 < x < 10

- (d) $x \le 3$, $x \ge 10$
- (2) If $x \in [6,7]$, then which of the following is true?
 - (a) x = 6, 7 only
- (b) $6 \le x \le 7$

(c) 6 < x < 7

- (d) x < 6 , x > 7
- (3) If $x \in (3,9]$, then which of the following is true?
 - (a) x = 3,9 only
- (b) $3 \le x \le 9$

(c) $3 \le x < 9$

(d) $3 < x \le 9$

GREATEST LOWER BOUND (g.l.b)

ANI

LEAST UPPER BOUND (I.u.b)

 $A = \{a, b\}$ is a set.

- (i) Lower bounds of $A: x \leq a$
- (ii) Greatest lower bound of A = a
- (iii) Upper bounds of $A: x \ge b$
- (iv) Least upper bound of A = b

MCQ- 2:

- If $B = \{x: 5 \le x < 7\}$, then what is the least upper bound of B?
 - (a) 5
- (b) 6
- (c) 7
- (d) None

Solution:

$$\therefore$$
 upper bounds of $B = x \ge 7$

 $\therefore l.u.b of B = 7$

The answer is (c).

EXERCISE-2

- (1) If $B = \{3,6,9,10\}$ is a set, then upper bounds of B = x = ?
 - (a) x = 10 only (b) $x \ge 10$
- (c) $x \le 3$
- (d) x = 3 only
- (2) If $B = \{5,6,7,8\}$, then least upper bound of B = x = ?

 - (a) x = 5 (b) x = 8
- (c) $x \le 5$
- (d) $x \ge 8$
- (3) If $B = \{-3,1,5\}$, then greatest lower bound of B = x = ?
- (a) x = -3 (b) x = 5 (c) $x \le -3$ (d) $x \ge 5$
- (4) If $B = \{-5,6,12\}$, then lower bounds of B = x = ?
 - (a) x = -5 only (b) x = 12 only (c) $x \le -5$ (d) $x \ge 12$



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FUNCTIONS

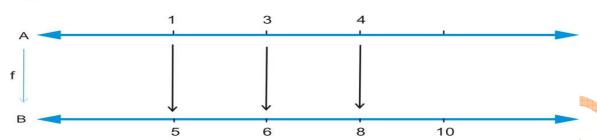
Definition 1:

A function f assigns to each element x of set A an element f(x) of another set B.

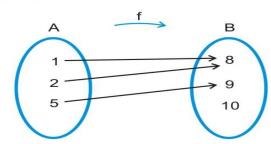
$$f: A \to B$$
 , $\forall x \in A$ and $f(x) \in B$

Explanation with Mapping: (Different style of mapping)

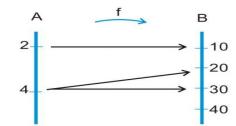
(1) f is a function A into B.



(2) f is a function A into B.

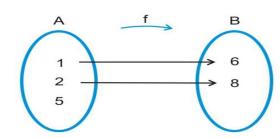


(3) f is not a function A into B.



because f(4)=20 and f(4)=30. 4 has two images 20 and 30.

(4) f is not a function A into B.



because 5 has no image.

RELATIONS

A relation is a set of ordered pairs.

or

Any subset of the set of Cartesian product $A \times B$ is called relation.

Examples:

If $A = \{1,3,5\}$ and $B = \{2,4,6\}$ then the following subsets are relation between A and B.

- (i) $\{(1,2), (1,4), (2,6)\}$
- (ii) $\{(3,4), (1,6), (3,4), (5,6)\}$

FUNCTIONS

Definition 2:

A relation f of between A and B is called a function if every $x \in A$ there is exactly one $y \in B$.

The element y is denoted by f(x).

Examples:

If $A = \{1,3,5\}$ and $B = \{2,4,6\}$ then the following subsets are relations are the functions A into B.

