

Chapter 17

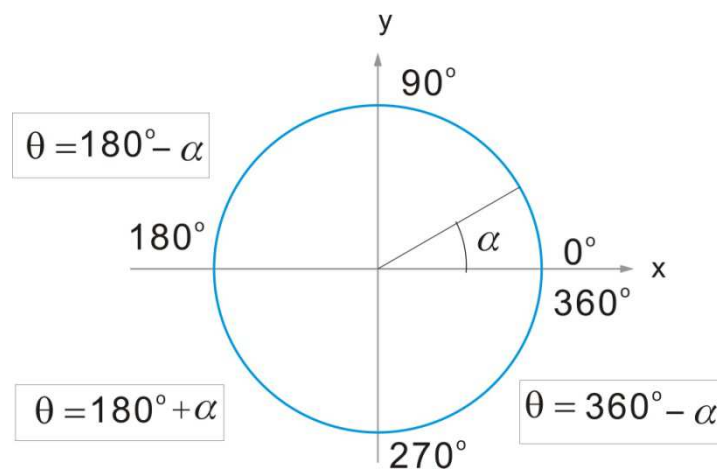
INVERSE TRIGONOMETRIC FUNCTIONS

DOMAIN AND RANGE

S. No.	Function	Domain	Range
1	$\sin^{-1}x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
2	$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
3	$\tan^{-1}x$	$(-\infty, +\infty)$ or \mathbb{R}	$(-\pi/2, \pi/2)$
4	$\operatorname{cosec}^{-1}x$	$(-\infty, -1] \cup [1, +\infty)$	$[-\pi/2, \pi/2] - \{0\}$
5	$\sec^{-1}x$	$(-\infty, -1] \cup [1, +\infty)$	$[0, \pi] - \{\pi/2\}$
6	$\cot^{-1}x$	$(-\infty, +\infty)$ or \mathbb{R}	$(0, \pi)$

EXERCISE-1

- (1) $f(x) = \sin^{-1}x$. What is the domain of f ?
- (a) $-1 < x < 1$ (b) $|x| < 1$ (c) $|x| \leq 1$ (d) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
- (2) $f(x) = \cos^{-1}x$. What is the range of f ?
- (a) $-\pi \leq x \leq \pi$ (b) $-1 \leq x \leq 1$ (c) $0 \leq x \leq 2\pi$ (d) $0 \leq x \leq \pi$
- (3) $f(x) = \tan^{-1}x$. What is the range of f ?
- (a) $0 \leq x < \frac{\pi}{2}$ (b) $-\frac{\pi}{2} < x < \frac{\pi}{2}$ (c) $-\infty < x < \infty$ (d) $0 < x < 2\pi$
- (4) $f(x) = \sin^{-1}x$. What is the range of f ?
- (a) $0 \leq x < 2\pi$ (b) $-1 < x < 1$ (c) $0 \leq x \leq \pi$ (d) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
- (5) $f(x) = \sec^{-1}x$. What is the domain of f ?
- (a) $(-\infty, \infty)$ (b) $(-\infty, -] \cup [1, \infty)$ (c) $[-1, 1]$ (d) $(-\infty, 0) \cup (0, \infty)$

SHIFTING THE BASIC ANGLE α TO 2nd, 3rd AND 4th QUADRANT**CALCULATION FOR ANGLES**

If p is a non-negative real number, θ is any angle and α is the basic angle (in first quadrant).

i) $\sin\theta = \pm p$

$\sin\theta = p$	$\sin\theta = -p$
θ is in the 1 st or 2 nd quadrant. $\alpha = \sin^{-1} p$ $\theta = \alpha, 180^\circ - \alpha$	θ is in the 3 rd or 4 th quadrant. $\alpha = \sin^{-1} p$ $\theta = 180^\circ + \alpha, 360^\circ - \alpha$

ii) $\cos\theta = \pm p$

$\cos\theta = p$	$\cos\theta = -p$
θ is in the 1 st or 4 th quadrant. $\alpha = \cos^{-1} p$ $\theta = \alpha, 360^\circ - \alpha$	θ is in the 2 nd or 3 rd quadrant. $\alpha = \cos^{-1} p$ $\theta = 180^\circ - \alpha, 180^\circ + \alpha$

iii) $\tan\theta = \pm p$

$\tan\theta = p$	$\tan\theta = -p$
θ is in the 1 st or 3 rd quadrant. $\alpha = \tan^{-1} p$ $\theta = \alpha, 180^\circ + \alpha$	θ is in the 2 nd or 4 th quadrant: $\alpha = \tan^{-1} p$ $\theta = 180^\circ - \alpha, 360^\circ - \alpha$

