

Chapter 13**FUNDAMENTALS OF TRIGONOMETRY****RADIANS AND DEGREE**

Relation between radians and degree

$$\pi \text{ radians} = 180^\circ$$

so that

$$(i) \quad 1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$(ii) \quad 1^\circ = \frac{\pi}{180} \text{ rad}$$

MCQ- 1:

$$210^\circ = ? \text{ rad}$$

(a) 3π

(b) 7π

(c) $7\pi/3$

(d) $7\pi/6$

Solution:

$$\begin{aligned} &= 210^\circ \times \frac{\pi}{180} \text{ rad} \\ &= \frac{7\pi}{3} \text{ rad} \end{aligned}$$

The answer is (c).

MCQ-2 :

$$\frac{8\pi}{3} = ? \text{ degree}$$

(a) 480°

(b) 240°

(c) 120°

(d) 540°

Solution:

$$\begin{aligned} &= \frac{8\pi}{3} \times \frac{180^\circ}{\pi} \\ &= 480^\circ \end{aligned}$$

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MCQ- 3:

45 minutes = ? radians

- (a) $3\pi/2$ (b) 45π (c) $5\pi/20$ (d) $\pi/240$

Solution:

$$\begin{aligned}
 & \boxed{45'} \\
 &= \frac{45^0}{60} \\
 &= \frac{3}{4} \times \frac{\pi}{180} \\
 &= \frac{\pi}{240}
 \end{aligned}$$

The answer is (d).

MCQ- 4:

300 seconds = ? rad

- (a) $\frac{\pi}{1240}$ (b) $\frac{\pi}{2160}$ (c) $\frac{\pi}{540}$ (d) $\frac{\pi}{326}$

Solution:

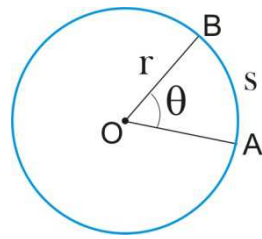
Firstly , convert 300 seconds into degree and than into radians.

$$\begin{aligned}
 & \boxed{300''} \\
 &= \frac{300^0}{3600} \\
 &= \frac{1^0}{12} \\
 &= \frac{1}{12} \times \frac{\pi}{180} \text{ rad} \\
 &= \frac{\pi}{2160} \text{ rad}
 \end{aligned}$$

The answer is (b).

EXERCISE-1

- (1) $80^\circ = \underline{\hspace{2cm}}$ radians.
(a) $\frac{3\pi}{8}$ (b) $\frac{2\pi}{5}$ (c) $\frac{4\pi}{9}$ (d) $\frac{7\pi}{9}$
- (2) $\frac{\pi}{10}$ radians = $\underline{\hspace{2cm}}$ degree.
(a) 24 (b) 6 (c) 18 (d) 9
- (3) 20 minutes = ? degree
(a) $\frac{1}{3}$ (b) $\frac{1}{180}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$
- (4) 50 minutes = ? radians
(a) $\frac{\pi}{12}$ (b) $\frac{2\pi}{27}$ (c) $\frac{\pi}{216}$ (d) $\frac{5\pi}{18}$
- (5) 120 seconds = ? radians
(a) $\frac{\pi}{5400}$ (b) $\frac{2\pi}{3}$ (c) $\frac{2\pi}{1243}$ (d) $\frac{3\pi}{1021}$

Relation between s , r and θ 

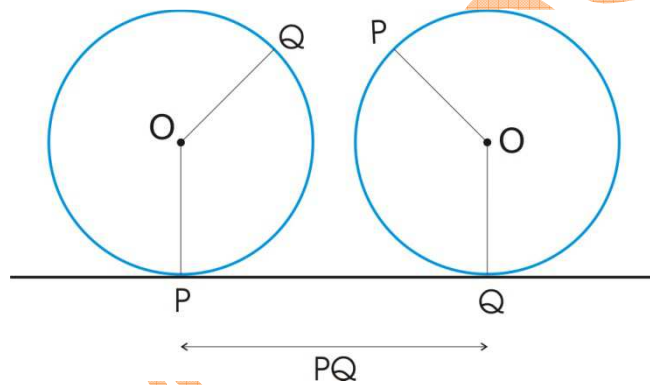
$$s = r\theta$$

where

s = arc length

r = radius

θ = angle (in radians)

Rotating circle on a horizontal line:

Point P is touching horizontal line. Rotate the circle on horizontal line till the point Q touches the line.

