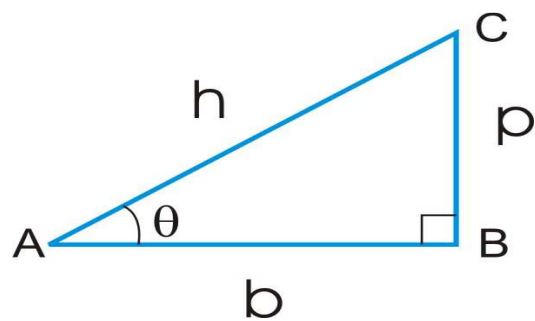


Chapter 16

SOLUTIONS OF TRIANGLES

TRIGONOMETRIC RATIOS



ABC is a rightangled triangle and $\hat{CAB} = \theta$

perpendicular: $BC = p$

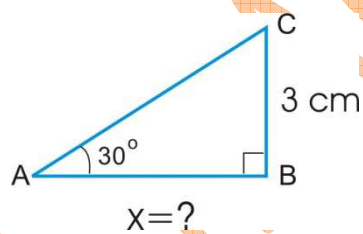
base: $AB = b$

hypotenuse: $AC = h$

The trigonometric ratios are

$$i) \sin \theta = \frac{p}{h} \quad ii) \cos \theta = \frac{b}{h} \quad iii) \tan \theta = \frac{p}{b}$$

MCQ- 1:



(a) $3\sqrt{3}$

(b) $\sqrt{3}$

(c) 6

(d) $\frac{1}{3\sqrt{3}}$

Solution:

$$b = x = ?$$

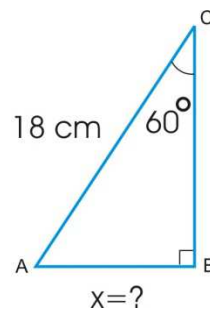
$$p = 3\text{cm}$$

$$\tan \theta = \frac{p}{b}$$

$$\begin{aligned}\tan 30^\circ &= \frac{3}{x} \\ \frac{1}{\sqrt{3}} &= \frac{3}{x} \\ x &= 3\sqrt{3} \text{ cm}\end{aligned}$$

The answer is (a).

MCQ-2 :



(a) $12\sqrt{3}$

(b) $18\sqrt{3}$

(c) $9\sqrt{3}$

(d) $\frac{6}{\sqrt{3}}$

Solution:

In this case perpendicular is x (opposite side to the angle 60°).

