





**HOW TO TEACH**  
**MATH MCQs**

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Graph of  $\sin x$

a  b 

c  d 

For Class XI

**Chapter 3****QUADRATIC AND CUBIC EQUATIONS****ROOTS OF A QUADRATIC EQUATION**

The value of  $x$  in a quadratic equation  $ax^2 + bx + c = 0$  is called the roots of the equation.

**Case-1: Coefficient of  $x^2$  is 1:****MCQ- 1:**

What are the roots of the equation  $x^2 + 5x - 14 = 0$ ?

- (a) 5, -1      (b) 14, -1      (c) -7, 2      (d) 2, -7

**Solution:****Long Method:**

$$\begin{aligned} x^2 + 5x - 14 &= 0 \\ x^2 + 7x - 2x - 14 &= 0 \\ x(x + 7) - 2(x + 7) &= 0 \\ (x - 2)(x + 7) &= 0 \\ \text{Either, } x - 2 = 0 &\text{ or } x + 7 = 0 \\ x = 2 &\text{ , } x = -7 \end{aligned}$$

The answer is (d).

**Shortcut:**

$$x^2 + 5x - 14 = 0$$

Break middle term  $5x$

$$7, -2$$

Change the signs and get the roots

$$x = -7, 2$$

The answer is (d).

**Case-2: Coefficient of  $x^2$  is not equal to 1:****MCQ-2:**

What are the roots of the equation  $4x^2 - 13x + 10 = 0$ ?

- (a)  $5, \frac{3}{2}$       (b)  $2, \frac{5}{4}$       (c)  $2, -4$       (d)  $\frac{1}{2}, \frac{7}{4}$

**Solution:****Long Method:**

$$\begin{aligned}
 4x^2 - 13x + 10 &= 0 \\
 4x^2 - 8x - 5x + 10 &= 0 \\
 4x(x - 2) - 5(x - 2) &= 0 \\
 (x - 2)(4x - 5) &= 0 \\
 \text{Either } x - 2 = 0 \text{ or } 4x - 5 &= 0 \\
 x = 2, & \quad x = \frac{5}{4}
 \end{aligned}$$

The answer is (b).

**Shortcut:**

$$4x^2 - 13x + 10 = 0$$

Break middle term  $-13x$

$$-8, -5$$

Change the sign and dividing by the coefficient of  $x^2$  (i.e. 4) to get the roots

$$x = \frac{8}{4}, \quad x = \frac{5}{4}$$

$$x = 2, \quad x = \frac{5}{4}$$

The answer is (b).

**FACTOR OF A QUADRATIC EXPRESSION****MCQ-3:**

What are the factors of the expression  $x^2 + 5x - 14$  ?

- (a)  $(x - 2)(x + 7)$                       (b)  $(x - 3)(x + 6)$   
 (c)  $(x - 6)(x + 8)$                       (d)  $(x - 1)(x + 6)$

**Solution:**

$$x^2 + 5x - 14 = 0$$

Break middle term  $5x$

$$7, -2$$

Change the signs and get the roots

$$x = -7, 2$$

Subtract both the roots from  $x$  and multiply them

$$(x - 2)(x - (-7))$$

$$(x - 2)(x + 7)$$

are the factors.

The answer is (a).

**MCQ-3:**

What are the factors of the expression  $4x^2 - 13x + 10 = 0$ ?

- (a)  $(x - 2)(4x - 5)$                       (b)  $(x - 1)(2x - 4)$   
 (c)  $(4x - 3)(x - 6)$                       (d)  $(x - 2)(4x - 5)$

**Solution:**

The roots of the equation  $4x^2 - 13x + 10 = 0$  are

$$x = 2 \text{ and } x = 5/4 \quad (\text{see example 2})$$

$$2, 5/4$$

Subtract both roots from  $x$  and multiply them

$$(x - 2)(x - 5/4)$$

Multiply second factor by 4

$$(x - 2)(4x - 5)$$

The answer is (a).

