



Chapter 14**TRIGONOMETRIC IDENTITIES****BASIC TRIGONOMETRIC IDENTITIES**

The angle is same that is θ for all trigonometric functions given below.

- 1) $\sin^2\theta + \cos^2\theta = 1$ {Relation between $\sin^2\theta$ and $\cos^2\theta$.
 - i) $\sin^2\theta = 1 - \cos^2\theta$
 - ii) $\cos^2\theta = 1 - \sin^2\theta$
- 2) $1 + \tan^2\theta = \sec^2\theta$ {Relation between $\tan^2\theta$ and $\sec^2\theta$.
 - i) $\sec^2\theta - \tan^2\theta = 1$
 - ii) $\sec^2\theta - 1 = \tan^2\theta$
- 3) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ {Relation between $\cot^2\theta$ and $\operatorname{cosec}^2\theta$.
 - i) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
 - ii) $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

Note: Above three formulae are used when the power of trigonometric function is square.

RECIPROCAL OF TRIGONOMETRIC FUNCTIONS

- 1) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$
- 2) $\sec\theta = \frac{1}{\cos\theta}$
- 3) $\cot\theta = \frac{1}{\tan\theta}$ or $\cot\theta = \frac{\cos\theta}{\sin\theta}$
- 4) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

MCQ-1:

$$\sec^2\theta + \operatorname{cosec}^2\theta - \tan^2\theta = ?$$

- | | |
|--|------------------------------------|
| (a) $1 + \sec^2\theta$ | (b) $\operatorname{cosec}^2\theta$ |
| (c) $1 + \operatorname{cosec}^2\theta$ | (d) $1 - \tan^2\theta$ |

Solution:

$$\sec^2\theta + \operatorname{cosec}^2\theta - \tan^2\theta$$

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$$\begin{aligned} &= \frac{\cos\theta \cos\theta}{\sin\theta} \\ &= \cos\theta \cot\theta \end{aligned}$$

The answer is (d).

EXERCISE-1

- (1) $\sin^3 \theta - \frac{1}{\operatorname{cosec}\theta} = ?$
 (a) $\sin \theta \sec \theta$ (b) $\sin \theta \cos^2 \theta$ (c) $-\sin \theta \cos^2 \theta$ (d) $-\sin^3 \theta$
- (2) $\sec^2 \theta + \operatorname{cosec}^2 \theta = ?$
 (a) $\cot^2 \theta$ (b) $\sec^2 \theta \operatorname{cosec}^2 \theta$ (c) $\tan^2 \theta + \cot^2 \theta$ (d) $\sin \theta \cos \theta$
- (3) $\tan^2 \theta - \cot^2 \theta = ?$
 (a) $1 + \operatorname{cosec}^4 \theta$ (b) $\sin^2 \theta - \cos^2 \theta$ (c) $\tan^2 \theta \cot^2 \theta$ (d) $\sec^2 \theta - \operatorname{cosec}^2 \theta$
- (4) $\cot^4 \theta \operatorname{cosec}^4 \theta = ?$
 (a) $\cos^4 \theta \operatorname{cosec}^8 \theta$ (b) $\operatorname{cosec}^4 \theta$ (c) $\operatorname{cosec}^2 \theta \tan^2 \theta$ (d) $\cos^2 \theta - \sin^2 \theta$
- (5) $\cot^4 \theta - \operatorname{cosec}^4 \theta = ?$
 (a) $\operatorname{cosec}^2 \theta - 2$ (b) $1 + 2\cot^2 \theta$ (c) $1 - 2\operatorname{cosec}^2 \theta$ (d) $-1 - \cot^4 \theta$
- (6) $\frac{1}{\sec^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta} = ?$
 (a) $2\cos^2 \theta + 1$ (b) 1 (c) $1 - 2\sin^2 \theta$ (d) $\cot^2 \theta + \tan^2 \theta$
- (7) $\left(\frac{\sin^2 \theta + \cos^2 \theta}{\sec^2 \theta - \tan^2 \theta} \right) (\operatorname{cosec}^2 \theta - \cot^2 \theta) = ?$
 (a) $\sec^2 \theta \operatorname{cosec}^2 \theta$ (b) $\tan^4 \theta$ (c) $2\sin^2 \theta$ (d) 1
- (8) $1 + \cos^2 \theta = ?$
 (a) $2\sin^2 \theta$ (b) $\sin^2 \theta$ (c) $2 - \sin^2 \theta$ (d) 2
- (9) $\frac{\sin \theta}{\operatorname{cosec} \theta} + \frac{\cos \theta}{\sec \theta} = ?$
 (a) $2\sin^2 \theta$ (b) 1 (c) $\sin^2 \theta \cos^2 \theta$ (d) $2\sec^2 \theta \operatorname{cosec}^2 \theta$
- (10) $\sqrt{3\sec^2 \theta + \tan^2 \theta + 1}$
 (a) $2 \sec \theta$ (b) $\sec \theta + \tan \theta$ (c) $\sqrt{2} \tan \theta$ (d) $\sqrt{4 + \tan^2 \theta}$