Examples:

- (i) $f = \{(1,2), (3,4), (5,6)\}$
- (ii) $f = \{(1,4), (3,6), (5,4)\}$
- (iii) $f = \{(1,2), (3,2), (5,2)\}$
- (iv) $f = \{(1,2), (3,4), (5,6)\}$

Following are not functions from A into B.

(i) $f = \{(1,2), (3,4), (3,6), (5,6)\}$

because 3 has two images 4 and 6.

(ii) $f = \{(1,2), (3,6)\}$

because image of 5 does not exist.

MCQ- 3:

 $f: A \to B$ is a function such that $A = \{2,5\}$ and $B = \{3,8,9\}$.

Which of the following is correct?

(a)
$$f = \{(2,3),(5,9)\}$$

(b)
$$f = \{(2,3),(2,5)\}$$

(c)
$$f = \{(2,3),(5,8),(5,9)\}$$
 (d) $f = \{(5,8)\}$

(d)
$$f = \{ (5,8) \}$$

Solution:

Option (a): Each element of set A has an element in set B.

The answer is (a).

Explanation:

Options (b), (c) and (d) are not correct because

Option (b): $\{(2,3),(2,5)\}$

 $2 \in A$ has two images in set B,

that are 3 and 5

option (c): $\{(2,3), (5,8), (5,9)\}$

5 ϵ A has two images in set B,

that are 8 and 9.

(2, ?)option (d): {(5, 8), }

 $2 \in A$ has no image in B, such that (2, ?).

MCQ- 4:

If $f: \mathbb{N} \to \mathbb{N}$, then which of the following is correct?

(a)
$$f(x) = \frac{1}{2}x^2$$

(b)
$$f(x) = \sqrt{x} + 3$$

(c)
$$f(x) = x^3 + 3$$

$$(d) f(x) = x^2 - 1$$

Solution:

 $f: \mathbb{N} \to \mathbb{N}$

 $\therefore f(x) \in \mathbb{N} = \{1,2,3,\dots\}$

Option(c):

$$x^3 \epsilon \mathbb{N}$$
 , $\forall \ x \epsilon \mathbb{N}$ and $(x^3+3) \epsilon \mathbb{N}$, $\forall \ x \epsilon \mathbb{N}$

The answer is (c).

Explanation:

Option(a): check at 3

$$f(3) = \frac{1}{2} \times 3^2 = \frac{9}{2} \notin \mathbb{N}$$

Option b: $f(5) = \sqrt{5} + 3 \notin \mathbb{N}$ Option d: $f(1) = 1^2 - 1 = 0 \notin \mathbb{N}$

EXERCISE-3

(1) $A = \{1,3,6\}$, $B = \{5,8\}$ and $f: A \to B$.

Which of the following is a function?

(a)
$$f = \{(1,5), (3,8), (3,5)\}$$

(b)
$$f = \{(1,8), (3,5)\}$$

(c)
$$f = \{(6,5), (1,8), (3,3)\}$$

(d)
$$f = \{(1,5),(3,5),(6,5)\}$$

(2) If $f: \mathbb{N} \to \mathbb{Z}$, then which of the following is true?

(a)
$$f(x) = \frac{1}{2}(2x+1)$$

(b)
$$f(x) = 2x^3 - 15x$$

(c)
$$f(x) = \sqrt{x+2}$$

(d)
$$f(x) = x + \sqrt{x}$$

VALUE OF THE FUNCTION

- If f is a function $f: A \rightarrow B$
- (i) f(a) is the value of the function at x = a, if $a \in A$ and the value of the function exist if $f(a) \in B$.
- (ii) f(a) is not the value of the function at x = a, if $a \not\in A$.

MCQ- 5:

If $f: \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = \sqrt{x}$$

then f(16) = ?

- (a) 16
- (b) 2
- (c) 4
- (d) 8

Solution:

$$f(x) = \sqrt{x}$$

$$f(16) = \sqrt{16}$$

The answer is (c).

MCQ-6:

If $f: \mathbb{Z} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} 2 + x^2 & when \ x \in \mathbb{Z}^+ \\ 5 - x & when \ x \in \mathbb{Z}^- \end{cases}$$

then f(-6) = ?

