

Chapter 5

DEVERMINANUS

A is a matrix of order 3×3 :

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Determinant of A:

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

Properties of Determinants:

- (i) $|A| = |A^t|$
- (ii) (a) To interchange any two rows:
 - (b) To interchange any two columns:

Multiply the determinant by (-1).

Example: Interchanging R_1 and R_2 .

$$|A| = (-1) \begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix}$$

- (iii) (a) Two rows are identical:
 - (b) Two columns are identical:

The value of the determinant is zero.

Example:

$$|A| = \begin{vmatrix} a & b & c \\ a & b & c \\ g & h & i \end{vmatrix} = 0$$

because R_1 and R_2 are identical.

- (iv) (a) All elements of a row are zero:
 - (b) All elements of a column are zero:

The value of the determinant is zero.

Example:

$$|A| = \begin{vmatrix} 0 & 0 & 0 \\ d & e & f \\ g & h & i \end{vmatrix} = 0$$

because all elements of R_1 are zero.

- (v) (a) To take common from any row:
 - (b) To take common from any column:

Example:

(1)
$$|A| = \begin{vmatrix} ka & kb & kc \\ d & e & f \\ g & h & i \end{vmatrix} \Rightarrow |A| = k \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$
(2)
$$|A| = \begin{vmatrix} ka & b & c \\ kd & e & f \\ kg & h & i \end{vmatrix} \Rightarrow |A| = k \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

- (vi) (a) To multiply any row by a number k:
 - (b) To multiply any column by a number k:

Divide the determinant by k.

Example:

Multiplying R_1 by k and dividing the determinant by k.

$$|A| = \frac{1}{k} \begin{vmatrix} ka & kb & kc \\ d & e & f \\ g & h & i \end{vmatrix}$$

- (vii) (a) To add one row or two rows in a row:
 - (b) To add one column or two columns in a column:

Example: Adding C_2 and C_3 in C_1 .

$$|A| = \begin{vmatrix} a+b+c & b & c \\ d+e+f & e & f \\ g+h+i & h & i \end{vmatrix}$$

- (viii) (a) To subtract one row or two rows from any row:
 - (b) To subtract one column or two columns from any column:

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Solution:

(c): 2 is taken common from each element, which is wrong.

The answer is (c).

VALUE OF DETERMINANT

MCQ- 4:

$$\begin{vmatrix} 5 & 9 & 8 \\ 0 & 2 & 1 \\ 0 & 4 & 3 \end{vmatrix} = ?$$

- (a) 6
- (b) 10
- (c) 18
- 24

Solution:

$$5\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} - 0 + 0$$

$$= 5(6 - 4)$$

$$= 10$$

The answer is (b).

MCQ-5:

$$\begin{vmatrix} 2 & 5 & 2 \\ 6 & 0 & 0 \\ 3 & 4 & 1 \end{vmatrix} = ?$$

(a) 30

(b) 18

(c) -18 (d) -30

Solution:

$$= -6 \begin{vmatrix} 5 & 2 \\ 4 & 1 \end{vmatrix} + 0 - 0$$
$$= -6(5 - 8)$$
$$= 18$$

The answer is (b).

MCQ-6:

$$\begin{vmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{vmatrix} = ?$$
(a) 0 (b) 5 (c) 9

Solution:

Note: Multiply the elements which are in the diagonals.

$$3 \times 2 \times 4$$

$$= 24$$

The answer is (d).

(1)
$$3\begin{vmatrix} 4 & -3 \\ 1 & 2 \end{vmatrix} = ?$$

which of the following is not right?

(a)
$$\begin{vmatrix} 12 & -9 \\ 1 & 2 \end{vmatrix}$$
 (b) $\begin{vmatrix} 12 & -9 \\ 3 & 6 \end{vmatrix}$ (c) $\begin{vmatrix} 12 \\ 3 \end{vmatrix}$

(b)
$$\begin{vmatrix} 12 & -9 \\ 3 & 6 \end{vmatrix}$$

(c)
$$\begin{vmatrix} 12 & -3 \\ 3 & 2 \end{vmatrix}$$

(d)
$$\begin{vmatrix} 4 & -9 \\ 1 & 6 \end{vmatrix}$$

(2)
$$\begin{vmatrix} 20 & 10 \\ 5 & 15 \end{vmatrix} = 5|A|$$

which of the following is not equal to |A| = ?