### COMPLETING THE SQUARE

#### Formulae:

$$(i) \quad a^2 + 2ab + b^2 = (a + b)^2$$

$$(ii) \quad a^2 - 2ab + b^2 = (a - b)^2$$

#### Lecture:

Fill in the blanks.

$$b^2 + 2ab + \underline{\hspace{2cm}} = (b + a)^2$$

$$b^2 + 2ab + \left(\frac{2a}{2}\right)^2 = (b + a)^2$$

$$b^2 + 2ab + a^2 = (b + a)^2$$

**Example:** Fill in the blanks  $x^2 + 10x + \underline{\hspace{2cm}} = (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})^2$ ?

**Solution:**

$$x^2 + 10x + \underline{\hspace{2cm}} = (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})^2$$

$$x^2 + 10x + \left(\frac{10}{2}\right)^2 = \left(x + \frac{10}{2}\right)^2$$

$$x^2 + 10x + 5^2 = (x + 5)^2$$

### EXERCISE-3.1

- (1) What are the roots of the equation  $x^2 - 8x - 20 = 0$ ?  
 (a) 4, -5      (b) 4, 5      (c) -2, 10      (d) -2, -6
- (2) What are the roots of the equation  $3x^2 - 10x + 8 = 0$ ?  
 (a) -1, 1/3      (b) 12, 2      (c) -2, -2/3      (d) 2, 4/3
- (3) What are the factors of the equation  $2x^2 + 5x - 18 = 0$ ?  
 (a)  $(x - 3)(2x + 9)$       (b)  $(x - 2)(2x + 9)$   
 (c)  $(x - 2)(2x + 1)$       (d)  $(2x + 1)(x - 9)$
- (4)  $x^2 - 6x + 9 = (x - \underline{\hspace{1cm}})^2$   
 (a) 3      (b) 2      (c) 6      (d) 9

(5)  $x^2 + 16x - 12 = (x + \underline{\quad ? \quad})^2 - 76$

(a) 12

(b) 64

(c) 16

(d) 8

(6)  $x^2 - 8x - 16 = (x - 4)^2 - \underline{\quad ? \quad}$

(a) 0

(b) 32

(c) 20

(d) 12

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**SOLUTIONS OF EQUATION****MCQ-4:**

$$x = ?, \text{ if } \sqrt{x+1} - \sqrt{x+13} = -2$$

- (a) 0                      (b) 1                      (c) 2                      (d) 3

**Solution:**

Do not solve the equation.

Put the value of  $x$ , from given four options one by one.

Option (d): For  $x = 3$

$$\begin{aligned} & \sqrt{3+1} - \sqrt{3+13} \\ &= \sqrt{4} - \sqrt{16} \\ &= 2 - 4 \\ &= -2 \end{aligned}$$

The answer is (d).

**MCQ-5:**

$$x = ?, \text{ if } 64^{2x} = 2$$

- (a)  $\frac{1}{6}$                       (b)  $\frac{1}{12}$                       (c)  $\frac{1}{2}$                       (d) 6

**Solution:**

$$64^{2x} = 2$$

On R.H.S: base = 2

On L.H.S: base = 64

Convert 64 into base 2.

$$64 = 2^6$$

$$(2^6)^{2x} = 2$$

$$2^{12x} = 2^1$$

$$12x = 1$$

$$x = 1/12$$

The answer is (b).

### EXERCISE-3.2

(1)  $x = ?$ , if  $2^{2x+1} + 4^x = 48$ .


- (a) 0                      (b) 1                      (c) 2                      (d) 3

(2)  $x = ?$ , if  $\sqrt{2x + 14} + \sqrt{x + 24} = 9$ .

- (a) 0                      (b) 1                      (c) 2                      (d) 3

(3)  $5^{2x+1} = 1$ , then  $x = ?$

- (a)  $5^{-\frac{1}{2}}$                       (b)  $-\frac{1}{2}$                       (c)  $-\frac{1}{2}$                       (d) 0

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**NATURE OF THE ROOTS****Lecture:**

The nature of roots of a quadratic equation

$$ax^2 + bx + c = 0$$

depend on discriminant

$$D = b^2 - 4ac$$

which is a part of quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The nature of the roots

- (i)  $D = 0$  , the roots are real and equal.
- (ii)  $D > 0$  , the roots are real and distinct (unequal).
- (iii)  $D < 0$  , the roots are complex and distinct (unequal).
- (iv)  $D$  is perfect square , the roots are rational and distinct, otherwise irrational and distinct.

