



Chapter 12

BINOMIAL THEOREM

Case-1: $n \in \mathbb{N}$.

i) $(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + {}^nC_3 a^{n-3}b^3 + \dots + b^n$

ii) $(a - b)^n = a^n - {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 - {}^nC_3 a^{n-3}b^3 + \dots \pm b^n$

or

i) $(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!} a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3}b^3 + \dots + b^n$

ii) $(a - b)^n = a^n - na^{n-1}b + \frac{n(n-1)}{2!} a^{n-2}b^2 - \frac{n(n-1)(n-2)}{3!} a^{n-3}b^3 + \dots \pm b^n$

MCQ- 1:What is the first three terms of the expansion $(x - 2y)^9$?

- a) $x^9 - 18x^8y + 144x^7y^2$ b) $x^9 - 9x^8y + 144x^7y^2$
 c) $x^9 - 18x^8y + 288x^7y^2$ d) $x^9 + 9x^8y + 144x^7y^2$

Solution:

$$(x - 2y)^9$$

$$= x^9 - 9x^8(2y)^1 + \frac{9(9-1)}{2!} x^7(2y)^2$$

$$= x^9 - 9x^8(2y) + \frac{9 \cdot 8}{2} x^7(4y^2)$$

$$= x^9 - 18x^8y + 144x^7y^2$$

The answer is (a).

MCQ- 2:

What is the first three terms of the expansion $(x^2 + 3y)^{10}$?

- a) $x^{20} + 30x^{18}y + 990x^8y^2$ b) $x^{20} + 30x^{18}y + 135x^{16}y^3$
 c) $x^{20} + 30x^{18}y + 405x^{16}y^2$ d) $x^{20} + 30x^{18}y + 120x^8y^4$

Solution:

Note: Do not expand to three terms. There is only third term is different in each option.

There are two options to confirm the answer.

- i) Coefficients are different in four options. Calculate the coefficient only.
 ii) Powers of x and y are different in four options. Calculate the power of x and y .

$$\begin{aligned} & \frac{10 \cdot (10 - 1)}{2!} (x^2)^8 (3y)^2 \\ &= \frac{10 \cdot 9}{2} \cdot x^{16} \cdot 9y^2 \\ &= 405x^{16}y^2 \end{aligned}$$

The answer is (c).

EXERCISE-1

- (1) $(x^2 + y)^{10} = ?$ to three terms.
 (a) $x^{20} + 10x^{18}y + 40x^{16}y^2$
 (b) $x^{20} + 10x^{18}y + 45x^{16}y^2$
 (c) $x^{20} + 8x^{16}y + 20x^{12}y$
 (d) $x^{20} + 10x^{19}y + 90x^{18}y^2$
- (2) $(x^3 - y^2)^8 = ?$ to three terms.
 (a) $x^{24} + 8x^{21}y^2 + 32x^{20}y^4$
 (b) $x^{24} + 8x^{21}y^2 + 56x^{18}y^4$
 (c) $x^{24} + 8x^{21}y^2 + 48x^{18}y^3$
 (d) $x^{24} + 8x^{21}y^2 + 28x^{18}y^4$

$(r + 1)$ th TERM OF $(a + b)^n$

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

MCQ-3 :

What is the fifth term of $(2x + y)^{10}$?

- (a) $64 \cdot {}^{10} C_4 x^4 y^4$ (b) $32 \cdot {}^{10} C_6 x^6 y^6$
 (c) $32 \cdot {}^{10} C_5 x^5 y^5$ (d) $64 \cdot {}^{10} C_4 x^6 y^4$

Solution:

$$(2x + y)^{10}$$

$$a = 2x, b = y, n = 10$$

For fifth term

$$r = 5 - 1 = 4 \text{ \{because } r + 1 = 5\}$$

$$n - r = 10 - 4 = 6$$

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

$$= {}^{10} C_4 \cdot 2^6 \cdot x^6 \cdot y^4$$

$$= 64 \cdot {}^{10} C_4 x^6 y^4$$

The answer is (d).

Shortcut:

$$r = 4 \text{ and } n - r = 10 - 4 = 6$$

$n - r$: Power of a and r : Power of b

$$\because a = 2x \text{ and } b = y$$

$$\therefore \text{Power of } x \text{ is } 6 \text{ and } y \text{ is } 4.$$

The answer is (d).

EXERCISE-2

(1) What is the 3rd term of the expansion $(a^2 + 2b)^{10}$?

