

Chapter 9**PROBABILITY**

If S is a finite sample space of equally likely outcomes and A is an event in S , the probability of A , denoted by $P(A)$, is

$$P(A) = \frac{n(A)}{n(S)}$$

TOTAL NUMBER OF OUT COMES

1 : Fair coin:



⇒

$$S = \{H, T\}$$
$$n(S) = 2$$

2 : Ordinary Die:



⇒

$$S = \{1, 2, 3, 4, 5, 6\}$$
$$n(S) = 6$$

3: Deck of cards:


Total cards = $n(S) = 52$


Total cards are divided into two colors.

Number of black cards = 26


Number of red cards = 26


Black cards are divided into two suits .

(a) clubs 

(b) spades 

Red cards are divided into two suits.

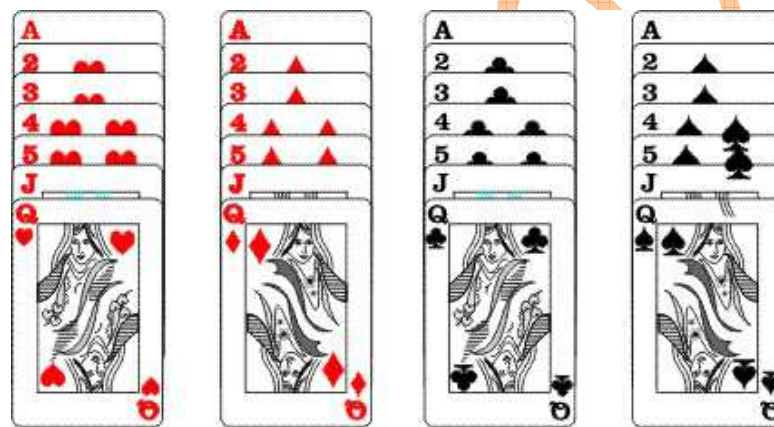
(a) diamond 

(b) heart 

Each suits contains 13 cards.

The out comes of each suits are

2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , 10 , J (jack) , Q (queen) , K (king) , A (ace).



SECTION A

Required outcome is exactly one, so do not use combination (nC_r).

Because ${}^nC_1 = n$

COIN IS TOSSED ONCE**OR****ONE COIN****MCQ-1 :**

A fair coin is tossed once. What is the probability of obtaining head?

- (a) 1 (b) $\frac{1}{2}$ (c) 0 (d) 2

Solution:

$$A = \{H\} \Rightarrow n(A) = 1$$

$$S = \{H, T\} \Rightarrow n(S) = 2$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(H) = \frac{1}{2}$$

COIN IS TOSSED TWICE**OR****TWO COINS ARE TOSSED**

Total outcomes:

$$S = \{HH, HT, TH, TT\}$$

Total Number of outcomes:

$$n(S) = 2^2 = 4$$

MCQ-2 :

A coin is tossed twice. What is the probability of one head and one tail?

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 0 (d) $\frac{3}{4}$

Solution:

$$A = \{HT, TH\} \Rightarrow n(A) = 2$$

$$n(S) = 2^2 = 4$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{2}{4} = \frac{1}{2}$$

The answer is (a).

A COIN IS TOSSED THRICE

Total Number of outcomes:

$$n(S) = 2^3 = 8$$

A COIN IS TOSSED FOUR TIMES

Total Number of outcomes:

$$n(S) = 2^4 = 16$$

A COIN IS TOSSED n TIMES

Total Number of outcomes:

$$n(S) = 2^n$$

EXERCISE-1

(1) A coin is tossed thrice. What is the probability of three heads?

- (a) -8 (b) $\frac{3}{2}$ (c) $\frac{1}{8}$ (d) $\frac{2}{3}$

(2) A coin is tossed twice. What is the probability of exactly one head?

- (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) 0.2

(3) A coin is tossed six times. What is the probability of getting all head?

- (a) $\frac{1}{64}$ (b) 3 (c) $\frac{3}{2}$ (d) $\frac{1}{6}$

A DIE IS ROLLED ONCE

Total outcomes:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Total Number of outcomes:

$$n(S) = 6$$

MCQ- 3:

A die is rolled once. What is the probability of getting number 5?