COMPOUND ANGLE FORMULA

- 1) $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$
- 2) $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$
- 3) $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$
- 4) $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$
- 5) $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$
- 6) $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$

MCQ- 3:

$$\cos 5\theta \cos\theta - \sin 5\theta \sin\theta = ?$$

- (a) $\cos 4\theta$ (b) $\cos 6\theta$ (c) $\sin 4\theta$ (d) $\sin\theta \cos\theta$

Solution:

$$\cos 5\theta \cos\theta - \sin 5\theta \sin\theta$$

$$\cos\alpha \cos\beta - \sin\alpha \sin\beta = \cos(\alpha + \beta)$$

$$= \cos(5\theta + \theta)$$

$$= \cos 6\theta$$

The answer is (b).

MCQ- 4:

$$\sin 200^\circ \cos 140^\circ - \cos 200^\circ \sin 140^\circ = ?$$

- (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{5}$

Solution:

$$\sin 200^\circ \cos 140^\circ - \cos 200^\circ \sin 140^\circ$$

$$\sin\alpha \cos\beta - \cos\alpha \sin\beta = \sin(\alpha - \beta)$$

$$= \sin(200^\circ - 140^\circ)$$

$$= \sin 60^\circ$$

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- (5) $\cos 100^\circ \sin 70^\circ - \sin 100^\circ \cos 70^\circ = ?$
(a) 0 (b) 0.5 (c) -0.5 (d) $\sqrt{\frac{3}{2}}$
- (6) $\frac{1 + \tan 135^\circ \tan 45^\circ}{\tan 135^\circ - \tan 45^\circ} = ?$
(a) 0 (b) 1 (c) ∞ (d) -1
- (7) $\frac{1 - \tan 20^\circ \tan 25^\circ}{\tan 20^\circ + \tan 25^\circ} = ?$
(a) 0 (b) ∞ (c) 1 (d) -1
- (8) $\sin (30^\circ + 45^\circ) = ?$
(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{1+\sqrt{3}}{2\sqrt{2}}$
- (9) $\cos (45^\circ - 60^\circ) = ?$
(a) $\frac{3\sqrt{3}}{2\sqrt{2}}$ (b) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ (c) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{3}}$
- (10) $\cos (\alpha + \beta) + \cos (\alpha - \beta)$
(a) $-2 \sin \alpha \cos \beta$ (b) $2 \cos \alpha \sin \beta$ (c) $2 \cos \alpha \cos \beta$ (d) $2 \sin \alpha \sin \beta$

DOUBLE ANGLE FORMULAE

- 1) $\sin 2\alpha = 2 \sin \alpha \cos \alpha$
- 2) i) $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$
 ii) $\cos 2\alpha = 2\cos^2 \alpha - 1$
 iii) $\cos 2\alpha = 1 - 2\sin^2 \alpha$
- 3) $\tan 2\alpha = \frac{2\tan \alpha}{1 - \tan^2 \alpha}$

HALF ANGLE FORMULAE

- 1) $\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$
- 2) $\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$
- 3) $\tan \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$

PRODUCT INTO SUM FORMULAE

- 1) $\sin \alpha \cos \beta = \frac{1}{2} \{\sin(\alpha + \beta) + \sin(\alpha - \beta)\}$
- 2) $\cos \alpha \sin \beta = \frac{1}{2} \{\sin(\alpha + \beta) - \sin(\alpha - \beta)\}$
- 3) $\cos \alpha \cos \beta = \frac{1}{2} \{\cos(\alpha + \beta) + \cos(\alpha - \beta)\}$
- 4) $\sin \alpha \sin \beta = -\frac{1}{2} \{\cos(\alpha + \beta) - \cos(\alpha - \beta)\}$