Arc length PQ = Linear distance PQ on the line

MCQ-5 :

What is the distance covered by a car if its wheel, radius 27 cm, turns 100° ?

- (a) 20π cm (b) $\frac{50\pi}{3}$ cm (c) 15π cm (d) $\frac{14\pi}{3}$ cm

Solution:

Angle θ must be in radians.

$$s = r\theta$$

$$= 27 \times 100 \times \frac{\pi}{180}$$

$$= 15\pi \text{ cm}$$

The answer is (c).

MCQ-6 :

What is the radius of the circle if an arc of length 12 cm subtended an angle 30° at the centre of the circle?

- (a) 2π cm (b) $\frac{72}{\pi}$ cm (c) $\frac{12}{\pi}$ cm (d) $\frac{6}{\pi}$ cm

Solution:

$$s = 12 \text{ cm}, \quad r = ?$$

Convert 30° into radians.

$$\theta = 30^\circ = \frac{\pi}{6} \text{ rad}$$

$$s = r\theta$$

$$12 = r \times \frac{\pi}{6}$$

$$r = \frac{72}{\pi} \text{ cm}$$

The answer is (b).

MCQ-7 :

What is the angle subtended an arc of length 320 cm at the centre of the circle of radius 1.6 metres?

- (a) 200° (b) 200 rad (c) 2° (d) 2 rad

Solution:

The units of length (cm) and radius (m) are not same, so

$$s = 320 \text{ cm} = 3.2 \text{ m}$$

$$s = r\theta$$

$$3.2 = 1.6\theta$$

$$\theta = 2 \text{ rad}$$

The answer is (d).

EXERCISE-2

(1) Arc length = 10 cm , angle = 50° , radius = ? cm

- (a) 2π cm (b) $\frac{72}{\pi}$ cm (c) $\frac{12}{\pi}$ cm (d) $\frac{36}{\pi}$ cm

(2) What is the distance covered by a bicycle if its wheel of radius 1.5 feet makes three complete revolutions?

- (a) 15π feet (b) 6π feet (c) 12π feet (d) 9π feet

(3) What is the angle subtended an arc of length 150 cm at the centre of the circle of radius 2 metres?

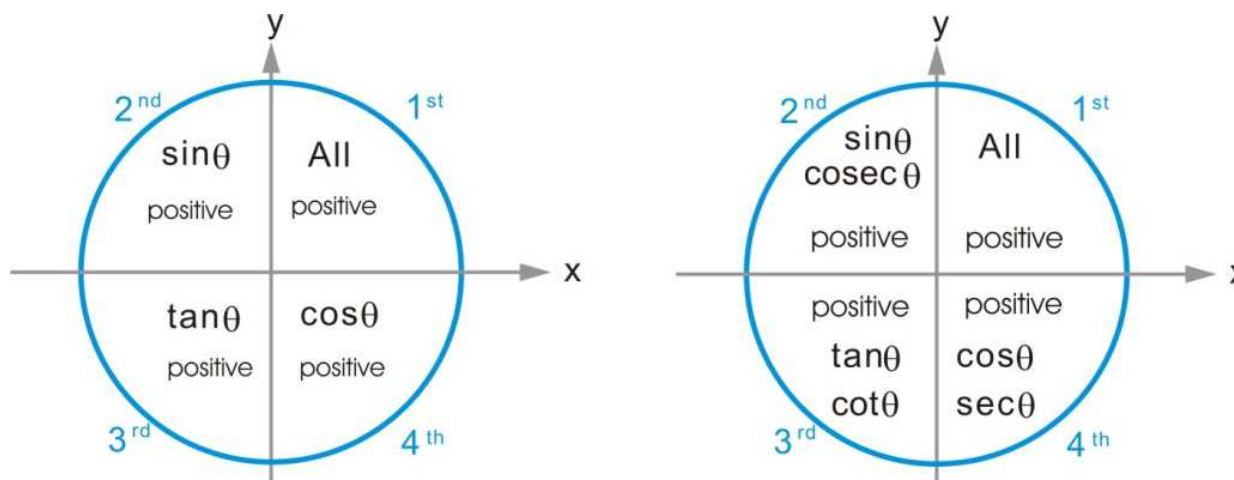
- (a) $\frac{3}{4}$ rad (b) 1.5 rad (c) 75 rad (d) 2 rad