- (a) $\frac{5}{6}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{1}{6}$

Solution:

$$A = \{5\} \Rightarrow n(A) = 1$$

$$n(S) = 6$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{6}$$

A DIE IS ROLLED TWICE (OR) TWO DICE ARE ROLLED

Possibility diagram:

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

$$n(S) = 6^2 = 36$$

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PLAYING CARDS

Total number of outcomes:

$$n(S) = 52$$

MCQ-5 :

A card is drawn at random from a deck of 52 cards. What is the probability that the card is a king?

- (a) $\frac{4}{13}$ (b) $\frac{1}{26}$ (c) $\frac{1}{13}$ (d) $\frac{1}{52}$

Solution:

$$\text{Total king} = 4 \Rightarrow n(A) = 4$$

$$n(S) = 52$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{4}{52} \\ = \frac{1}{13}$$

The answer is (c).

EXERCISE-3

- (1) A card is drawn at random from a deck of 52 well shuffled cards. What is the probability that the card is a black queen?
- (a) $\frac{2}{13}$ (b) $\frac{1}{13}$ (c) $\frac{1}{52}$ (d) $\frac{1}{26}$
- (2) A card is drawn at random from a deck of 52 well shuffled cards. What is the probability that the card is a diamond.
- (a) $\frac{1}{4}$ (b) $\frac{1}{13}$ (c) $\frac{2}{13}$ (d) $\frac{1}{52}$
- (3) A card is drawn at random from a deck of 52 well shuffled cards. What is the probability that the drawing card is a king or jack?
- (a) $\frac{1}{8}$ (b) $\frac{1}{26}$ (c) $\frac{2}{13}$ (d) $\frac{1}{13}$
- (4) A card is drawn at random from a deck of 52 well shuffled cards. What is the probability that the drawing card is an ace, a king or a queen?
- (a) $\frac{1}{52}$ (b) $\frac{3}{26}$ (c) $\frac{1}{12}$ (d) $\frac{3}{13}$
- (5) A card is drawn at random from a deck of 52 cards. What is the probability the card is a queen or a red card?
- (a) $\frac{1}{13}$ (b) $\frac{15}{26}$ (c) $\frac{7}{13}$ (d) $\frac{1}{2}$
- (6) A card is drawn at random from a deck of 52 well shuffled cards. What is the probability the drawing card is a club or a jack?
- (a) $\frac{2}{13}$ (b) $\frac{1}{13}$ (c) $\frac{17}{52}$ (d) $\frac{4}{13}$

BALLS OF DIFFERENT COLORS**MCQ-6 :**

A bag has 5 green, 2 white and 3 black balls. If one ball is drawn at random. What is the probability that it is green ball?

- (a) $\frac{1}{5}$ (b) $\frac{1}{2}$ (c) $\frac{1}{10}$ (d) $\frac{5}{8}$

Solution:

$$\text{Total balls} = 5 + 2 + 3 = 10 \Rightarrow n(S) = 10$$

$$\text{Green balls} = 5 \Rightarrow n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{5}{10} \\ = \frac{1}{2}$$

The answer is (b).

EXERCISE-4

(1) A bag has 6 red and 3 green balls. A ball is drawn at random. What is the probability that the ball is green?

- (a) $\frac{3}{2}$ (b) $\frac{2}{9}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

(2) A box contains 3 white, 2 black and 5 red balls. A ball is drawn at random. What is the probability that the ball is white or black?

- (a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{1}{5}$ (d) $\frac{1}{2}$

$$P(A \cup B) = P(A) + P(B) \text{ , where } A \cap B = \emptyset$$

Note:

U : Use for (in words) "or"

MCQ- 7:

A die is rolled. What is the probability that the number comes up is an even number or 5?

(a) $\frac{2}{3}$

(b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) $\frac{1}{3}$

Solution:

Even numbers = 2, 4, 6

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

$$P(5) = \frac{1}{6}$$

$$P(A) = P(E) + P(5)$$

$$\begin{aligned} P(A) &= \frac{1}{2} + \frac{1}{6} \\ &= \frac{2}{3} \end{aligned}$$

The answer is (a).