**Roots are equal if  $D = 0$ :****MCQ-6:**

What is the nature of the roots of  $3x^2 - 6x + 3 = 0$  ?

- (a) real and equal
- (b) real and unequal
- (c) complex and unequal
- (d) irrational and unequal

**Solution:**

$$3x^2 - 6x + 3 = 0$$

$$a = 3 \quad b = -6 \quad c = 3$$

$$D = b^2 - 4ac$$

$$D = (-6)^2 - 4(3)(3)$$

$$D = 36 - 36$$

$$= 0$$

The answer is (a).

**MCQ-7:**

What is the value of  $m$  if the roots of the equation  $5x^2 - 2mx + 6 = 0$  are equal?

- (a)  $m < \pm\sqrt{30}$       (b)  $m > \pm\sqrt{30}$       (c)  $m = \pm\sqrt{30}$       (d) None

**Solution:**

$$5x^2 - 2mx + 6 = 0$$

$$a = 5, \quad b = -2m, \quad c = 6$$

$$b^2 - 4ac = 0$$

$$(-2m)^2 - 4 \cdot 5 \cdot 6 = 0$$

$$4m^2 - 120 = 0$$

$$m^2 = 30$$

$$m = \pm\sqrt{30}$$

The answer is (c).

**Roots are real and Distinct if  $D > 0$ :**

**MCQ-8:**


What is the nature of the roots of  $5x^2 - 7x + 1 = 0$ ?

- (a) real and equal      (b) rational and unequal  
(c) complex and unequal      (d) irrational

**Solution:**

$$5x^2 - 7x + 1 = 0$$

$$a = 5 \quad b = -7 \quad c = 1$$

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**MCQ-9:**

What is the value of  $m$  if the roots of the equation  $4x^2 - 6x + 3m = 0$  are real and unequal?

- (a)  $m > 2/3$       (b)  $m < 1/4$       (c)  $m < 2/3$       (d)  $m < 3/4$

**Solution:**

$$4x^2 - 6x + 3m = 0$$

$$a = 4, \quad b = -6, \quad c = 3m$$

$$D > 0$$

$$b^2 - 4ac > 0$$

$$(-6)^2 - 4(4)(3m) > 0$$

$$36 - 48m > 0$$

$$3 - 4m > 0$$

$$-4m > -3$$

$$4m < 3$$

$$m < 3/4$$

The answer is (d).

**Roots are complex and unequal if  $D < 0$ :****MCQ-10:**

What is the nature of the roots of the equation  $5x^2 - 8x + 4 = 0$  ?

- (a) real and equal    (b) rational and unequal  
(c) irrational and unequal                                      (d) complex and unequal

**Solution:**

$$5x^2 - 8x + 4 = 0$$

$$a = 5, b = -8, c = 4$$

$$D = b^2 - 4ac$$

$$\begin{aligned} D &= (-8)^2 - 4(5)(4) \\ &= 64 - 80 \\ &= -16 < 0 \end{aligned}$$

The answer is (d).

**MCQ-11:** What is the value of  $m$  if the nature of the roots of the equation

$$4x^2 - 2mx + 9 = 0 \text{ are complex and distinct?}$$

- (a)  $m < -6, m > 6$     (b)  $m > 6$   
(c)  $m < \pm 6$     (d)  $-6 < m < 6$

**Solution:**

$$4x^2 - 2mx + 9 = 0$$

$$a = 4, b = -2m, c = 9$$

$$b^2 - 4ac < 0$$

$$(-2m)^2 - 4(4)(9) < 0$$

$$4m^2 - 144 < 0$$

$$m^2 - 36 < 0$$

$$m^2 < 36$$

$$-6 < m < 6 \quad \{ \because m^2 = 36 \Rightarrow m = \pm 6$$

The answer is (d).

**Roots are rational and distinct if d is perfect square:**

**MCQ-12:**

What is the nature of the roots of the equation  $3x^2 - 8x + 4 = 0$ ?

