

## Chapter 2

**COMPLEX NUMBERS**

Complex numbers are written as

$$(i) \ a + ib$$

$$(ii) \ (a, b)$$

where  $a$  and  $b$  are real numbers.

$a$  = real part and  $b$  = imaginary part.

**ADDITION**

$$i) \ (a + ib) + (c + id) = (a + c) + i(b + d)$$

$$ii) \ (a, b) + (c, d) = (a + c, b + d)$$

**MCQ- 1:**

$$(3 + 5i) + (8 + 16i) = ?$$

$$(a) \ (11, 11)$$

$$(b) \ 3 + 16i$$

$$(c) \ 11 + 21i$$

$$(d) \ 5 + 5i$$

**Solution:**

$$(3 + 5i) + (8 + 16i)$$

$$= 3 + 8 + 5i + 16i$$

$$= 11 + 21i$$

The answer is (c).

**MCQ- 2:**

$$(12, -8) + (-3, 20) = ?$$

$$(a) \ (9, 12)$$

$$(b) \ 15 - 28i$$

$$(c) \ 15 + 28i$$

$$(d) \ 9 - 12i$$

**Solution:**

$$(12, -8) + (-3, 20)$$

$$= (12 - 3, -8 + 20)$$

$$= (9, 12)$$

The answer is (b).

**SUBTRACTION**

i)  $(a + ib) - (c + id) = (a - c) + i(b - d)$

ii)  $(a, b) - (c, d) = (a - c, b - d)$

**MCQ-3:**

$(10 + 2i) - (16 - 5i) = ?$

- (a)
- $26 - 3i$
- (b)
- $(-6, -7)$
- (c)
- $6 - 3i$
- (d)
- $-6 + 7i$

**Solution:**

$$(10 + 2i) - (16 - 5i)$$

$$= (10 - 16) + (2 + 5)i$$

$$= -6 + 7i$$

The answer is (d).

**MCQ-4:**

$(13, -5) - (8, 12) = ?$

- (a)
- $21 + 7i$
- (b)
- $(5, -17)$
- (c)
- $5 + 7i$
- (d)
- $21 - 17i$

**Solution:**

$$(13, -5) - (8, 12)$$

$$= (13 - 8, -5 - 12)$$

$$= (5, -17)$$

The answer is (b).

**MULTIPLICATION**

i)  $(a, b) \cdot (c, d) = (ac - bd, bc + ad)$

ii)  $(a + ib) \cdot (c + id) = (ac - bd) + i(bc + ad)$

**Shortcut:****Step-1****Real part**

$$\begin{array}{cc} a & b \\ \downarrow & \downarrow \\ c & d \\ & ac - bd \end{array}$$

**Step-2****Imaginary part**

$$\begin{array}{cc} a & b \\ \swarrow & \searrow \\ c & d \\ & bc + ad \end{array}$$

**Combined Step-1 and Step-2:**

$$\begin{array}{cc} a & b \\ \downarrow & \swarrow \searrow \\ c & d \\ & (ac - bd, bc + ad) \end{array}$$

**MCQ-5:**

$(5 + 2i) \cdot (9 + 4i) = ?$

- (a) (45, 8)      (b) (20, 18)      (c) (37, 38)      (d) (24, 14)

**Solution:**

$$\begin{array}{cc} 5 & 2 \\ \downarrow & \swarrow \searrow \\ 9 & 4 \end{array}$$

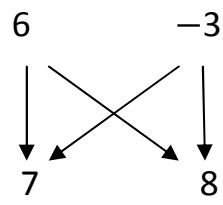
$$\begin{aligned} & (5 \times 9 - 2 \times 4, 2 \times 9 + 5 \times 4) \\ & = (45 - 8, 18 + 20) \\ & = (37, 38) \end{aligned}$$

The answer is (c).

**MCQ-6:**

$$(6, -3) \cdot (7, 8) = ?$$

- (a) (66, 27)    (b) (42, -24)    (c) (48, -21)    (d) (30, 26)

**Solution:**

$$\begin{aligned} & (6 \times 7 - (-3) \times 8, -3 \times 7 + 6 \times 8) \\ &= (42 + 24, -21 + 48) \\ &= (66, 27) \end{aligned}$$

The answer is (a).