(4) What is the radius of the circle if an arc of length 20 cm subtended an angle 60° at the centre of the circle?

- (a) $\frac{30}{\pi}$ cm (b) $\frac{60}{\pi}$ cm (c) $\frac{40}{\pi}$ cm (d) $\frac{1}{3}$ cm

THE SIGNS OF THE VALUES OF TRIGONOMETRIC FUNCTIONS

A circle is divided into four equal parts. Each part is called quadrant.



$$\text{Since } \operatorname{cosec} \theta = \frac{1}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \cot \theta = \frac{1}{\tan \theta}$$

Therefore:

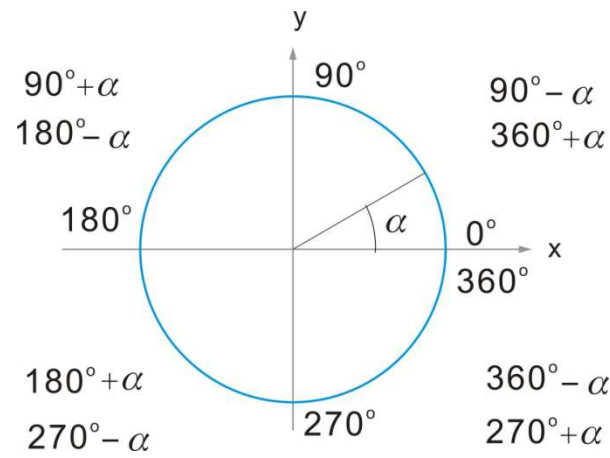
- in 1st quadrant, all trigonometric functions are positive.
- in 2nd quadrant, sin θ and cosec θ are positive.
- in 3rd quadrant, tan θ and cot θ are positive.
- in 4th quadrant, cos θ and sec θ are positive.

EXERCISE-3

- $\cos \theta = -\frac{\sqrt{3}}{2}$ and sin θ is negative. In which quadrant does $\rho(\theta)$ lie?
 (a) 1st (b) 2nd (c) 3rd (d) 4th
- $\cot \theta = 5$ and cos θ is negative. In which quadrant does $\rho(\theta)$ lie?
 (a) 1st (b) 2nd (c) 3rd (d) 4th
- $\operatorname{cosec} \theta = -2$ and tan θ is positive. In which quadrant does $\rho(\theta)$ lie?
 (a) 1st (b) 2nd (c) 3rd (d) 4th
- sin $\theta = 5$ and sec θ is positive. In which quadrant does $\rho(\theta)$ lie?
 (a) 1st (b) 2nd (c) 3rd (d) 4th
- $\operatorname{cosec} \theta = -5$ and sec θ is positive. In which quadrant does $\rho(\theta)$ lie?
 (a) 1st (b) 2nd (c) 3rd (d) 4th

α IS BASIC ANGLE

If α is the basic angle, so it is in the first quadrant.



$f(180^\circ \pm \alpha)$ and $f(360^\circ \pm \alpha)$ into $f(\alpha)$

180° and 360° are on x-axis.