- (a) real and equal                      (b) rational and unequal  
(c) irrational and unequal            (d) complex and unequal

**Solution:**

$$3x^2 - 8x + 4 = 0$$

$$a = 3, \quad b = -8, \quad c = 4$$

$$D = b^2 - 4ac$$

$$= (8)^2 - 4(3)(4) = 64 - 48 = 16 = 4^2$$

16 is a perfect square of 4.

The answer is (b).

**Lecture:**

**If  $D > 0$ , the roots are rational or irrational:**

If  $D > 0$ , the roots are real and distinct (unequal).

Since

$$\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}'$$

$\mathbb{Q}$  : set of rational numbers

$\mathbb{Q}'$  : set of irrational numbers

so that if  $D > 0$


There are two cases.

**Case-1:** D is perfect square, the roots are rational and distinct.

**Case-2:** D is not perfect square, the roots are irrational and distinct.

**EXERCISE-3.3**

- (1) What is the nature of the roots of the equation  $x^2 - 5x + 6 = 0$ ?  
(a) irrational      (b) real equal      (c) not real      (d) rational
- (2) Let D be the discriminant of the quadratic equation  
 $ax^2 + bx + c = 0$   
The roots of the equation are real if  
(a)  $D = 0$       (b)  $D \geq 0$       (c)  $D < 0$       (d) None
- (3) The roots of a quadratic equation are not real if  
(a)  $D = 0$       (b)  $D < 0$       (c)  $D \geq 0$       (d)  $D > 0$
- (4) The roots of a quadratic equation are real and distinct if  
(a)  $D > 0$       (b)  $D < 0$       (c)  $D = 0$       (d) None
- (5) What are the nature of the roots of the equation  $4x^2 - 12x + 9 = 0$ ?  
(a) equal      (b) real distinct      (c) complex      (d) rational
- (6) What are the nature of the roots of the equation  $x^2 + 2x - 3 = 0$ ?  
(a) equal      (b) real distinct      (c) complex      (d) rational

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**SUM AND PRODUCT OF THE ROOTS**

$$\text{Sum of the roots} = -b/a$$

$$\text{Product of the roots} = \frac{c}{a}$$

**MCQ-13:**

What are the sum and product of the roots of the equation  $5x^2 - 8x + 9 = 0$ ?

- (a)  $\frac{5}{8}, \frac{9}{8}$       (b)  $\frac{8}{5}, \frac{9}{5}$       (c)  $\frac{5}{9}, \frac{8}{9}$       (d)  $\frac{-5}{8}, \frac{1}{8}$

**Solution:**

$$5x^2 - 8x + 9 = 0$$

$$a = 5, b = -8, c = 9$$

$$\text{sum of the roots} = -b/a$$

$$\begin{aligned} \text{sum of the roots} &= -\frac{(-8)}{5} \\ &= \frac{8}{5} \end{aligned}$$

$$\text{Product of the roots} = \frac{c}{a}$$

$$= \frac{9}{5}$$

The answer is (b).

**Lecture:**

Explanations:

**Sum of the roots and product of the roots without using formulae:**

Firstly, find the roots of the equation

$$x^2 - 5x + 6 = 0$$

$$x^2 - 2x - 3x + 6 = 0$$

$$x(x - 2) - 3(x - 2) = 0$$

$$(x - 2)(x - 3) = 0$$

Either  $x - 2 = 0$  or  $x - 3 = 0$

$$x = 2, \quad x = 3$$

2, 3 are the roots.

Now,

$$\text{Sum of the roots} = 2 + 3 = 5$$

$$\text{Products of the roots} = 2 \times 3 = 6$$

**Using formula:**

$$x^2 - 5x + 6 = 0$$

$$a = 1, \quad b = -5, \quad c = 6$$

$$\text{Sum of the roots} = \frac{-b}{a} = \frac{-(-5)}{1} = 5$$

$$\text{product of the roots} = \frac{c}{a} = \frac{6}{1} = 6$$

**MCQ-14:**

What is the value of m if the sum of the roots of the equation

$$5x^2 - (m + 6)x + 9 = 0 \text{ is } 8?$$

(a) 34

(b) 28

(c) 14

(d) 6

**Solution:**

$$5x^2 - (m + 6)x + 9 = 0$$

$$a = 5, \quad b = -(m + 6), \quad c = 9$$

$$\text{sum of the roots} = 8$$



$$\frac{-b}{a} = 8$$

$$\frac{-\{-(m+6)\}}{5} = 8$$

$$m + 6 = 40$$

$$m = 34$$

The answer is (a).