$$\begin{aligned}p &= x = ? \\ h &= 18\end{aligned}$$

$$\sin\theta = \frac{p}{h}$$

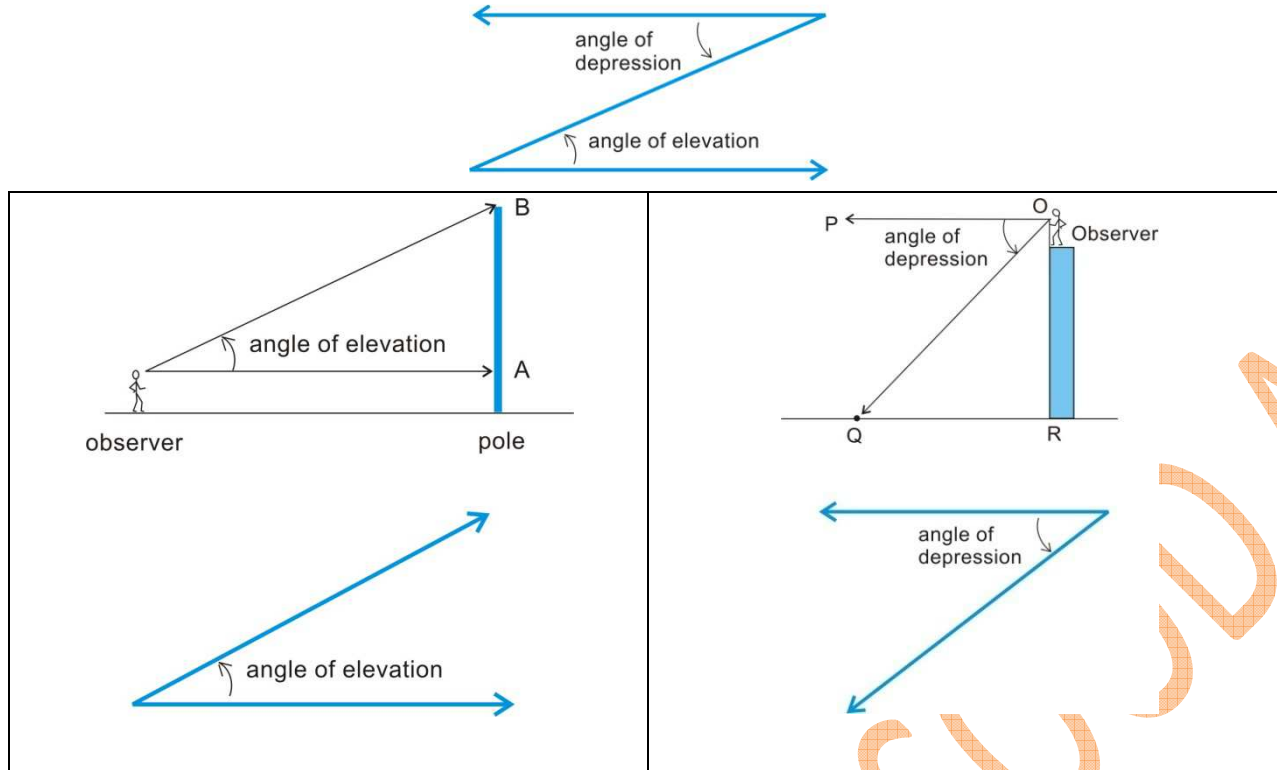
$$\sin 60^\circ = \frac{x}{18}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{18}$$

$$x = 9\sqrt{3}$$

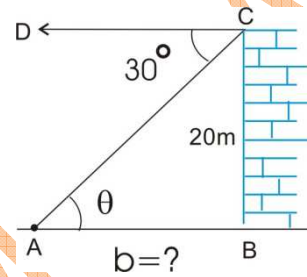
The answer is (c).

ANGLE OF ELEVATION AND DEPRESSION



MCQ-3 :

The angle of depression from the top of a building 20m high to a car on the ground is 30° . What is the distance between the building and the car?



- (a) $10\sqrt{3}$ m (b) 40 m (c) $20\sqrt{3}$ m (d) $\frac{40}{\sqrt{3}}$ m

Solution:

$\therefore \angle CAB = \angle ACD = 30^\circ$

$\therefore \theta = 30^\circ$

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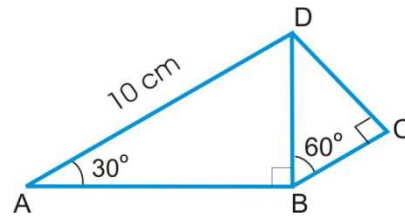
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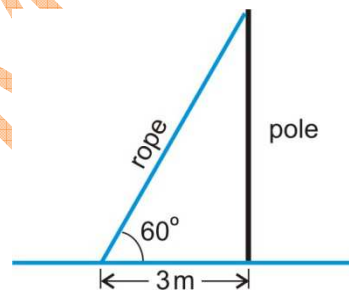
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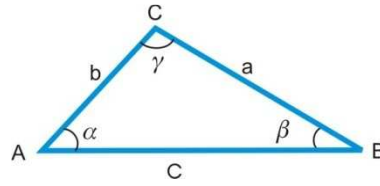
(4) $BC = ?$



- (a) 5cm (b) $\frac{5\sqrt{3}}{2}$ cm (c) 2.5cm (d) $5\sqrt{3}$ m
- (5) A ladder of length 6 m and angle 30° with the ground. What is the height of the top of the ladder from the ground?
 (a) $2\sqrt{3}$ m (b) 2 m (c) $3\sqrt{3}$ m (d) 3 m
- (6) A cyclist travels heading east 12 km and then 5 km heading south. How far is he from his initial position?
 (a) 10 km (b) 13 km (c) $\sqrt{119}$ km (d) 18 m
- (7) A tower is erected at 16 m from a point on the ground. The angle of elevation of top of tower from the point is 45° . What is the height, in metres, of the tower?
 (a) 12.5 (b) $16\sqrt{2}$ (c) $16/\sqrt{2}$ (d) 16
- (8) The angle of depression from the top of an apartment 70 feet high to the base of a house is 45° . how far is the house from the foot of the apartment?
 (a) $35\sqrt{3}$ ft (b) 35 ft (c) 70 ft (d) 100 ft
- (9) A vertical pole is supported by a rope makes an angle 60° . What is the length of the rope?