The functions of \sin , \cos , and \tan at $(180^\circ \pm \alpha)$ and $(360^\circ \pm \alpha)$ will not change, only the sign will change according to the quadrant, when the angles $(180^\circ \pm \alpha)$ and $(360^\circ \pm \alpha)$ are shifted in first quadrant.

$\theta = (180^\circ - \alpha)$ $\rho(\theta)$ is in the 2 nd quadrant.	$\theta = 180^\circ + \alpha$ $\rho(\theta)$ is in the third quadrant
$\sin\theta$ is positive and $\cos\theta$ and $\tan\theta$ are negative. i) $\sin(180^\circ - \alpha) = \sin \alpha$ ii) $\cos(180^\circ - \alpha) = -\cos \alpha$ iii) $\tan(180^\circ - \alpha) = -\tan \alpha$	$\sin\theta$ and $\cos\theta$ are negative and $\tan\theta$ is positive. i) $\sin(180^\circ + \alpha) = -\sin \alpha$ ii) $\cos(180^\circ + \alpha) = -\cos \alpha$ iii) $\tan(180^\circ + \alpha) = \tan \alpha$

$\theta = 360^\circ - \alpha$ $\rho(\theta)$ is in the fourth quadrant	$\theta = 360^\circ + \alpha$ $\rho(\theta)$ is in the first quadrant
$\cos\theta$ is positive and $\sin\theta$ and $\tan\theta$ are negative. i) $\sin(360^\circ - \alpha) = -\sin \alpha$ ii) $\cos(360^\circ - \alpha) = \cos \alpha$ iii) $\tan(360^\circ - \alpha) = -\tan \alpha$	$\sin\theta$, $\cos\theta$ and $\tan\theta$ are positive. i) $\sin(360^\circ + \alpha) = \sin \alpha$ ii) $\cos(360^\circ + \alpha) = \cos \alpha$ iii) $\tan(360^\circ + \alpha) = \tan \alpha$

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MCQ- 8:

$\sin(720^\circ - \theta) = ?$

- (a) $-\tan \theta$ (b) $-\sin \theta$ (c) $-\cos \theta$ (d) $\cos \theta$

Solution:

$$\begin{aligned} & \sin(720^\circ - \theta) \\ &= \sin(2 \times 360^\circ - \theta) \\ &= -\sin \theta \end{aligned}$$

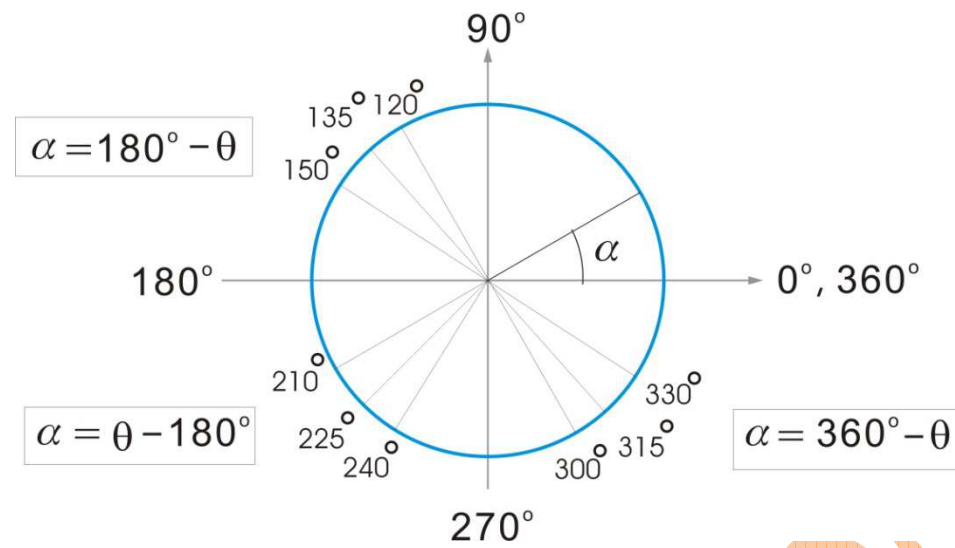
The answer is (b).