AUTHOR

M. MAQSOOD ALI

ASSISTANT PROFESSOR OF MATHEMATICS



FREE DOWNLOAD

ALL BOOKS AND CD ON MATHEMATICS

BY

M. MAQSOOD ALI

FROM WEBSITE

www.mathbunch.com

METHOD-2 (PARTICULARLY FOR CLASS XI):

The period of $\sin\theta$ and $\cos\theta$ is 2π and $\tan\theta$ is π .

$\sin\theta = p > 0$	$\cos\theta = p > 0$
$\theta = \sin^{-1}p, \pi - \sin^{-1}p$ and then add $2n\pi, n \in \mathbb{Z}$	$\theta = \pm \cos^{-1}p$ and then add $2\pi, n \in \mathbb{Z}$

$\tan\theta = p > 0$	$\tan\theta = -p < 0$
$\theta = \tan^{-1}p$ Calculate only one angle and then add $n\pi, n \in \mathbb{Z}$.	$\theta = \tan^{-1}(-p)$ Calculate only one angle and then add $n\pi, n \in \mathbb{Z}$.

Examples:

i) $\sin\theta = 0.5$

$$\theta = \sin^{-1}0.5 = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\text{Solution set: } \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\} ; n \in \mathbb{Z}$$

ii) $\cos\theta = 0.5$

$$\theta = \cos^{-1}0.5 = \pm \frac{\pi}{3}$$

$$\text{Solution set: } \left\{ \pm \frac{\pi}{3} + 2n\pi \right\} ; n \in \mathbb{Z}$$

iii) $\tan\theta = 1$

$$\theta = \tan^{-1}1 = \frac{\pi}{4}$$

$$\text{Solution set: } \left\{ \frac{\pi}{4} + n\pi \right\} ; n \in \mathbb{Z}$$

iv) $\tan\theta = -1$

$$\theta = \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\text{Solution set: } \left\{ -\frac{\pi}{4} + n\pi \right\} ; n \in \mathbb{Z}$$

MCQ- 3:

$\theta = ?$, if $\sin(3\theta/2) = 0$, where $0^\circ \leq \theta \leq 360^\circ$

- (a) $0^\circ, 120^\circ, 180^\circ$ (b) $0^\circ, 180^\circ, 300^\circ$
 (c) $0^\circ, 180^\circ, 360^\circ$ (d) $0^\circ, 120^\circ, 240^\circ$

Solution:

$$\begin{aligned}\frac{3\theta}{2} &= \sin^{-1} 0 \\ &= 0^\circ, 180^\circ - 0^\circ \\ &= 0^\circ, 180^\circ\end{aligned}$$

Since θ is multiplying by 3, so write another angle by adding 360° in first value that is 0° .

$$\frac{3\theta}{2} = 0^\circ, 180^\circ, 360^\circ + 0^\circ$$

$$\frac{3\theta}{2} = 0^\circ, 180^\circ, 360^\circ$$

$$\theta = 0^\circ, 120^\circ, 240^\circ$$

All the above angles lie between $0^\circ - 360^\circ$.

The answer is (d).