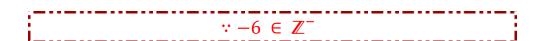
- (a) 49
- (b) 38
- (c) 0
- (d) 11

Solution:

 \mathbb{Z} : set of integers

 \mathbb{Z}^- : set of negative integers

 \mathbb{Z}^+ : set of positive integers



$$f(x) = 5 - x$$

$$f(-6) = 5 - (-6)$$

$$= 11$$

The answer is (d).

MCQ- 7:

If
$$f: \mathbb{N} \to \mathbb{R}$$
 is defined by $f(x) = 8x^2 + 3$

then f(-2) = ?

(a) -29

- (b) 35
- (c) 0
- (d) None

Solution:

$$\mathbb{N} = \{1,2,3,4,\dots\}$$

$$\mathbb{N} \text{ is the set of natural numbers.}$$

 $-2
otin \mathbb{N}$ and $f: \mathbb{N} \to \mathbb{R}$

f(-2) does not exist.

The answer is (d).

MCQ- 8:

If $f: [-3,8] \to \mathbb{R}$ is defined by

$$f(x) = 2\sqrt{x} + 3$$

then f(9) = ?

(a) 3

(b) 21

(c) 9

(d) None

Solution:

$$f: [-3,8] \to \mathbb{R}$$

9 ∉ [-3,8]∴ f(9) does not exist.

The answer is (d).

EXERCISE-4

- (1) A function $f: [2,8] \to \mathbb{R}$ is defined by $f(x) = 2x^3 5$. What is the value of f(1)?
 - (a) 12
- (b) 6
- (c) -3
- (d) None
- (2) A function $f: [6,12] \rightarrow \mathbb{N}$ is defined by $f(x) = 2x^2 + 6x$. What is the value x when $f(x) = \frac{7}{2}$?
 - (a) -1/2

- (b) 5/2 (c) 3/2 (d) -7/2

(3)

$$f(x) = \begin{cases} 2x^2 - 1 & , & 2 \le x < 5 \\ x^2 + 2 & , & 5 \le x \le 10 \end{cases}$$

f(5) = ?

- (a) 27
- (b) 49 (c) 76
- (d) None



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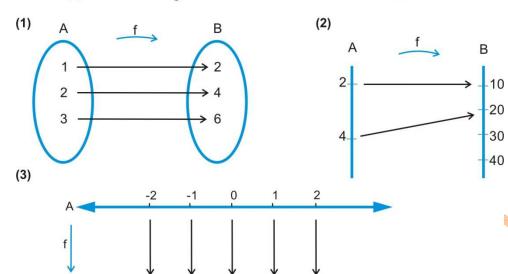
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ONE TO ONE FUNCTIONS

A function $f: A \to B$ is said to be one to one if $x_1 \neq x_2$ implies that $f(x_1) \neq f(x_2)$ such that $x_1, x_2 \in A$ and $f(x_1), f(x_2) \in B$.

Examples:

(a) Following functions are one to one functions.



3

(4) If $A = \{1, 2, 4\}$ and $B = \{1, 10, 20\}$ and $f = \{(1, 1), (2, 10), (4, 20)\}$

-6

-3

- (5) f(x) = 5x, $\forall x \in \mathbb{R}$
- (6) $f(x) = x^2 + 3x, \forall x \in \mathbb{R}$
- (b) Following functions, are not one one functions.
- $(1) \qquad f(x) = |x|$
- (2) f(x) = 6 (or any constant function)

MCQ- 9:

If $f: A \to B$ such that $A = \{2,6,8\}$ and $B = \{1,3,5,7,9\}$ then which of the following is one to one function?

(a)
$$f = \{(2,1),(8,3),(6,1)\}$$
 (b) $f = \{(2,3),(8,7),(6,5),(2,9)\}$ (c) $f = \{(2,1),(8,9)\}$ (d) $f = \{(2,1),(8,9),(6,5)\}$

Solution:

```
Option(d): {(2,1),(8,9),(6,5)}
```

f is a function and the images of 2,8 and 6 are different.