SUM INTO PRODUCT FORMULAE

- 1) $\sin u + \sin v = 2 \sin \frac{u+v}{2} \cos \frac{u-v}{2}$
- 2) $\sin u - \sin v = 2 \cos \frac{u+v}{2} \sin \frac{u-v}{2}$
- 3) $\cos u + \cos v = 2 \cos \frac{u+v}{2} \cos \frac{u-v}{2}$
- 4) $\cos u - \cos v = -2 \sin \frac{u+v}{2} \sin \frac{u-v}{2}$

MCQ- 6:

$$2\sin 15^\circ \cos 15^\circ = ?$$

- (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

Solution:

$$2\sin 15^\circ \cos 15^\circ$$

$$2 \sin \alpha \cos \alpha = \sin 2\alpha$$

$$= \sin 2(15^\circ)$$

$$= \sin 30^\circ$$

$$= \frac{1}{2}$$

The answer is (b).

EXERCISE-3

- (1) $2\cos \alpha - \sin \alpha \sin 2\alpha = ?$
 (a) $2\cos^2 \alpha - \sin \alpha$ (b) $2\sin^3 \alpha$ (c) $2(\cos \alpha - \sin^2 \alpha)$ (d) $2\cos^3 \alpha$
- (2) $\sin^2(\alpha/2) + \frac{\cos \alpha}{2}$
 (a) $\frac{1}{2}$ (b) 1 (c) $\tan \alpha/2$ (d) $1 + \operatorname{cosec}^2 \alpha$
- (3) $\sqrt{1 + \cos 2\alpha} = ?$
 (a) $\sqrt{\cos^2 \alpha - \sin^2 \alpha}$ (b) $\sqrt{2}\cos \alpha$ (c) $\sqrt{1 + 2\cos^2 \alpha}$ (d) $\sqrt{2} \sin \alpha$
- (4) $2 \sin 15^\circ \cos 15^\circ = ?$
 (a) 0.25 (b) 0.5 (c) 0.87 (d) 0.17
- (5) $2 \sin 3\alpha \cos \alpha = ?$
 (a) $\cos 4\alpha + \cos 2\alpha$ (b) $\sin 4\alpha - \sin 2\alpha$
 (c) $\sin 4\alpha + \sin 2\alpha$ (d) $\cos 4\alpha - \sin 2\alpha$

- (6) $2 \cos 4\alpha \cos 2\alpha = ?$
- (a) $\cos 6\alpha - \cos 2\alpha$ (b) $\sin 6\alpha + \sin 2\alpha$
(c) $\sin 6\alpha - \sin 2\alpha$ (d) $\cos 6\alpha + \cos 2\alpha$
- (7) $\sin 5\alpha + \sin \alpha = ?$
- (a) $2 \cos 3\alpha \sin 2\alpha$ (b) $2 \sin 3\alpha \cos 2\alpha$
(c) $2 \sin 3\alpha \sin \alpha$ (d) $2 \cos 3\alpha \cos \alpha$
- (8) $\cos 3\alpha - \cos \alpha = ?$
- (a) $2 \sin 2\alpha \cos \alpha$ (b) $-2 \sin 2\alpha \sin \alpha$
(c) $2 \cos 2\alpha \cos \alpha$ (d) $-2 \sin^2 \alpha \cos \alpha$
- (9) $\tan \frac{\alpha}{2} = ?$
- (a) $\frac{\sin \alpha}{1 + \cos \alpha}$ (b) $\frac{1 + \cos \alpha}{\sin \alpha}$
(c) $\frac{2 \tan \alpha}{1 - \tan^2 \alpha}$ (d) $1 + \cot \frac{\alpha}{2}$
- (10) $\frac{1 - \cos \alpha}{1 + \cos \alpha} = ?$
- (a) $2 \cos^2 \frac{\alpha}{2}$ (b) $\tan^2 \frac{\alpha}{2}$ (c) $1 + \cot \alpha$ (d) $\cos \alpha - \sin \alpha$

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