- (a) $3\sqrt{3}$ m (b) 3 m (c) 6 m (d) $3\sqrt{3}/2$ m

LAWS OF SINE

Suppose that α , β and γ are the angles opposite to the sides of lengths a , b and c respectively.

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

so that

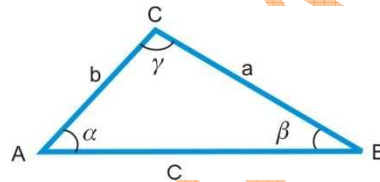
$$\text{i) } \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\text{ii) } \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\text{iii) } \frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

Condition:

This law is used when angle and the length of its opposite side are given.

LAWS OF COSINE

$$\text{i) } a^2 = b^2 + c^2 - 2bc \cos\alpha$$

$$\text{ii) } b^2 = a^2 + c^2 - 2ac \cos\beta$$

$$\text{iii) } c^2 = a^2 + b^2 - 2ab \cos\gamma$$

Conditions:

This law is used when

- i) Two sides and included angle are given.
- ii) Three sides are given.

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$$\theta = 180^\circ - 150^\circ$$

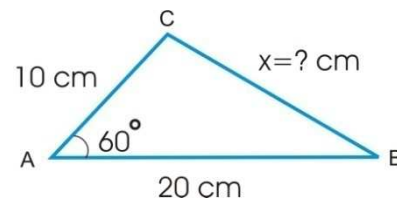
$$\theta = 30^\circ$$

$$\frac{x}{\sin 30^\circ} = \frac{12}{\sin 30^\circ}$$

$$x = 12 \text{ cm}$$

The answer is (c).

MCQ-6 :



(a) $18\sqrt{2}$

(b) 30

(c) 20

(d) $10\sqrt{3}$

Solution:

Use law of cosine, because two sides and included angle are given.

$$x^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \cos 60^\circ$$

$$= 500 - 200$$

$$= 300$$

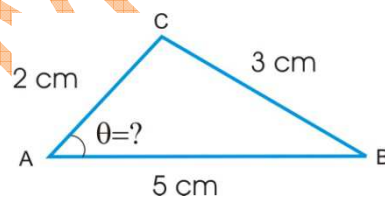
$$x = \sqrt{300}$$

$$= \sqrt{3 \times 100}$$

$$= 10\sqrt{3} \text{ cm}$$

The answer is (d).

MCQ- 7:



(a) 0°

(b) 30°

(c) 45°

(d) 60°

Solution:

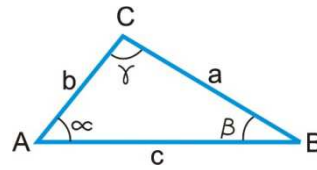
Use law of cosine, because three sides are given.

$$\cos\theta = \frac{5^2 + 2^2 - 3^2}{2 \times 5 \times 2}$$

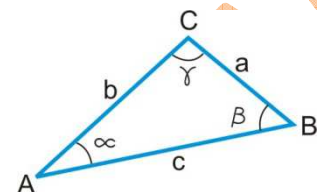
$$\cos\theta = 1$$

$$\theta = 0^\circ$$

The answer is (a).