Shortcut:

Out comes:

Even numbers = 2, 4, 6
or 5

$$A = \{2, 4, 6, 5\}$$

$$n(A) = 4$$

$$P(A) = \frac{P(A)}{P(S)}$$

$$\begin{aligned} P(A) &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

The answer is (a).

MCQ- 8:

A card is drawn from a deck of 52 cards. What is the probability that the card is a king or a queen?

- (a) $\frac{4}{13}$ (b) $\frac{1}{26}$ (c) $\frac{2}{13}$ (d) $\frac{1}{13}$

Solution:

$$P(K) = \frac{4}{52} = \frac{1}{13}$$

$$P(Q) = \frac{4}{52} = \frac{1}{13}$$

$$P(K \cup Q) = P(K) + P(Q)$$

$$= \frac{1}{13} + \frac{1}{13}$$

$$= \frac{2}{13}$$

The answer is (c).

Shortcut:

$$A = \{K, Q\} \Rightarrow n(A) = 8$$

$$n(S) = 52$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{8}{52}$$

$$= \frac{2}{13}$$

The answer is (c).

EXERCISE-5

- (1) A die is rolled. What is the probability that the number comes up is an odd number or 4?
 (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$
- (2) A card is drawn from a deck of 52 cards. What is the probability that the card is a ace or a jack?
 (a) $\frac{4}{13}$ (b) $\frac{1}{26}$ (c) $\frac{2}{13}$ (d) $\frac{1}{13}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B), \text{ where } A \cap B \neq \emptyset$$

$A \cap B \neq \emptyset$: Set A and set B are not disjoint.

MCQ-9 :

A die is rolled. What is the probability that the number comes up is an even or a prime number?

- (a) $\frac{5}{6}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{1}{2}$

Solution:

$$E = \{2, 4, 6\} \Rightarrow P(E) = \frac{3}{6} = \frac{1}{2}$$

$$P = \{2, 3, 5\} \Rightarrow P(P) = \frac{3}{6} = \frac{1}{2}$$

$$E \cap P = \{2\} \Rightarrow P(E \cap P) = \frac{1}{6}$$

$$P(E \cup P) = P(E) + P(P) - P(E \cap P)$$

$$\begin{aligned} P(E \cup P) &= \frac{1}{2} + \frac{1}{2} - \frac{1}{6} \\ &= \frac{5}{6} \end{aligned}$$

The answer is (a).

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TWO OBJECTS ONE BY ONE**Case-1: Replaced****MCQ-11 :**

A bag has 5 green, 2 white and 3 black balls. If one ball is drawn at random and replaced. Another ball is drawn, what is the probability that both the balls are green?

(a) $\frac{1}{4}$

(b) $\frac{3}{5}$

(c) $\frac{5}{12}$

(d) $\frac{2}{9}$

Solution:

Green balls = 5

White balls = 2

Black balls = 3

$$\text{Total balls} = 5 + 2 + 3 = 10$$

$$P(\text{both balls are green}) = P(\text{ball is green}) \cdot P(\text{ball is green after replacement})$$

$$P(G \cap G) = P(G) \cdot P(G)$$

$$P(G \cap G) = \frac{5}{10} \cdot \frac{5}{10}$$

$$= \frac{1}{4}$$

The answer is (a).

Case-2: Not replaced**MCQ-12 :**

A bag has 5 green, 2 white and 3 black balls. If one ball is drawn at random and not replaced. Another ball is drawn, what is the probability that both the balls are green?