EXERCISE-4

- (1) $\sin(180^\circ - \theta) \cos(180^\circ + \theta) \tan(360^\circ - \theta) = ?$
 (a) $\sin^2 \theta$ (b) $\cos^2 \theta$ (c) $\sin \theta \cos \theta$ (d) $-\sin \theta \cos \theta$
- (2) $\sec(180^\circ + \theta) \cos(180^\circ - \theta) = ?$
 (a) $\tan \theta$ (b) $-\cot \theta$ (c) $-\tan \theta$ (d) 1
- (3) $\sin(90^\circ + \theta) \tan(270^\circ + \theta) = ?$
 (a) $\frac{\sin \theta}{\cos^2 \theta}$ (b) $\frac{-\cos^2 \theta}{\sin \theta}$ (c) $\sin \theta \cos^2 \theta$ (d) $-\sin^2 \theta \cos \theta$
- (4) $\sin(180^\circ + \theta) \cos(270^\circ + \theta) = ?$
 (a) $\sin^2 \theta$ (b) $\cos^2 \theta$ (c) $\sin \theta \cos \theta$ (d) $-\sin^2 \theta$
- (5) $\operatorname{cosec}(270^\circ + \theta) \tan(90^\circ + \theta) = ?$
 (a) $\sin^2 \theta$ (b) $-\sec \theta$ (c) $\operatorname{cosec} \theta$ (d) $\cot \theta$

VALUES OF TRIGONOMETRIC FUNCTIONS

(SHIFTING THE ANGLE IN FIRST QUADRANT)



Learn following values by heart.

$\sin 30^\circ = \frac{1}{2} = 0.5$	$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$	$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$
$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$	$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$	$\cos 60^\circ = \frac{1}{2} = 0.5$
$\tan 30^\circ = \frac{1}{\sqrt{3}}$	$\tan 45^\circ = 1$	$\tan 60^\circ = \sqrt{3}$

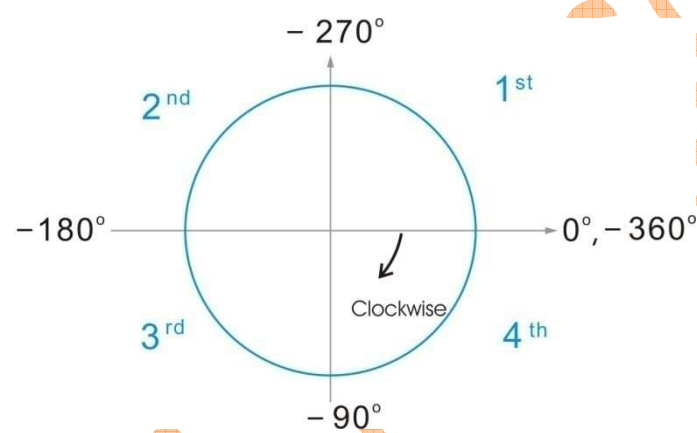
θ : in 2 nd quadrant	θ : in 3 rd quadrant	θ : in 4 th quadrant
$\alpha = 180^\circ - \theta$	$\alpha = \theta - 180^\circ$	$\alpha = 360^\circ - \theta$
$180^\circ - 120^\circ = 60^\circ$	$210^\circ - 180^\circ = 30^\circ$	$360^\circ - 300^\circ = 60^\circ$
$180^\circ - 135^\circ = 45^\circ$	$225^\circ - 180^\circ = 45^\circ$	$360^\circ - 315^\circ = 45^\circ$
$180^\circ - 150^\circ = 30^\circ$	$240^\circ - 180^\circ = 60^\circ$	$360^\circ - 330^\circ = 30^\circ$

Explanation:

Firstly write of the signs (+ or -) of the value according to the quadrant, and than shift the angle in 1st quadrant from any other quadrant.