- (a) 0 (b) $4 \binom{10}{2} a^{12} b^6$ (c) $180 a^{16} b^2$ (d) $-75 a^{10} b^5$

(2) What is the 7th term of the expression $(a^3 + 2b^2)^9$?

- (a) $-\binom{9}{7} a^9 b^6$ (b) $64 \binom{9}{7} a^9 b^{12}$ (c) $64 \binom{9}{6} a^9 b^{12}$ (d) $-64 \binom{9}{7} a^9 b^{12}$

(3) What is the 5th term of the expansion $(x^2 + 2)^8$?

- (a) $16\binom{8}{4}x^6$ (b) $32\binom{8}{5}x^6$ (c) $-8\binom{8}{4}x^6$ (d) $32\binom{8}{6}x^{10}$

(4) What is the 4th term of the expansion $(1+3b^2)^{12}$?

- (a) $81\binom{12}{4}b^8$ (b) $27\binom{12}{3}b^6$ (c) $81\binom{12}{3}b^4$ (d) $9\binom{12}{4}b^6$

MIDDLE TERM

Middle term of $(a + b)^n$.

Case-1: n is even.

There is only one middle term.

$\left(\frac{n+2}{2}\right)$ th term is the middle term.

Case-2: n is odd:

There are two middle terms:

$\left(\frac{n+1}{2}\right)$ th and $\left(\frac{n+3}{2}\right)$ th terms are middle terms.

MCQ- 4:

What is the middle term of $(x + 2y^3)^8$?

- (a) (b) $16\binom{8}{4}x^4y^{12}$ (c) (d)

$$a = x, b = 2y^3, n = 8$$

$$\frac{n+2}{2}$$

$$= \frac{8+2}{2}$$

$$= \frac{10}{2}$$

$$= 5$$

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MCQ- 7:

Which term of the expansion $(6x^3 + 2/x^3)^8$ is independent of x ?

- (a) 3th (b) 4th (c) 5th (d) 6th

Solution:

$$a = 6x^3, \quad b = 2/x^3 = 2x^{-3}, \quad n = 8$$

$$n - r = 8 - r$$

$$3(8 - r) - 3r = 0$$

$$24 - 3r - 3r = 0$$

$$r = 4$$

$$T_{r+1} = T_{4+1} = T_5$$

The answer is (c).

EXERCISE-4

(1) What is the coefficient of x^6 in the expansion of $(1 + 2x^2)^6$?

- (a) $4\binom{5}{3}$ (b) $8\binom{6}{2}$ (c) $8\binom{6}{3}$ (d) $4\binom{6}{2}$

(2) What is the coefficient of x^{12} in the expansion of $(1 + 3x^3)^{10}$?

- (a) $27\binom{9}{3}$ (b) $81\binom{10}{4}$ (c) $27\binom{9}{4}$ (d) $81\binom{10}{3}$

(3) What is the term independent of x in the expansion of $(x + \frac{1}{x})^8$?

- (a) $\binom{8}{4}$ (b) $\binom{8}{3}$ (c) $\binom{7}{4}$ (d) $\binom{8}{5}$

(4) What is the term independent of x in the expansion of $(x^2 + \frac{2}{x^2})^6$?

- (a) $16\binom{6}{5}$ (b) $8\binom{5}{4}$ (c) $16\binom{6}{4}$ (d) $8\binom{6}{3}$

(5) Which term of the expansion $(x^2 + \frac{1}{x})^{20}$ involving x^7 ?

- (a) 9^{th} (b) 10^{th} (c) 11^{th} (d) 12^{th}

(6) Which term of the expansion $(x^3 + \frac{1}{2x^2})^{10}$ involving x^5 ?

- (a) 5^{th} (b) 6^{th} (c) 7^{th} (d) 8^{th}

Case-2: $n = \pm \frac{p}{q}$; ($q \neq 1, q \neq 0$) or n is negative integers:

$$(a) (1 + b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \dots$$

$$(b) (1 - b)^n = 1 - nb + \frac{n(n-1)}{2!}b^2 - \frac{n(n-1)(n-2)}{3!}b^3 + \dots$$

MCQ-8 :

What are the first three terms of $(1 + 3x^5)^{-8}$?