(a) $\frac{1}{4}$

(b) $\frac{3}{5}$

(c) $\frac{5}{12}$

(d) $\frac{2}{9}$

Solution:

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EXERCISE-7

- (1) A card is drawn at random from a deck of well shuffled cards, and is not replaced. Another card is drawn, what is the probability that both cards are kings?
- (a) $\frac{1}{13 \times 17}$ (b) $\frac{3}{13 \times 17}$ (c) $\frac{2}{52^2}$ (d) $\frac{7}{51 \times 52}$
- (2) A card is drawn at random from a deck of 52 well shuffled cards and is replaced. Another card is drawn at random. What is the probability that both the cards are queens.
- (a) $\frac{1}{8}$ (b) $\frac{1}{169}$ (c) $\frac{2}{13}$ (d) $\frac{3}{26}$
- (3) A bag has 5 green, 2 white and 3 black balls. If two balls are drawn at random, one at a time and replaced. What is the probability that both the balls are black?
- (a) $\frac{3}{10}$ (b) $\frac{3}{5}$ (c) $\frac{9}{100}$ (d) $\frac{2}{3}$
- (4) A bag contains 6 red and 4 black balls. Two balls are drawn at random one by one without replacement. What is the probability that first ball is black and second green.
- (a) $\frac{4}{15}$ (b) $\frac{5}{90}$ (c) $\frac{3}{10}$ (d) $\frac{16}{15}$
- (5) Two members are chosen one by one, without replacement, out of 5 men and 4 women. What is the probability that first is man and second is women?
- (a) $\frac{7}{9}$ (b) $\frac{5}{18}$ (c) $\frac{8}{9}$ (d) $\frac{19}{8}$

SECTION B

**PROBABILITY USING COMBINATION
(TWO OBJECTS SIMULTANEOUSELY)**

Number of required outcomes two or more than two.

In this case use COMBINATION.

MCQ- 13:

Two cards are drawn at random from a deck of 52 cards. What is the probability that the cards are Jack or ace?

(a) $\frac{{}^8C_2}{{}^{52}C_2}$ (b) $\frac{{}^4C_2 \cdot {}^4C_2}{{}^{52}C_2}$ (c) $\frac{2 \cdot {}^4C_2}{{}^{52}C_2}$ (d) $\frac{{}^4C_1 \cdot {}^4C_1}{{}^{52}C_2}$

Solution:

No. of cards Jack and ace = 8

No. of drawing cards = 2

Total cards = 52

NOTE: The cards may be both Jack, both ace or one Jack and one ace.

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{{}^8C_2}{{}^{52}C_2}$$

The answer is (a).

MCQ- 14:

Two cards are drawn at random from a deck of 52 cards. What is the probability that the cards are one Jack and one ace?

- (a) $\frac{{}^8C_2}{{}^{52}C_2}$ (b) $\frac{2 \cdot {}^4C_2}{{}^{52}C_2}$ (c) $\frac{2 \cdot {}^4C_1}{{}^{52}C_2}$ (d) $\frac{{}^4C_1 \cdot {}^4C_1}{{}^{52}C_2}$

Solution:

No. of jack = 4
No of ace = 4

NOTE: One card is jack and one is ace, neither both are jack nor both ace.

$$P(A) = \frac{n(A)}{n(s)}$$

$$P(A) = \frac{{}^4C_1 \cdot {}^4C_1}{{}^{52}C_2}$$

The answer is (d).

MCQ-15 :

A bag has 6 red, 10 green and 12 blue balls. Six balls are drawn at random. What is the probability of 2 red and 4 blue balls?

- (a) $\frac{{}^{18}C_6}{{}^6C_2 \cdot {}^{12}C_4}$ (b) $\frac{{}^6C_2 + {}^{12}C_4}{{}^{28}C_6}$ (c) $\frac{{}^8C_6}{{}^{28}C_6}$ (d) $\frac{{}^6C_2 \cdot {}^{12}C_4}{{}^{28}C_6}$

Solution:

Total red balls = 6 , Required red balls = 2

Total blue balls = 12 , Required blue balls = 4

Total balls = 6 + 10 + 12 = 28

$$P(A) = \frac{n(A)}{n(s)}$$

$$P(A) = \frac{{}^6C_2 \cdot {}^{12}C_4}{{}^{28}C_6}$$

The answer is (d).

EXERCISE-8

- (1) Two cards are drawn simultaneously at random from a deck 52 well shuffled cards. What is the probability that the drawing cards are king or queen?