For example

- i) $\cos 210^\circ = -\cos(210^\circ - 180^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$
- ii) $\cos 330^\circ = \cos(360^\circ - 330^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$
- iii) $\cos 120^\circ = -\cos(180^\circ - 120^\circ) = -\cos 60^\circ = -\frac{1}{2}$
- iv) $\sin 240^\circ = -\sin(240^\circ - 180^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$
- v) $\tan 150^\circ = -\tan(180^\circ - 150^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$

FOR NEGATIVE ANGLES

- i) $\sin(-\theta) = -\sin\theta$
- ii) $\cos(-\theta) = \cos\theta$
- iii) $\tan(-\theta) = -\tan\theta$

so that

- i) $\sin(-150^\circ) = -\sin 150^\circ = -\sin(180^\circ - 150^\circ) = -\sin 30^\circ = -\frac{1}{2}$
- ii) $\cos(-150^\circ) = \cos 150^\circ = -\cos(180^\circ - 150^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$
- iii) $\tan(-300^\circ) = -\tan 300^\circ = -\{-\tan(360^\circ - 300^\circ)\} = \tan 60^\circ = \sqrt{3}$

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MCQ-12 :

$\sin \frac{\theta}{2} = ?$ if $\sin \theta = \frac{3}{5}$, $\rho(\theta)$ is in the second quadrant.

- (a) $-\frac{3}{\sqrt{5}}$ (b) $\frac{4}{\sqrt{10}}$ (c) $\frac{4}{5}$ (d) $\frac{3}{\sqrt{10}}$

Solution:

$$\therefore \sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

First find the value of $\cos \theta$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\cos^2 \theta = 1 - \frac{9}{25}$$

$$= \frac{16}{25}$$

$$\cos \theta = \pm \frac{4}{5}$$

$\therefore \rho(\theta)$ is in the 2nd quadrant.

$$\cos \theta = -\frac{4}{5}$$

$\therefore \rho\left(\frac{\theta}{2}\right)$ is in the 1st quadrant.

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$= \sqrt{\frac{1 + \frac{4}{5}}{2}}$$

$$= \frac{3}{\sqrt{10}}$$

The answer is (d).

EXERCISE-6

- (1) $\sin \theta = -\frac{1}{5}$ and $\rho(\theta)$ is in the 4th quadrant. What is the value of $\tan \theta$?
- (a) $\frac{1}{2\sqrt{6}}$ (b) $-\frac{1}{2\sqrt{6}}$ (c) $-\frac{1}{\sqrt{26}}$ (d) $\frac{1}{\sqrt{26}}$
- (2) $\tan \theta = \frac{3}{2}$ and $\rho(\theta)$ is in the 3rd quadrant. What is the value of $\cos \theta$?
- (a) $\frac{-3}{\sqrt{5}}$ (b) $\frac{-2}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{13}}$ (d) $\frac{-2}{\sqrt{13}}$
- (3) $\cos \theta = \frac{-2}{3}$ and $\rho(\theta)$ is in the 2nd quadrant. What is the value of $\sin \theta$?
- (a) $\frac{\sqrt{5}}{3}$ (b) $\frac{-\sqrt{5}}{3}$ (c) $\frac{\sqrt{13}}{3}$ (d) $\frac{2}{\sqrt{13}}$
- (4) Given that $\cos \theta = \frac{-2}{7}$ and $\tan \theta = \frac{-3\sqrt{5}}{2}$, $\rho(\theta)$ is in the 2nd quadrant, what is the value of $\sin \theta$?
- (a) $\frac{7}{3\sqrt{5}}$ (b) $\frac{3\sqrt{5}}{7}$ (c) $\frac{-3\sqrt{5}}{7}$ (d) $\frac{5}{\sqrt{7}}$
- (5) Given that $\sin \theta = \frac{-3}{5}$ and $\rho(\theta)$ is in the 4th quadrant, what is the value of $\cot \theta$?
- (a) $\frac{-2}{\sqrt{6}}$ (b) $\frac{-\sqrt{34}}{3}$ (c) $\frac{-4}{5}$ (d) $\frac{-4}{3}$

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