**MCQ-15:**

$kx^2 - 5x + 6 = 0$ , what is the value of k if the product of the roots is 18?

- (a) 3                      (b)  $\frac{1}{3}$                       (c) -3                      (d)  $-\frac{1}{3}$

**Solution:**

$$kx^2 - 5x + 6 = 0$$

$$a = k, \quad b = -5, \quad c = 6$$

Product of the roots = 18

$$\frac{c}{a} = 18$$

$$\frac{6}{k} = 18$$

$$k = 6/18$$

$$k = 1/3$$

The answer is (b).

**MCQ-16:**

What is the equation whose roots are additive inverse of the roots 5 and 8 of an equation?

- (a)  $x^2 + 8x + 5 = 0$                       (b)  $x^2 + 3x + 13 = 0$   
 (c)  $x^2 + 2x + 8 = 0$                       (d)  $x^2 + 13x + 40 = 0$

**Solution:**

$$\text{Additive inverse of 5 and 8 are } -5 \text{ and } -8$$

$$\text{Sum of the roots} = (-5) + (-8)$$

$$= -5 - 8$$

$$= -13$$

$$\text{Product of the roots} = (-5)(-8)$$

$$= (-5)(-8)$$

$$= 40$$

Required Equation:

$$x^2 - (\text{sum of the roots})x + (\text{product of the roots}) = 0$$

$$x^2 - (-13)x + 40 = 0$$

$$x^2 + 13x + 40 = 0$$

The answer is (d).

### EXERCISE-3.4

- (1) What is the sum of two roots of the equation  $5x^2 - 8x - 4 = 0$ ?  
 (a)  $\frac{1}{2}$                       (b)  $\frac{5}{8}$                       (c)  $\frac{8}{5}$                       (d)  $\frac{4}{5}$
- (2) The product of the roots of the equation  $2x^2 - 5kx + k = 2$  is 8. What is the value of  $k$ ?  
 (a) 12                      (b) 18                      (c) 16                      (d) 20
- (3) The square of the sum of the roots of the equation  $3x^2 - 6kx + 10 = 0$  is 36. What is the value of  $k$ ?  
 (a) -3                      (b) 9                      (c)  $\sqrt{18}$                       (d) -6
- (4) The sum of the roots of the equation  $x^2 - 2kx + 12 = 0$  is equal to 4 times the product of the roots. What is the value of  $k$ ?  
 (a) 12                      (b) 24                      (c) 48                      (d) 6
- (5) The product of the roots of the equation  $2x^2 + 6x + k = 1$  is equal to the square of the sum of the roots. What is  $k$ ?  
 (a) -6                      (b) 7                      (c) 19                      (d) 18

- (6) The product of the roots of the equation  $3x^2 - 12x + p = 6$  is equal to the square root of sum of the roots. What is the value of  $p$ ?
- (a) 12                      (b) 6                      (c) 18                      (d) 54
- (7)  $\alpha$  and  $\beta$  are roots of the equation  $5x^2 + 3kx + 20 = 5x$ . What is the value of  $k$  if  $(\alpha + \beta)^2 = \alpha\beta$ ?
- (a) 15                      (b) -5                      (c) -25                      (d) None
- (8) What is the equation whose roots are  $3i$  and  $5i$ ?
- (a)  $x^2 - 8x - 15 = 0$                       (b)  $x^2 + 8ix - 15 = 0$   
(c)  $x^2 - 15x + 8 = 0$                       (d) None
- (9) What is the equation whose roots are three times the roots of the equation  $(x - 2)(x - 5) = 0$ ?
- (a)  $x^2 - 21x + 30 = 0$                       (b)  $x^2 - 7x + 10 = 0$   
(c)  $x^2 - 21x + 90 = 0$                       (d)  $x^2 - 81 = 0$
- (10) Sum and product of the roots of the equation  $x^2 + bx + c = 0$  are 18 and  $-115$  respectively. What are the values of  $b$  and  $c$ ?
- (a) 18, 115                      (b) 115, -18                      (c) 18, -115                      (d) -18, -115

**CUBIC EQUATIONS**

$\omega$  is a cube root of 1 (unity).