**DIVISION**

$$i) (a + i b) \div (c + i d) = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2} i$$

$$ii) (a, b) \div (c, d) = \left( \frac{ac + bd}{c^2 + d^2}, \frac{bc - ad}{c^2 + d^2} \right)$$

**Shortcut:**

$$\begin{array}{cc} a & b \\ \downarrow & \swarrow \downarrow \\ c & d \end{array}$$

$$\left( \frac{ac + bd}{c^2 + d^2}, \frac{bc - ad}{c^2 + d^2} \right)$$

**MCQ-7:**

$$(3 + 5i) \div (2 + 7i) = ?$$

$$(a) \left( \frac{3}{2}, \frac{5}{7} \right) \quad (b) \left( \frac{7}{24}, \frac{16}{3} \right) \quad (c) \left( \frac{41}{53}, \frac{-11}{53} \right) \quad (d) \left( \frac{23}{53}, -2 \right)$$

**Solution:**

$$\begin{array}{cc} 3 & 5 \\ \downarrow & \swarrow \downarrow \\ 2 & 7 \end{array}$$

$$\left( \frac{3 \times 2 + 5 \times 7}{2^2 + 7^2}, \frac{5 \times 2 - 3 \times 7}{2^2 + 7^2} \right)$$

$$= \left( \frac{6+35}{4+49}, \frac{10-21}{4+49} \right)$$

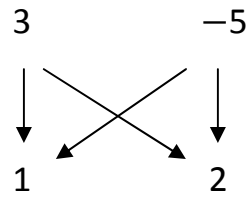
$$= \left( \frac{41}{53}, \frac{-11}{53} \right)$$

The answer is (c).

**MCQ-8:**

$(3, -5) \div (1, 2) = ?$

- (a)  $(\frac{2}{5}, \frac{-8}{5})$       (b)  $(\frac{-13}{12}, \frac{7}{6})$       (c)  $(\frac{23}{12}, \frac{-1}{6})$       (d)  $(\frac{-7}{5}, \frac{-11}{5})$

**Solution:**

$$\left( \frac{3 \times 1 + (-5) \times 2}{1^2 + 2^2}, \frac{-5 \times 1 - 3 \times 2}{1^2 + 2^2} \right)$$

$$= \left( \frac{3-10}{1+4}, \frac{-5-6}{1+4} \right)$$

$$= \left( \frac{-7}{5}, \frac{-11}{5} \right)$$

The answer is (d).

**FORMULAE**

(i)  $(a + ib)(a - ib) = a^2 + b^2$

(ii)  $(a + ib)^2 = a^2 - b^2 + 2abi$

(iii)  $(a - ib)^2 = a^2 - b^2 - 2abi$

**EXERCISE-2.1**

(1)  $(3, -5) + (2, 8) = ?$

- (a)  $(11, -3)$       (b)  $(5, 3)$       (c)  $(6, -40)$       (d)  $(1, -13)$

(2)  $(8, -3) - (-2, 6) = ?$


- (a)  $(6, 3)$       (b)  $(10, 3)$       (c)  $(10, -9)$       (d)  $(-6, -3)$

(3)  $(5 + 3i) - (2 - 6i) = ?$

- (a)  $3 - 3i$       (b)  $3 + 9i$       (c)  $7 - 3i$       (d)  $3 - 7i$

(4) What is the imaginary part of the number  $(2i + 5)(5 - 2i)$  ?