(a)
$$\begin{vmatrix} 4 & 2 \\ 5 & 15 \end{vmatrix}$$

(b)
$$\begin{vmatrix} 4 & 10 \\ 1 & 15 \end{vmatrix}$$
 (c) $\begin{vmatrix} 20 & 10 \\ 1 & 3 \end{vmatrix}$ (d) $\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix}$

(d)
$$\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix}$$

$$(3) \begin{vmatrix} 4 & 3 & 2 \\ 8 & 1 & 4 \\ 20 & 6 & 10 \end{vmatrix} = ?$$

$$(c) -10$$

$$\begin{vmatrix} 3 & 8 & 5 \\ 0 & 1 & 2 \\ 0 & 3 & 4 \end{vmatrix} = ?$$

(a)
$$-25$$

(d)
$$-24$$

$$\begin{vmatrix}
5 & 0 & 0 \\
0 & 3 & 0 \\
0 & 0 & -2
\end{vmatrix} = 3$$

(b)
$$-30$$

(c)
$$-6$$

(7) $A = \begin{bmatrix} 3 & 8 \\ 6 & 2 \end{bmatrix}$ and |A| = -42 then $|A^t| = ?$

- (a) 0
- (b) 42
- (c) -42
- (d) None

(8) If $\det A = 5$, then $\det(A^t) = ?$

- (a) 5
- (b) 1/5
- (c) -5
- (d) None

(9) If |A| = 6, then $|A^t| =$ _____.

- (a) 1/6
- (b) 6
- (c) 1
- (d) None

(10) If |A| = 2, then $|A| \cdot \frac{1}{|A^t|}$. I = ?

- (a) I
- (b)4 I
- (c) 4
- (d) 1/4

 $(11)\begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 7 \end{vmatrix} = ?$

- (a) 39
- (b) 26
- (c) 0
- (d) None

(12) The value of $\begin{vmatrix} 8 & 5 & 4 \\ 14 & 9 & 7 \\ 12 & 6 & 6 \end{vmatrix}$ is _____.

- (a) 38
- (b) 50
- (c) 0
- (d) None

(13) The value of determinant of a unit matrix of order 6 is ______.

- (a) 6
- (b) 1
- (c) 0
- (d) None

SINGULAR AND NON SINGULAR MATRIX

Singular Matrix:

A square matrix X is called singular, if

$$|X| = 0$$

Non Singular Matrix:

A square matrix X is called non singular, if

$$|X| \neq 0$$

MCQ- 7:

What is the value of x if following matrix is singular

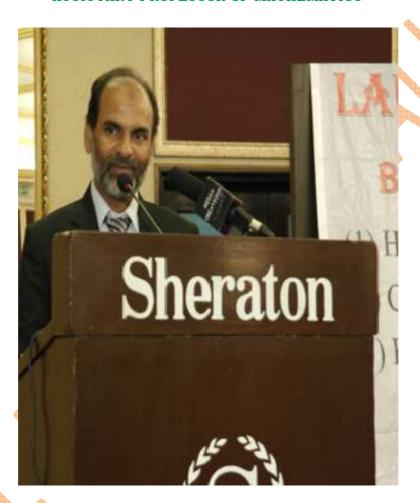
 $\begin{bmatrix} 5 & -6 \\ 10 & x \end{bmatrix}$?

- (a) 10
- (b) -12
- (c) 8
- (d) -30

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Let A be a matrix. The adjoint of A is

$$\text{Adjoint of } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

where A_{ij} are cofactors.

MCQ- 9:

$$\begin{bmatrix} -1 & 7 & 5 \\ 2 & 4 & -6 \\ 3 & -2 & 1 \end{bmatrix}$$

What is the cofactor of 2?

(b)
$$-10$$

(c)
$$-17$$

Solution:

 A_{12} is the cofactor of 2.

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 7 \\ -2 \end{vmatrix}$$
$$= (-1)(7+10)$$
$$= -17$$

The answer is (c).

EXERCISE-3

- (1) If $\begin{bmatrix} 3 & 2 & 1 \\ 5 & 4 & 6 \\ 8 & 9 & 7 \end{bmatrix}$, the cofactor of 5 is _____.
 - (a) -5
- (b) 5
- (c) 6
- (d) 9
- (2) If $\begin{bmatrix} 3 & -1 & 4 \\ 2 & -3 & -6 \\ 4 & 1 & 7 \end{bmatrix}$, the cofactor of -1 is _____.
 - (a) 38
- (b) -38
- (c) 1
- (d) None

INVERSE OF A MATRIX OF ORDER 2

If
$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
 then

$$B^{-1} = \frac{Adj B}{|B|}$$
$$= \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

MCQ-10:

What is the inverse of $B = \begin{bmatrix} 3 & -5 \\ 2 & 1 \end{bmatrix}$?

(a)
$$\frac{1}{13} \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$$

(b)
$$\frac{1}{15} \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$$

(c)
$$\frac{-1}{6} \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$$

(d)
$$\frac{1}{12}\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$$

Solution:

Note: Do not find the inverse of B. compare the four options, all the matrices are same except the scalars, which is multiplying with the matrix. So only find the determinant of the matrix.

$$\begin{vmatrix} 3 & -5 \\ 2 & 1 \end{vmatrix}$$
$$= 3 + 10$$
$$= 13$$

The answer is (a).