- (a) $1 - 24x^5 + 32x^{10}$ (b) $1 + 24x^5 + 108x^{10}$
 (c) $1 - 24x^5 + 324x^{10}$ (d) $1 - 8x^5 + 648x^{10}$

Solution:

$$(1 + 3x^5)^{-8}$$

$$= 1 + (-8)(3x^5) + \frac{(-8)(-8-1)}{2!}(3x^5)^2$$

$$= 1 + (-8) \cdot (3x^5) + \frac{(-8)(-9)}{2}(9x^{10})$$

$$= 1 - 24x^5 + 324x^{10}$$

The answer is (c).

EXERCISE-5

(1) $(1 + x)^{-6} = ?$ to three terms.

- (a) $1 - 6x + 21x^2$ (b) $1 - 6x + 15x^2$
 (c) $1 - 6x + 42x^2$ (d) $1 - 6x + 12x^2$

(2) $(1 - x)^{-8} = ?$ to three terms.

- (a) $1 + 6x + 24x^2$ (b) $1 + 4x + 48x^2$
 (c) $1 + 8x + 28x^2$ (d) $1 + 8x + 36x^2$

$(r + 1)$ th TERM OF $(1 + b)^n$

$$T_{r+1} = \frac{n(n-1)(n-2) \dots \{n-(r-1)\}}{r!} b^r$$

FIRST NEGATIVE TERM

For first negative term

$$n - (r - 1) < 0$$

$$n - r + 1 < 0$$

$$r > n + 1$$

Shortcut:

$$r = [n] + 1$$

Note: (i) $[x]$ is called "least integer function".

For example,

$$[3.05] = 4, [6.97] = 7, [54.2] = 55, [0.4] = 1$$

(ii) $\lfloor x \rfloor$ is called "greatest integer function".

For example,

$$\lfloor 3.05 \rfloor = 3, \lfloor 6.97 \rfloor = 6, \lfloor 54.2 \rfloor = 54, \lfloor 0.4 \rfloor = 0$$

MCQ-9 :What is the first negative term of $(1 + 2x^3)^{7/2}$?

(a) $-\frac{7}{8} x^{21}$

(b) $-\frac{7}{8} x^{18}$

(c) $-\frac{7}{8} x^{12}$

(d) $-\frac{7}{8} x^{15}$

Solution:**Note:** First negative term depends on

$$n(n-1)(n-2) \dots \{n-(n-r)\}$$

these are r factors.The term will be negative, when a factor is negative. This factor can be calculated by last the factor $\{n - (r - 1)\}$.

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EXERCISE-6

- (1) What is the first negative term of $(1 + 2x^3)^{5/2}$?
- (a) $-\frac{3}{7}x^{12}$ (b) $-\frac{5}{8}x^{12}$ (c) $-\frac{7}{8}x^{12}$ (d) $-\frac{2}{5}x^{12}$
- (2) Which term of the expansion $(1 + 5x^2)^{15/2}$ is the first negative term?
- (a) 8th (b) 9th (c) 10th (d) 11th

TERM INVOLVING x^m

Term involving x^m , ($m \in \mathbb{N}$) in the expansion of $(1 + x^k)^n$ can be found as

$$k r = m$$

MCQ-11 :

What is the term involving x^{12} in the expansion of $(1 + 2x^3)^{3/2}$?

- (a) $\frac{9}{32}x^{12}$ (b) $\frac{1}{18}x^{12}$ (c) $\frac{3}{8}x^{12}$ (d) $-\frac{3}{128}x^{12}$

Solution:

$$(1 + 2x^3)^{3/2}$$

$$3r = 12$$

$$r = 4$$

There are 4 factors in T_{r+1} formula when $b = 2x^3$

$$T_{4+1} = \frac{3}{2} \cdot \frac{1}{2} \cdot \frac{-1}{2} \cdot \frac{-3}{2} \cdot (2x^3)^4$$

$$T_5 = \frac{9}{4 \cdot 3 \cdot 2 \cdot 1} x^{12}$$

$$= \frac{3}{8} x^{12}$$

The answer is (c).

EXERCISE-7

- (1) What is the term involving x^6 in the expansion of $(1 + x^2)^{5/2}$?
- (a) $\frac{5}{16}x^6$ (b) $\frac{3}{8}x^6$ (c) $\frac{7}{5}x^6$ (d) $\frac{9}{10}x^6$
- (2) Which term of the expansion $(1 - 2x^5)^{-12/5}$ involving x^5 ?
- (a) 4th (b) 5th (c) 1st (d) 2nd

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