- (a) 0      (b) 2      (c) -10      (d) -2

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- (10) What is the real part of  $\frac{2-i}{2+i}$ ?
- (a) 2                      (b) 1                      (c)  $\frac{-4}{5}$                       (d)  $\frac{3}{5}$
- (11) What is the imaginary part of  $\frac{4i+6}{3i}$ ?
- (a) 2                      (b) -2                      (c)  $\frac{4}{3}$                       (d)  $\frac{2}{i}$
- (12)  $(4+3i)(4-3i) = ?$
- (a) 5                      (b)  $16-9i$                       (c) 25                      (d) 7
- (13) What is the real part of the complex number  $(3-2i)^2$ ?
- (a) -12                      (b) 5                      (c) 13                      (d) 3



**ADDITIVE INVERSE**Additive inverse of  $(a, b) = (-a, -b)$ **Examples:**

- 1) Additive inverse of  $3 - 5i = -3 + 5i$
- 2) Additive inverse of  $(-8, 6) = (8, -6)$

**MULTIPLICATIVE INVERSE**Multiplicative inverse of  $(a, b) = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$ **MCQ-9:**What is the multiplicative inverse of  $3 + 5i$  ?

- (a)  $\frac{3}{34} - \frac{5}{34}i$     (b)  $\frac{3}{26} + \frac{7}{26}i$     (c)  $\frac{9}{34} + \frac{6}{17}i$     (d)  $\frac{5}{26} - \frac{5}{26}i$

**Solution:**

$$\text{Multiplicative inverse of } a + bi = \frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}i$$

$$\begin{aligned} \text{Multiplicative inverse of } (3 + 5i) &= \frac{3}{3^2+5^2} + \frac{-5}{3^2+5^2}i \\ &= \frac{3}{34} - \frac{5}{34}i \end{aligned}$$

The answer is (a).

**EXERCISE-2.2**

- (1) What is the conjugate of  $3i - 5$  ?  
(a)  $-5 + 3i$     (b)  $3i + 5$     (c)  $-5 - 3i$     (d)  $5 - 3i$
- (2) What is the conjugate of complex number  $(3, -8)$ ?  
(a)  $(3, 8)$     (b)  $(-3, -8)$     (c)  $(-3, 8)$     (d)  $(-8, 3)$
- (3) What is the conjugate of  $2 + 3i$ ?  
(a)  $-2 - 3i$     (b)  $\sqrt{13}$     (c)  $-2 + 3i$     (d)  $2 - 3i$
- (4) What is the magnitude of the complex number  $3 - i$ ?  
(a) 3    (b)  $2\sqrt{2}$     (c)  $\sqrt{10}$     (d)  $\sqrt{2}$
- (5) What is the value of  $|(4 - 3i)(8 + 6i)|$ ?  
(a)  $10\sqrt{7}$     (b) 50    (c) 6    (d) None
- (6) What is the value of  $|\frac{25 + 8i}{8 - 25i}|$ ?  
(a)  $4\sqrt{5}$     (b) 8    (c) 5    (d) 1
- (7) What is the magnitude of  $\frac{(2 + i)(1 + 3i)}{i}$ ?  
(a)  $\sqrt{15}$     (b)  $-8$     (c) 5    (d)  $5\sqrt{2}$
- (8) What is the magnitude of the conjugate of  $2 - i$ ?  
(a)  $\sqrt{3}$     (b)  $-\sqrt{5}$     (c)  $2 + i$     (d)  $\sqrt{5}$

**CONJUGATE OF COMPLEX NUMBERS**

- i) Conjugate of  $a + ib = a - ib$   
 ii) Conjugate of  $(a, b) = (a, -b)$   
 iii) Conjugate of  $z = a + ib$ :

$$\bar{z} = a - ib$$

**Examples:**

- i) Conjugate of  $3 + 5i = 3 - 5i$   
 ii) Conjugate of  $8 - 9i = 8 + 9i$   
 iii) Conjugate of  $6i = -6i$   
 iv) Conjugate of  $(-3, -7) = (-3, 7)$

**MODULUS OF COMPLEX NUMBERS**

- i) Modulus of  $z = a + ib$

$$|z| = \sqrt{a^2 + b^2}$$

- ii)  $|a + ib| = \sqrt{a^2 + b^2}$

- iii) Modulus of  $a + ib = \sqrt{a^2 + b^2}$

**MCQ-10:**

What is the modulus of  $3 - 4i$  ?