 $\therefore f$ is one to one.

The answer is (d).

Explanation:

```
Option (a): { (2,1),(8,3),(6,1)}
```

f is not one to one, the images of 2 and 6 are same.

Option (b): $\{(2,3), (8,7), (6,5), (2,9)\}$

f is not a function.

∴ 2 is repeated in the domain.

Option (c): {(2, 1), (8, 9)} (6, ?)

 \Rightarrow f(6) does not exist.

 $\therefore f$ is not a function.

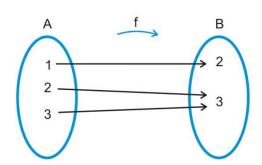
ONTO FUNCTIONS

A function $f:A\to B$ is said to be onto if for every $y\in B$ there is some $x\in A$ such that y=f(x).

Examples:

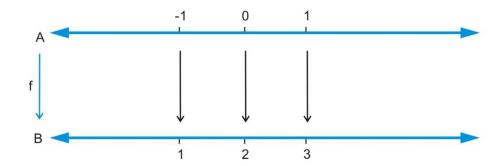
(A) Following functions are onto functions.

(1)



because each element of B is the image of some element of A, under f.

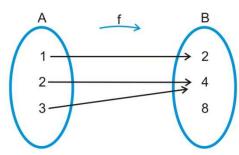
(2)



because each element of B is the image of an element of A under f.

(B) Following functions are not onto functions.

(1)



because $8 \in B$ is not the image of any element of A under f.

MCQ- 10:

If $f: A \to B$ such that $A = \{5,6,7\}$ and $B = \{2,4\}$ then which of the following is onto function?

(a)
$$f = \{(5,2), (6,4)\}$$

(a)
$$f = \{(5,2), (6,4)\}$$
 (b) $f = \{(5,2), (6,2), (7,4)\}$

(c)
$$f = \{(5,2), (6,4), (6,2)\}$$
 (d) $f = \{(5,2), (6,2), (7,2)\}$

d)
$$f = \{(5,2), (6,2), (7,2)\}$$

Solution:

Option (b) f is a function and onto.

Because all the elements in B are the images of the elements in A.

The answer is (b).

Explanation:

Option (a) and option (c) are not functions.

Option (d) is a function but not onto because for 4 ϵ B is not the image of any element in A.

EXERCISE-5



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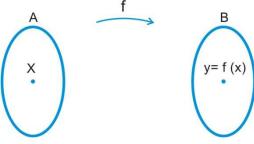
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COMPOSITE FUNCTIONS

Suppose that A, B and C are three sets. Let f be function A intoB

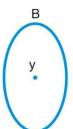
$$y = f(x), \ \forall x \in A \ where \ y \in B.$$

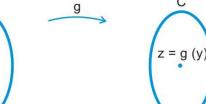
Geometrically it can be represented as



Let g be a function B into C, then

z = g(y), $\forall y \in B \text{ where } z \in C$



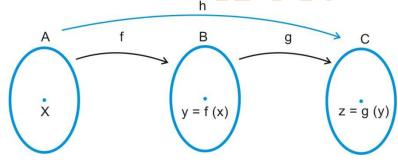


h is a new function A into C defined as

$$h(x) = g[f(x)]$$

is called composite function.

Geometrically it can be represented as



The composite function g[f(x)] is written as

 $(g_o f)(x)$

read "g composed with f".

Shortcut"

- (i) $f_o g = f(g)$
- (ii) $g_o f = g(f)$
- (iii) $f_o f = f(f)$

MCQ- 11:

If
$$f(x) = 5x - 3$$
 and $g(x) = 2x^2$, then $f \circ g = ?$
(a) $10x^2 - 3$ (b) $10x^3 - 6x^2$ (c) $2(5x - 3)^2$ (d) $2x^2 + 5x - 3$

Solution:

$$fog = f(g)$$

$$= 5g - 3 \qquad \text{{Replace } x \text{ with } g}$$

$$= 5(2x^2) - 3 \qquad \text{{Put the value of } g(x)}$$

$$= 10x^2 - 3$$

The answer is (a).