All cube roots of 1 are

1,  $\omega$  and  $\omega^2$ .

where  $\omega = \frac{-1+\sqrt{3}i}{2}$  and  $\omega^2 = \frac{-1-\sqrt{3}i}{2}$

**PROPERTIES OF  $\omega$** 

1)  $\omega^3 = 1$

2)  $1 + \omega + \omega^2 = 0$

i)  $1 + \omega = -\omega^2$

ii)  $1 + \omega^2 = -\omega$

iii)  $\omega + \omega^2 = -1$

**VALUE OF  $\omega^n$** 

i)  $n$  is exactly divisible by 3:

$$\omega^n = 1$$

ii) The remainder is 1, when  $n$  is divided by 3:

$$\omega^n = \omega$$

iii) The remainder is 2, when  $n$  is divided by 3:

$$\omega^n = \omega^2$$

**MCQ-17:**

$\omega^4 = ?$

- (a) 1      (b)  $\omega$       (c)  $\omega^2$       (d) 0

**Solution:**

Remainder is 1, when 4 is divided by 3

$$\omega^4 = \omega$$

**Note:**  $\omega^4 = \omega^3 \cdot \omega = 1 \cdot \omega = \omega$

The answer is (b).

**MCQ-18:**

$$\omega^{38} = ?$$

- (a) 1                      (b)  $\omega$                       (c)  $\omega^2$                       (d) 0

**Solution:**

Remainder is 2, when 38 is divided by 3

$$\omega^{38} = \omega^2$$

**Note:**  $\omega^{38} = \omega^{36} \cdot \omega^2 = (\omega^3)^{12} \cdot \omega^2 = (1)^{12} \cdot \omega^2 = \omega^2$

The answer is (c).

**MCQ-19:**

$$\omega^{19} = ?$$

- (a) 1                      (b)  $\omega$                       (c)  $\omega^2$                       (d) 0

**Solution:**

Remainder is 1, when 19 is divided by 3

$$\omega^{19} = \omega$$

**Note:**  $\omega^{19} = \omega^{18} \cdot \omega = (\omega^3)^6 \cdot \omega = (1)^6 \cdot \omega = \omega$

The answer is (b).

**MCQ-20:**

$$\omega^5 + \omega^{18} + \omega^{25} = ?$$

- (a) 1                      (b)  $\omega$                       (c)  $\omega^2$                       (d) 0

**Solution:**

Remainder is 2 when 5 is divided by 3  $\Rightarrow \omega^5 = \omega^2$

18 is exactly divisible by 3  $\Rightarrow \omega^{18} = 1$

Remainder is 1 when 25 is divided by 3  $\Rightarrow \omega^{25} = \omega$

$$\omega^5 + \omega^{18} + \omega^{25}$$

$$= \omega^2 + 1 + \omega$$

$$= 1 + \omega + \omega^2$$

$$= 0$$

The answer is (d).

**EXERCISE-3.5**

(1)  $\omega^{50} = ?$

(a) 0

(b) 1

(c)  $\omega$

(d)  $\omega^2$


(2)  $\omega^{37} = ?$

(a) 0

(b) 1

(c)  $\omega$

(d)  $\omega^2$

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(8)  $(\omega^{10} + \omega^6)^8 = ?$