- (a) 4                      (b) 5                      (c) 1                      (d) 9

**Solution:**

$$\text{Modulus of } a + ib = \sqrt{a^2 + b^2}$$

$$\begin{aligned} \text{Modulus of } 3 - 4i &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

The answer is (b).

**Properties:**

i)  $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

ii)  $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$

**MCQ-11:**

$| (4 + 2i)(2 - 4i)(4 + 3i) | = ?$

(a) 52

(b) 84

(c) 100


(d) 124

**Solution:**

$$|(4 + 2i)(2 - 4i)(4 + 3i)|$$

**Note:** Do not multiply the complex numbers  $4 + 2i$ ,  $2 - 4i$  and  $4 + 3i$ .Using property  $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$  calculate the modulus of each complex number and then multiply them.

$$\begin{aligned} &= |4 + 2i| \cdot |2 - 4i| \cdot |4 + 3i| \\ &= \sqrt{4^2 + 2^2} \cdot \sqrt{2^2 + (-4)^2} \cdot \sqrt{4^2 + 3^2} \end{aligned}$$

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(a) 224

(b) 148

(c) 268

(d) 130

**Solution:**

$$|(3 + 2i)(2 - 3i)(-6 + 8i)|$$

**Note:** Do not multiply the complex numbers  $3 + 2i$ ,  $2 - 3i$  and  $-6 + 8i$ .Using property  $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$  calculate the modulus of each complex number and then multiply them.

$$\begin{aligned}
 &= |3 + 2i| \cdot |2 - 3i| \cdot |-6 + 8i| \\
 &= \sqrt{3^2 + (2)^2} \cdot \sqrt{2^2 + (-3)^2} \cdot \sqrt{(-6)^2 + 8^2} \\
 &= \sqrt{9 + 4} \cdot \sqrt{4 + 9} \cdot \sqrt{36 + 64} \\
 &= \sqrt{13} \cdot \sqrt{13} \cdot \sqrt{100} \\
 &= 130
 \end{aligned}$$

The answer is (d).

**MCQ-13:**

$$\left| \frac{(7 - i)(i + 7)}{3 - 4i} \right| = ?$$

- (a) 8                      (b) 10                      (c) 24                      (d) 18

**Solution:**

$$\left| \frac{(7 - i)(i + 7)}{3 - 4i} \right|$$

$$\begin{aligned}
 &= \frac{|7 - i| \cdot |i + 7|}{|3 - 4i|} \\
 &= \frac{\sqrt{7^2 + (-1)^2} \cdot \sqrt{1^2 + 7^2}}{\sqrt{3^2 + (-4)^2}} \\
 &= \frac{\sqrt{49 + 1} \cdot \sqrt{1 + 49}}{\sqrt{9 + 16}}
 \end{aligned}$$

$$= \frac{\sqrt{50} \cdot \sqrt{50}}{\sqrt{25}}$$

$$\begin{aligned}
 &= \frac{50}{5} \\
 &= 10
 \end{aligned}$$

The answer is (b).

**MCQ-14:**

$$\left| \frac{2-4i}{i-2} \right| = ?$$

(a) 2

(b) 8

(c) 6

(d) 14

**Solution:**

$$\left| \frac{2-4i}{i-2} \right|$$

$$= \frac{|2-4i|}{|i-2|}$$

$$= \frac{\sqrt{2^2+(-4)^2}}{\sqrt{1^2+(-2)^2}}$$

$$= \frac{\sqrt{4+16}}{\sqrt{1+4}}$$

$$= \frac{\sqrt{20}}{\sqrt{5}}$$

$$= \sqrt{\frac{20}{5}}$$

$$= \sqrt{4}$$

$$= 2$$

The answer is (a).