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POLYNOMIAL FUNCTIONS

A function is of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

is called polynomial function in x of degree n, where n is non-negative integer and the highest power of x, where $a_0, a_{n-1}, \ldots, a_1, a_0$ are real numbers.

Particular Polynomial Functions:

- (i) If $n = 0 \Rightarrow P(x) = a_0$ (constant function, degree 0)
- (ii) If $n = 1 \Rightarrow P(x) = a_1 x + a_0$ (linear function, degree 1)
- (iii) If $n = 2 \Rightarrow P(x) = a_2x^2 + a_1x + a_0$ (quadratic function, degree 2)

MCQ- 12:

Which of the following is a polynomial function?

- (a) $f(x) = 2x + \sqrt{x}$
- (b) $f(x) = \frac{3}{2}x^2 + 2x$
- (c) $f(x) = 5x^{\frac{2}{3}} + 2$
- (d) $f(x) = 2x^{-3} + 6x^2$

Solution:

Option(b): the powers of x are non-negative integers.

The answer is (b).

MCQ-13:

What is the degree of the polynomial function $f(x) = 5x^4 + 3x^2 + 9x$?

- (a) 5
- (b) 9
- (c) 4
- (d) 1

Solution;

The highest power of x is 4.

The degree of the polynomial function is 4.

The answer is (c).

EXERCISE-7

- (1) Which of the following is a polynomial function?
 - (a) f(x) = 5

- (b) $f(x) = 2\sqrt{x} + 3$
- (c) $f(x) = \frac{2}{x^2} + 7x$
- (d) $f(x) = 5x^{-6} + 6$

EVEN FUNCTIONS

A function is said to be even function, if

$$f(-x) = f(x)$$

ODD FUNCTIONS

A function is said to be odd function, if

$$f(-x) = -f(x)$$

Note:

- (1) $f(x) = g(x) + h(x) + t(x) + \cdots$
 - (i) f is even function if g, h, t ... are even functions.
 - (ii) f is odd function if g, h, t ... are odd functions.
- (2) f(x) = g(x).f(x)
 - (i) f is even function if g and h both are even or both are odd functions.
 - (ii) f is odd function if one of them g or h are odd function and other is even function.
- (3) $f(x) = \frac{g(x)}{f(x)}$
 - (i) f is even function if g and h both are even or both are odd functions.
 - (ii) f is odd function if one of them g or h are odd function and other is even function.
- (4) A constant function is an even function.

MCQ- 14:

Which of the function is an odd function?

(a)
$$f(x) = x^3 + 5$$

(b)
$$f(x) = 2x^2$$

$$(c) f(x) = 3x^3 + \sin x$$

(d)
$$f(x) = x^5 + \cos x$$

Solution:

 $f(x) = 3x^3 + \sin x$

$$f(-x) = 3(-x)^3 + \sin(-x)$$
$$= -3x^3 - \sin x$$
$$= -(3x^3 + \sin x)$$
$$= -f(x)$$

The answer is (c).

Shortcut:

Since x^3 and sinx are odd functions. So that f(x) is an odd function.

The answer is (c).

Explanation:

Option (a): There is a constant term 5, so it can not be an odd function.

Option (b): The power of x is an even number, which is 2, so it is not an odd function.

option (d): $cos(-x) = cosx \neq -cosx$ ∴ it is not an odd function.

Option (c): the power of x is odd number, that is 3 so $(-x)^3 = -x^3$ and sin(-x) = -sinx.So f is an odd function.

EXERCISE-8

(1) Which of the following is neither even nor odd function?

(a)
$$f(x) = 3x^3 + 2x$$

(a)
$$f(x) = 3x^3 + 2x$$
 (b) $f(x) = 2x^2 + 9$

(c)
$$f(x) = 5x^3 + 6$$
 (d) $f(x) = x^2 + \cos x$

(d)
$$f(x) = x^2 + \cos x$$

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