EXERCISE-2

(1) $\sin^{-1} \frac{1}{2} + \tan^{-1} 1 = ?$

- (a) 75° (b) 105° (c) 120° (d) 90°

(2) $\sin^{-1} \frac{1}{\sqrt{2}} + \cos^{-1} \frac{\sqrt{3}}{2} = ?$

- (a) 120° (b) 105° (c) 75° (d) 45°

(3) $\sec^{-1} 2 + \cot^{-1} 1 = ?$

- (a) 75° (b) 105° (c) 85° (d) 60°

(4) $\operatorname{cosec}^{-1} 2 + \sec^{-1} \frac{2}{\sqrt{3}} = ?$

- (a) 120° (b) 105° (c) 90° (d) 60°

- (5) $3 \operatorname{cosec}^{-1} 2 - 2 \operatorname{sec}^{-1} \sqrt{2} = ?$
 (a) 0° (b) 60° (c) 30° (d) 90°
- (6) $\cos \left\{ 5 \operatorname{sec}^{-1} \frac{2}{\sqrt{3}} - 2 \operatorname{cosec}^{-1} \frac{2}{\sqrt{3}} \right\} = ?$
 (a) $\frac{\pi}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$
- (7) $\sin (2 \operatorname{sec}^{-1} 2 - 2 \operatorname{cot}^{-1} 1) = ?$
 (a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$
- (8) $\cos (3 \cos^{-1} \frac{1}{2} - 3 \sin^{-1} \frac{1}{2}) = ?$
 (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) 0
- (9) What is the value of x if $\sin \frac{x}{2} = \frac{1}{2}$?
 (a) 30° (b) 300° (c) 240° (d) 370°
- (10) Given that $\tan \frac{x}{2} = 1$. What is the value of x ?
 (a) 380° (b) 710° (c) 810° (d) 150°
- (11) Given that: $2 \cos 2x = 1$. What is the value of x ?
 (a) 720° (b) 420° (c) 120° (d) 330°
- (12) Given that $\sin^{-1} \frac{\sqrt{3}}{2} = 60^\circ$.
 What is the value of x if $2 \sin 2x = -\sqrt{3}$?
 (a) 510° (b) 240° (c) 315° (d) 210°
- (13) Given that $\sin^{-1} 0.5 = 30^\circ$.
 What is the value of x if $2 \sin 5x + 1 = 0$?
 (a) 108° (b) 138° (c) 240° (d) 300°
- (14) Given that $\cos^{-1} \frac{1}{\sqrt{2}} = 45^\circ$.
 What is the value of x if $2 \cos^2 \frac{x}{2} - 1 = 0$?
 (a) 45° (b) 450° (c) 135° (d) 225°

SIMPLE AND INVERSE TRIGONOMETRIC FUNCTIONS**Formulae:**

$$(i) \quad \theta = \operatorname{cosec}^{-1}x \quad \Rightarrow \quad \theta = \sin^{-1}\frac{1}{x}$$

$$(ii) \quad \theta = \operatorname{sec}^{-1}x \quad \Rightarrow \quad \theta = \cos^{-1}\frac{1}{x}$$

$$(iii) \quad \theta = \operatorname{cot}^{-1}x \quad \Rightarrow \quad \theta = \tan^{-1}\frac{1}{x}$$

Shortcuts For $\sin\left(\cos^{-1}\frac{-\sqrt{3}}{2}\right)$, $\tan(\tan^{-1}\sqrt{3})$, $\cot(\tan^{-1}\sqrt{3})$ etc:

For the MCQs like $x = g(f^{-1}a) = ?$

where f and g are trigonometric functions.

Case-1: g and f are not same:

- i) Find basic angle α .
- ii) $x = \pm g(\alpha)$

Case-2: g and f are same :

$$x = a$$

Case-3: g is reciprocal of f:

$$x = 1/a$$

MCQ- 4:

$$\sin\left(\cos^{-1}\frac{-\sqrt{3}}{2}\right) = ?$$

- (a) $-\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) None

Solution: (Long Method)

$$\theta = \cos^{-1}\frac{-\sqrt{3}}{2}$$

Basic angle α

$$\alpha = \cos^{-1}\frac{\sqrt{3}}{2}$$

AUTHOR
M. MAQSOOD ALI
ASSISTANT PROFESSOR OF MATHEMATICS



FREE DOWNLOAD
ALL BOOKS AND CD ON MATHEMATICS
BY
M. MAQSOOD ALI
FROM WEBSITE

www.mathbunch.com

$$\begin{aligned}\pm \tan \alpha &= \pm \tan 45^\circ \\ &= \pm 1\end{aligned}$$

The answer is (b).

MCQ-6 :

$$\tan(\tan^{-1}\sqrt{3}) = ?$$

- (a) $-\sqrt{3}$ (b) -1 (c) $-\frac{1}{\sqrt{3}}$ (d) None

Solution:

$$\begin{aligned}\tan(\tan^{-1}\sqrt{3}) \\ &= \sqrt{3}\end{aligned}$$

The answer is (d).