MCQ- 11:

What is the inverse of matrix $\begin{bmatrix} -3 & 2 \\ -8 & 6 \end{bmatrix}$?

(a)
$$\frac{1}{3}\begin{bmatrix} 3 & -8 \\ 2 & -6 \end{bmatrix}$$

(b)
$$\frac{1}{3} \begin{bmatrix} 3 & -2 \\ 8 & -6 \end{bmatrix}$$

(a)
$$\frac{1}{3}\begin{bmatrix} 3 & -8 \\ 2 & -6 \end{bmatrix}$$
 (b) $\frac{1}{3}\begin{bmatrix} 3 & -2 \\ 8 & -6 \end{bmatrix}$ (c) $\frac{1}{3}\begin{bmatrix} 3 & -2 \\ 8 & -6 \end{bmatrix}$ (d) $\frac{1}{3}\begin{bmatrix} 6 & -2 \\ 8 & -3 \end{bmatrix}$

(d)
$$\frac{1}{3}\begin{bmatrix} 6 & -2 \\ 8 & -3 \end{bmatrix}$$

Solution:

Note: The scalars multiplying with the matrices are same in each option that is $\frac{1}{3}$. So only find the adjoint of the matrix.

Adjoint of the matrix is

$$\begin{bmatrix} 6 & -2 \\ 8 & -3 \end{bmatrix}$$

The answer is (d).

EXERCISE-4

(1) If $O((A) = m \times n \text{ and } O(B) = m \times n$, then $(A - B)^2$

(a)
$$A^2 - AB - BA + B^2$$

(b)
$$A^2 - 2AB + B^2$$

(c)
$$A^2 - B^2$$

(d) None

(2) P, Q and R are matrices. P^{-1} is possible, if PQ = R then,

(a)
$$Q = RP^{-1}$$

(b)
$$Q = P^{-1}R$$

(a)
$$Q = RP^{-1}$$
 (b) $Q = P^{-1}R$ (c) $Q = R - P$ (d) None

(3) What is the value of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$?

(d) None

(4) $AA^{-1}\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} = ?$

(b)
$$\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 7 & -2 \\ -1 & 3 \end{bmatrix}$$
 (d) None

(5) $AIA^{-1} =$ ____, where I is unit matrix.

(d) None

(6) $\begin{bmatrix} 1 & 5 \\ 3 & 1 \end{bmatrix} P^{-1}P = ?$

(a)
$$\begin{bmatrix} 1 & 5 \\ 3 & 1 \end{bmatrix}$$

(a)
$$\begin{bmatrix} 1 & 5 \\ 3 & 1 \end{bmatrix}$$
 (b) $\begin{bmatrix} 1 & -5 \\ -3 & 1 \end{bmatrix}$ (c) I

(d) None

(7) If $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$, then $A^{-1} = ?$

(a)
$$-\frac{1}{7}\begin{bmatrix} -2 & 3\\ 3 & -4 \end{bmatrix}$$

(a)
$$-\frac{1}{7}\begin{bmatrix} -2 & 3\\ 3 & -4 \end{bmatrix}$$
 (b) $-\frac{1}{7}\begin{bmatrix} 4 & -5\\ -3 & 2 \end{bmatrix}$

(c)
$$-\frac{1}{7}\begin{bmatrix} 4 & -3 \\ -5 & 2 \end{bmatrix}$$
 (d) $-\frac{1}{7}\begin{bmatrix} -4 & -3 \\ -5 & -2 \end{bmatrix}$

(d)
$$-\frac{1}{7}\begin{bmatrix} -4 & -3 \\ -5 & -2 \end{bmatrix}$$

(8) If $A = \begin{bmatrix} 10 & 2 \\ 5 & 6 \end{bmatrix}$, then $A^{-1} = ?$

$$(a)\frac{1}{60}\begin{bmatrix} 6 & -2\\ -5 & 10 \end{bmatrix}$$
 $(b)\frac{1}{40}\begin{bmatrix} 6\\ -5 \end{bmatrix}$

$$(a) \frac{1}{60} \begin{bmatrix} 6 & -2 \\ -5 & 10 \end{bmatrix} (b) \frac{1}{40} \begin{bmatrix} 6 & -2 \\ -5 & 10 \end{bmatrix} (c) \frac{1}{30} \begin{bmatrix} 6 & -2 \\ -5 & 10 \end{bmatrix} (d) \frac{1}{50} \begin{bmatrix} 6 & -2 \\ -5 & 10 \end{bmatrix}$$

(9) If $A = \begin{bmatrix} 5 & 10 \\ 3 & 6 \end{bmatrix}$, then $A^{-1} = ?$

(a)
$$\begin{bmatrix} 6 & -10 \\ -3 & 5 \end{bmatrix}$$
 (b) $\begin{bmatrix} 6 & -2 \\ -1 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} \frac{6}{5} & -2 \\ \frac{-3}{5} & 1 \end{bmatrix}$ (d) None



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