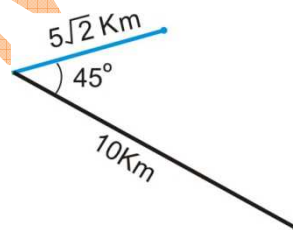
EXERCISE-2(1) $\alpha = ?$ 

(a) $\sin^{-1}\left(\frac{b}{a} \sin\beta\right)$ (b) $\frac{a}{b} \sin\beta$ (c) $\sin^{-1}\left(\frac{a}{b \sin\beta}\right)$ (d) $\sin^{-1}\left(\frac{a}{b} \sin\beta\right)$

(2) $\cos\beta = ?$ 

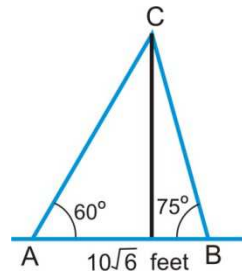
(a) $\frac{a^2 + b^2 + c^2}{2ac}$ (b) $\frac{a^2 + b^2 - c^2}{2ac}$ (c) $\frac{a^2 - b^2 + c^2}{2ac}$ (d) $\frac{2ac}{a^2 + b^2 - c^2}$

(3) Two cars start from the same point make an angle 45° . What is the distance between the cars if they cover distance of 10 km and $5\sqrt{2}$ km respectively?

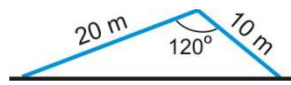


(a) 8 km (b) 22 km (c) 50 km (d) $5\sqrt{2}$ km

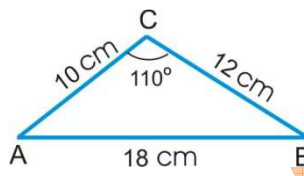
- (4) two ropes support a pole making angle 75° and 60° with the horizontal at a distance $10\sqrt{6}$ feet. What is the length of smaller rope?



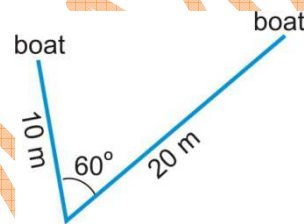
- (a) 15 ft (b) 30 ft (c) 35 ft (d) 20 ft
- (5) Two sides of length 10 m and 20 m are joined at an angle 120° . What is the distance between other two ends?



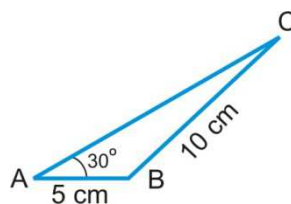
- (a) 17 m (b) $10\sqrt{7}$ m (c) $10\sqrt{3}$ m (d) 22 m
- (6) Which is the smallest angle of the triangle ABC? ($\sin 110^\circ = 0.9$)



- (a) 20° (b) 30° (c) 60° (d) 15°
- (7) Two boats are tied by two ropes, as shown in the figure. What is the distance between the boats?



- (a) 300 m (b) $10\sqrt{3}$ m (c) 30 m (d) 25 m
- (8) $\hat{A}CB = ?$



- (a) $\cos^{-1}0.25$ (b) $\sin^{-1}0.25$ (c) $\sin^{-1}0.5$ (d) $\cos^{-1}0.5$

AREA OF TRIANGLE

Area of a triangle is denoted by Δ .

There are three cases:

Case-1: Two sides and included angle are given:

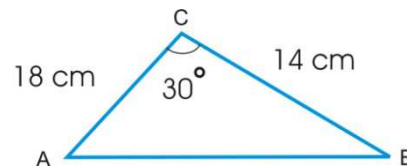
$$1) \Delta = \frac{1}{2} bc \sin \alpha$$

$$2) \Delta = \frac{1}{2} ac \sin \beta$$

$$3) \Delta = \frac{1}{2} ab \sin \gamma$$

MCQ- :

What is the area of the triangle ABC ?



(a) 48 cm^2

(b) 63 cm^2

(c) 84 cm^2

(d) 36 cm^2

Solution:

Two sides and included angle are given.

$$\begin{aligned} \Delta &= \frac{1}{2} \cdot 18 \cdot 14 \sin 30^\circ \\ &= 63 \text{ cm}^2 \end{aligned}$$

The answer is (b).

Case-2: One side and two angles are given:

$$(1) \Delta = \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha}$$

$$(2) \Delta = \frac{b^2 \sin \alpha \sin \gamma}{2 \sin \beta}$$

$$(3) \Delta = \frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma}$$

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MCQ- 9:

What is the area of the triangle ABC of lengths 5 cm, 7 cm and 10 cm?