MCQ- 7:

$$\cot(\tan^{-1}\sqrt{3}) = ?$$

- (a) $-\sqrt{3}$ (b) -1 (c) $-\frac{1}{\sqrt{3}}$ (d) None

Solution:

$\cot\theta$ is reciprocal of $\tan\theta$.

$$\begin{aligned}\cot(\tan^{-1}\sqrt{3}) \\ &= \frac{1}{\tan(\tan^{-1}\sqrt{3})} \\ &= \frac{1}{\sqrt{3}}\end{aligned}$$

The answer is (d).

MCQ- 8:

$$\sin(\operatorname{cosec}^{-1}\sqrt{2}) = ?$$

- (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) None

Solution:

$$\sin(\operatorname{cosec}^{-1}\sqrt{2})$$

$$= \sin(\sin^{-1} 1/\sqrt{2})$$

Note: $\theta = \operatorname{cosec}^{-1} \sqrt{2}$
 $\operatorname{cosec} \theta = \sqrt{2}$
 $\frac{1}{\sin \theta} = \sqrt{2}$
 $\sin \theta = 1/\sqrt{2}$
 $\theta = \sin^{-1} 1/\sqrt{2}$

$$= 1/\sqrt{2}$$

The answer is (c).

EXERCISE-3

(1) $\tan(\sin^{-1} 0.5) = ?$

- (a) $1/\sqrt{2}$ (b) $1/2$ (c) $-\frac{1}{\sqrt{3}}$ (d) $-\sqrt{3}$

(2) $\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = ?$

- (a) $\frac{\sqrt{3}}{2}$ (b) $1/2$ (c) $1/\sqrt{2}$ (d) $-\frac{\sqrt{3}}{2}$

(3) $\sec(\cos^{-1} 0.5) = ?$

- (a) 2 (b) $1/2$ (c) $\frac{\sqrt{3}}{2}$ (d) $1/\sqrt{2}$

(4) $\tan(\cot^{-1} \sqrt{3}) = ?$

- (a) 1 (b) $1/\sqrt{2}$ (c) $1/\sqrt{3}$ (d) $\sqrt{3}$

SOLUTION OF EQUATIONS

Mostly MCQs "SOLUTION OF EQUATIONS" depends on that do you know the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ at $\theta = 0, \pi/6, \pi/4, \pi/3, \pi/2, \pi, 2\pi$.

MCQ-9 :

$x = ?$, if $\sin x + \cos x = -1$

- (a) 0 (b) 2π (c) $\pi/2$ (d) π

Solution:

Do not solve the equation.
Check four given option one by one.
Option (d): $x = \pi$

$$\begin{aligned} \sin\pi + \cos\pi & \\ &= 0 + (-1) \\ &= -1 \end{aligned}$$

The answer is (d).

MCQ- 10:

$x = ?$, if $\cos^2 x + \tan x = 3/2$

- (a) 2π (b) $\pi/4$ (c) $\pi/2$ (d) $\pi/6$

Solution:

Do not solve the equation.
Check four given option one by one.
Option (a): $x = 2\pi$

$$\begin{aligned} \cos^2 2\pi + \tan 2\pi & \\ &= 1 + 0 \\ &= 1 \neq 3/2 \end{aligned}$$

Option (d): $x = \pi/4$

$$\begin{aligned} \cos^2 \pi/4 + \tan \pi/4 & \\ &= \frac{1}{2} + 1 \end{aligned}$$

$$= \frac{3}{2}$$

The answer is (b).

EXERCISE-4

- (1) $\sin x - \cos x = 0$ then $x = ?$
(a) 0° (b) 30° (c) 45° (d) 90°
- (2) $\sqrt{2} \operatorname{cosec} x - \tan x = 1$. What is the value of x ?
(a) 60° (b) 0° (c) 30° (d) 45°
- (3) $\sec^2 x - \operatorname{cosec}^2 x = 0$
(a) 90° (b) 0° (c) 30° (d) 45°

AUTHOR

M. MAQSOOD ALI

ASSISTANT PROFESSOR OF MATHEMATICS



FREE DOWNLOAD

ALL BOOKS AND CD ON MATHEMATICS

BY

M. MAQSOOD ALI

FROM WEBSITE

www.mathbunch.com