(a) $\frac{{}^8C_2}{{}^{52}C_2}$ (b) $\frac{{}^4C_2 \cdot {}^4C_2}{{}^{52}C_2}$ (c) $\frac{2 \cdot {}^4C_2}{{}^{52}C_2}$ (d) $\frac{{}^4C_1 \cdot {}^4C_1}{{}^{52}C_2}$

- (2) Two cards are drawn simultaneously from a deck of 52 well shuffled cards. What is the probability that the drawing cards are both king or both queen?

(a) $\frac{{}^8C_2}{{}^{52}C_2}$ (b) $\frac{{}^4C_2 \cdot {}^4C_2}{{}^{52}C_2}$ (c) $\frac{2 \cdot {}^4C_2}{{}^{52}C_2}$ (d) $\frac{{}^4C_1 \cdot {}^4C_1}{{}^{52}C_2}$

- (3) Two cards are drawn at random from a deck of 52 well shuffled cards. What is the probability of one king and one queen?

(a) $\frac{{}^8C_2}{{}^{52}C_2}$ (b) $\frac{{}^4C_2 \cdot {}^4C_2}{{}^{52}C_2}$ (c) $\frac{2 \cdot {}^4C_2}{{}^{52}C_2}$ (d) $\frac{{}^4C_1 \cdot {}^4C_1}{{}^{52}C_2}$

- (4) Three students are selected at random from 6 boys and 3 girls. What is the probability of selecting two boys and one girl?

(a) $\frac{2 \cdot {}^6C_2 \cdot {}^3C_1}{{}^9C_3}$ (b) $\frac{{}^6C_3}{{}^9C_3}$ (c) $\frac{{}^6C_2 \cdot {}^3C_1}{{}^9C_2}$ (d) $\frac{{}^6C_2 + {}^3C_1}{{}^9C_2}$

- (5) A bag has 5 red, 8 green and 9 blue balls. Five balls are drawn at random. What is the probability of 2 blue and 3 green balls?

(a) $\frac{{}^8C_2 \cdot {}^9C_3}{{}^{22}C_5}$ (b) $\frac{{}^8C_3 + {}^9C_2}{{}^{22}C_5}$ (c) $\frac{{}^8C_3 \cdot {}^9C_2}{{}^{22}C_3 \cdot {}^{22}C_2}$ (d) $\frac{{}^8C_3 \cdot {}^9C_2}{{}^{22}C_5}$

SECTION C

Some probabilities are given. To find the other probabilities with the help of given probabilities.

$$P(A \cap B) = P(A).P(B) \text{ and } P(A') = 1 - P(A)$$

\cap : in words "and"

A' : not in A

MCQ- 16:

The probability that Ali passes the test is 0.8. What is the probability he will fail in test?

- (a) 1 (b) 0.4 (c) 0.2 (d) 0.8

Solution:

$$P(\text{Ali passes the test}) = P(A) = 0.8$$

$$P(\text{Ali will not pass the test}) = P(A') = ?$$

$$P(A') = 1 - P(A)$$

$$P(A') = 1 - 0.8 \\ = 0.2$$

The answer is (c).

MCQ-17 :

The probability that Ali will solve a problem is $\frac{2}{7}$ and Sarim will solve it $\frac{1}{6}$. What is the probability that both will solve it?

- (a) $\frac{1}{42}$ (b) $\frac{1}{21}$ (c) $\frac{19}{42}$ (d) 1

Solution:

$$P(\text{Ali will solve the problem}) = P(A) = \frac{2}{7}$$

$$P(\text{Sarim will solve the problem}) = P(S) = \frac{1}{6}$$

$$P(\text{both solve the problem}) = P(A \cap S)$$

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EXERCISE-9

- (1) The probability that Ali passes each test is 0.6. What is the probability, he will fail in first two test?
- (a) $\frac{4}{5}$ (b) 0 (c) 0.16 (d) 0.8
- (2) The probability that Atif goes to school each day is 0.7. What is the probability that he will go to school on Monday and not Tuesday?
- (a) 0.3 (b) 0.4 (c) 1 (d) 0.21
- (3) The probability that team A wins a cricket test match is $\frac{2}{5}$ and not win is $\frac{1}{2}$. What is the probability that match will be drawn?
- (a) 0.5 (b) 1.5 (c) $\frac{1}{5}$ (d) 0.1

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