- (a) $6\sqrt{31}$ (b) $7\sqrt{21}$ (c) $9\sqrt{23}$ (d) $2\sqrt{61}$

Solution:

Firstly, calculate s , by adding three sides and dividing by 2.

$$s = \frac{5+7+10}{2}$$

$$s = 11\text{cm}$$

Now calculate $s - a$, $s - b$ and $s - c$ and use formula case-3

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Delta = \sqrt{11 \cdot 6.4 \cdot 1}$$

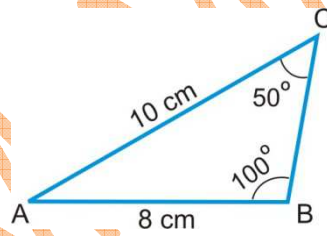
$$= \sqrt{264}$$

$$= 2\sqrt{61} \text{ cm}^2$$

The answer is (d).

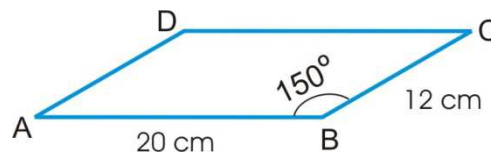
EXERCISE-3

- (1) What is the area of the triangle ABC?



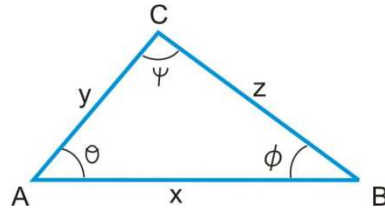
- (a) 35 cm^2 (b) 30 cm^2 (c) 20 cm^2 (d) 40 cm^2

- (2) What is the area of the parallelogram ABCD?

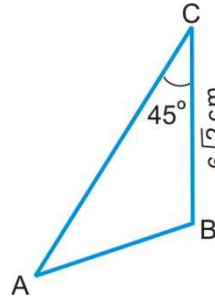


- (a) 60 cm^2 (b) 150 cm^2 (c) 208 cm^2 (d) 120 cm^2

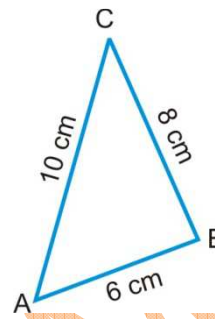
(3) What is the area of the triangle ABC?



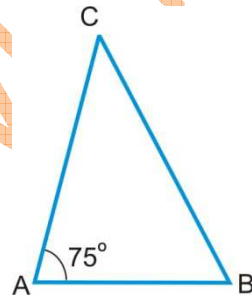
- (a) $\frac{1}{2} yz \sin\theta$ (b) $\frac{1}{2} xz \sin\psi$ (c) $\frac{1}{2} yz \sin\psi$ (d) $\frac{1}{2} xy \sin\phi$
- (4) The area of the triangle ABC is 60 cm^2 . What is the length of side \overline{AC} ?



- (a) 16 cm (b) 20 cm (c) 10 cm (d) 32 cm
- (5) What is the area of triangle ABC?

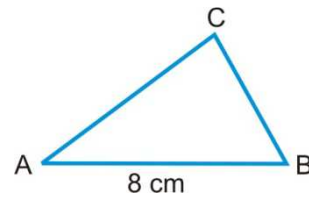


- (a) 24 cm^2 (b) 30 cm^2 (c) 18 cm^2 (d) 576 cm^2
- (6) What is the area of isosceles triangle ABC such that $AC=BC=12 \text{ cm}$?

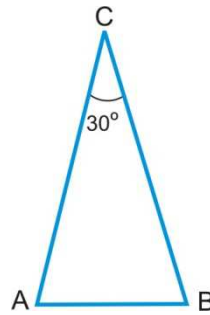


- (a) 64 cm^2 (b) 36 cm^2 (c) 24 cm^2 (d) 32 cm^2

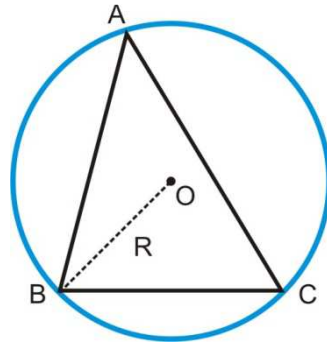
- (7) What is the area of equilateral triangle ABC whose length of a side is 8 cm?