(a) 1

(b)  $\omega$

(c)  $\omega^2$

(d)  $2\omega$

**CUBIC EQUATIONS****Lecture:**Real cube root of  $x^3 - a^3 = 0$ :

$$x = \sqrt[3]{a^3} = a$$

**Example-1:**Find real cube root of  $x^3 - 27 = 0$ 

$$x^3 = 27$$

$$x = \sqrt[3]{27} = \sqrt[3]{3^3} = 3$$

All cube roots of  $a^3$ :

$$x^3 = a^3$$

$$x^3 - a^3 = 0$$

$$(x - a)(x^2 + ax + a^2) = 0$$

$$\text{Either } x - a = 0 \Rightarrow x = a$$

$$\text{or } x^2 + ax + a^2 = 0$$

$$x = \frac{-a \pm \sqrt{a^2 - 4a^2}}{2}$$

$$x = \frac{-a \pm \sqrt{-3}a}{2}$$

$$x = \left( \frac{-1 \pm \sqrt{3}i}{2} \right) a$$

All cube roots are

$$a, \left( \frac{-1 + \sqrt{3}i}{2} \right) a, \left( \frac{-1 - \sqrt{3}i}{2} \right) a$$

or

$$a, a\omega, a\omega^2$$

All cube roots of  $-a^3$ :

$$x^3 = -a^3$$

$$x^3 - (-a)^3 = 0$$

All cube roots are

$$(-a), (-a)\omega, (-a)\omega^2$$

or

$$-a, -a\omega, -a\omega^2$$

where  $\omega$  is cube root of 1.

**Result:**

(i) One cube root is real and other two are complex of the equation

$$x^3 - a^3 = 0 \quad \text{or} \quad x^3 + a^3 = 0.$$

(ii) The cube roots of  $a^3$  are  $a, a\omega, a\omega^2$

and the cube roots of  $-a^3$  are  $-a, -a\omega, -a\omega^2$

where  $\omega = \frac{-1+\sqrt{3}i}{2}$  and  $\omega^2 = \frac{-1-\sqrt{3}i}{2}$  are complex numbers.

This shows that one cube root of a real number is real and other two roots are complex. This result helps us to find all the cube roots of a real number.

**MCQ-21:**

What are all cube roots of 1 (unity).

- (a)  $1, \omega, \omega^2$       (b) 1      (c)  $1, -1$       (d)  $1, -1, \omega$

**Solution:**

$$x^3 = 1$$

$$x^3 = 1^3$$

$$\Rightarrow a = 1$$

All cube roots of 1 (unity) are

$$a, a\omega, a\omega^2$$

$$\Rightarrow 1, \omega, \omega^2$$

The answer is (a).



**MCQ-22:**

What are the cube roots of 64?

- (a) 8    (b) 4, -4,  $4\omega$   
(c) 8, -8,  $8\omega$                                       (d) 4,  $4\omega$ ,  $4\omega^2$

**Solution:**

$$64 = 4^3$$
$$\Rightarrow a = 4$$

All cube roots of 64 are

$$a, a\omega, a\omega^2$$
$$\Rightarrow 4, 4\omega, 4\omega^2$$

The answer is (d).

**MCQ-23:**

What are the cube roots of -27?

- (a) 9    (b) 3,  $3\omega$ ,  $3\omega^2$   
(c) -3,  $-3\omega$ ,  $-3\omega^2$                                       (d) -3,  $-3\omega$ , -3

**Solution:**

$$-27 = (-3)^3$$
$$\Rightarrow a = -3$$

All cube roots of -27 are

$$a, a\omega, a\omega^2$$
$$\Rightarrow -3, -3\omega, -3\omega^2$$

The answer is (c).

**MCQ-24:**

What are the cube roots of 1728?

- (a)  $-12, 12, 12\omega$                       (b)  $-6, 6, 6\omega^2$   
(c)  $-8, 8, 4(-1 + \sqrt{3}i)$               (d)  $-12, -6 + 6\sqrt{3}i, -6 - 6\sqrt{3}i$

**Solution:**

Do not find cube roots of 1728.

Read the given options carefully.

- i) There are two real and one complex roots in first three option (a), (b), (c).  
ii) Option (d) has one real and two complex roots.

The answer is (d).

**EXERCISE-3.6**

(1) What is a cube root of 64?

- (a)  $-8$       (b)  $-4\omega^2$       (c)  $8\omega^2$       (d)  $4\omega$

(2) What is a cube root of  $-27$ ?

- (a)  $3\sqrt{3}i$       (b)  $-3\omega^2$       (c)  $3\omega$       (d)  $3\omega^2$

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