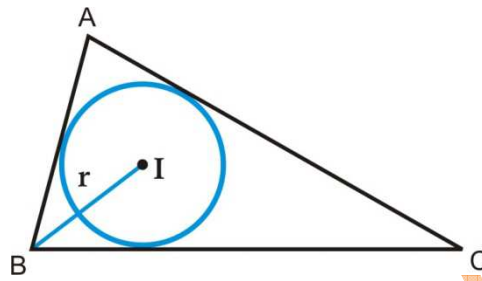
- (a) $32\sqrt{3} \text{ cm}^2$ (b) $20\sqrt{3} \text{ cm}^2$ (c) $12\sqrt{3} \text{ cm}^2$ (d) $16\sqrt{3} \text{ cm}^2$
- (8) What is the length of an equilateral triangle whose area is $36\sqrt{3} \text{ cm}^2$?
- (a) 12 cm (b) 8 cm (c) 9 cm (d) 18 cm
- (9) The area of an isosceles triangle ABC is 9 cm^2 . Given that $AC=BC$. What is the length of side \overline{BC} ?



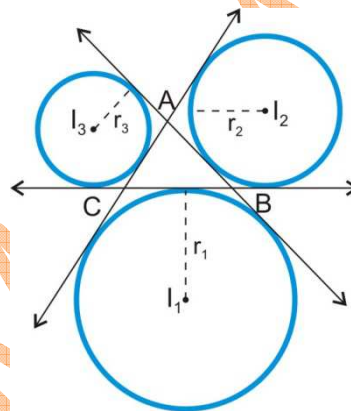
- (a) 12 cm (b) 6 cm (c) 9 cm (d) 15 cm
- (10) What is the length of a side of an equilateral triangle whose area is $36\sqrt{3} \text{ cm}^2$?
- (a) 18 cm (b) $15\sqrt{3} \text{ cm}$ (c) 12 cm (d) 6 cm

CIRCUM RADIUS

$$R = \frac{abc}{4\Delta}$$

IN-RADIUS

$$r = \frac{\Delta}{s}$$

ESCRIBED-RADII

$$i) r_1 = \frac{\Delta}{s-a}$$

$$ii) r_2 = \frac{\Delta}{s-b}$$

$$iii) r_3 = \frac{\Delta}{s-c}$$

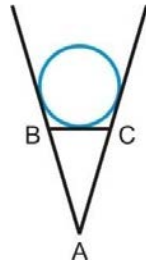
Note: All above formulae of radius are in term of Δ (area of triangle).

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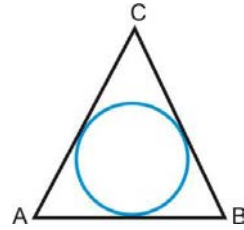


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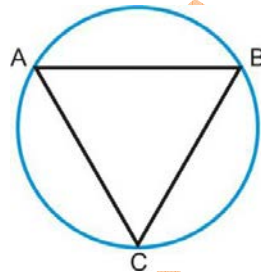
- (7) $AB=12$ cm, $BC=6$ cm, $AC=14$ cm and $\Delta ABC=36$ cm^2 . What is the radius in cm of the circle?



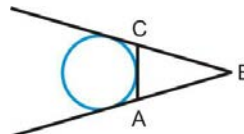
- (a) 2.25 (b) 9 (c) 3.6 (d) 7
- (8) $AB=3$ cm, $BC=4$ cm, $AC=5$ cm and $\Delta ABC=6$ cm^2 . What is the radius in cm of the circle?



- (a) 2.5 (b) 1 (c) 2 (d) 3
- (9) $AB=12$ cm, $BC=6$ cm, $AC=14$ cm and $\Delta ABC=36$ cm^2 . What is the radius in cm of the circle?



- (a) 2.25 (b) 9 (c) 3.6 (d) 7
- (10) $AB=3$ cm, $BC=4$ cm, $AC=5$ cm and $\Delta ABC=6$ cm^2 . What is the radius in cm of the circle?



- (a) 6 (b) 2.5 (c) 1 (d) 2

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