**EXERCISE-2.3**

- (1) What is the additive inverse of complex numbers  $a + ib$ ?  
(a)  $a - ib$       (b)  $0$       (c)  $-a + ib$       (d)  $-a - ib$
- (2) What is the additive inverse of  $5 - 3i$ ?  
(a)  $0$       (b)  $-5 - 3i$       (c)  $-5 + 3i$       (d)  $5 + 3i$
- (3) What is the multiplicative inverse of  $a + ib$ ?  
(a)  $\frac{a+ib}{a^2+b^2}$       (b)  $\frac{-a-ib}{a^2+b^2}$       (c)  $\frac{-a+ib}{a^2+b^2}$       (d)  $\frac{a-ib}{a^2+b^2}$
- (4) What is the multiplicative inverse of  $2 + 5i$ ?  
(a)  $\frac{2+5i}{29}$       (b)  $\frac{2-5i}{29}$       (c)  $\frac{-2-5i}{29}$       (d)  $\frac{-2+5i}{29}$
- (5) What is the multiplicative inverse of  $3 - i$ ?  
(a)  $\frac{3+i}{10}$       (b)  $\frac{-3+i}{8}$       (c)  $\frac{-3-i}{10}$       (d)  $\frac{-3+i}{10}$
- (6) What is the identity element with respect to multiplication of complex number  $(2, 8)$ ?  
(a)  $(-2, -8)$       (b)  $(-2, 8)$       (c)  $(0, 1)$       (d)  $(1, 0)$

**SOLUTION OF THE EQUATIONS**

Mostly equations involving complex numbers are solved by comparing the real and imaginary parts.

**MCQ-15:**

What are the values of  $x$  and  $y$  for the equation

$$(x + 5) + i(2 - y) = 7 + 10i$$

- (a) 6, 4      (b) -3, 8      (c) 5, 9      (d) 2, -8

**Solution:**

$$(x + 5) + i(2 - y) = 7 + 10i$$

$$x + 5 = 7, \quad 2 - y = 10$$

$$x = 7 - 5, \quad y = 2 - 10$$

$$x = 2, \quad y = -8$$

The answer is (d).


**MCQ-16:**

What is the value of  $x$  for equation  $x + 5 + 9yi = 3y + 6i$ ?

- (a) 6      (b) -2      (c) -8      (d) -3

**Solution:**

$$x + 5 + 9yi = 3y + 6i$$

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The answer is (d).



**EXERCISE-2.4**

- (1) What is the value of  $x$  if  $2x + iy = (5 + i)^2$   
(a) 13                      (b) 24                      (c) 12                      (d) 5
- (2) What is the value of  $b$  if  $(a + ib)^2 = 4b + 10ai$   
(a) 10                      (b) 5                      (c) 2                      (d) None

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**VALUE OF  $i^n$** 

The value of  $i^n$  ( $n \in \mathbb{Z}^+$ ) can be found directly by using the following methods.

**Case-1:  $n$  is even:**

$$i^n = +1 \text{ or } -1$$

If  $n$  is exactly divisible by 4, then  $i^n = +1$

If  $n$  is not exactly divisible by 4 but divisible by 2, then  $i^n = -1$

**Case-1:  $n$  is odd:**

$$i^n = +i \text{ or } -i$$

If  $(n - 1)$  is exactly divisible by 4, then  $i^n = +i$

If  $(n - 1)$  is not exactly divisible by 4 but divisible by 2, then  $i^n = -i$

**MCQ-17:**

$$i^{24} = ?$$

- (a) 1                      (b) -1                      (c)  $i$                       (d)  $-i$

**Solution:**

24 is exactly divisible by 4.

$$i^{24} = 1$$

**Note:**  $i^{24} = (i^2)^{12} = (-1)^{12} = 1$

The answer is (a).

**MCQ-18:**

$$i^{18} = ?$$

- (a) 1                      (b) -1                      (c)  $i$                       (d)  $-i$

**Solution:**

18 is not exactly divisible by 4 but divisible by 2.

$$i^{18} = -1$$

**Note:**  $i^{18} = (i^2)^9 = (-1)^9 = -1$

The answer is (b).

**MCQ-19:**

$$i^{37} = ?$$

- (a) 1                      (b) -1                      (c)  $i$                       (d)  $-i$

**Solution:**

$37 - 1 = 36$  is exactly divisible by 4.

$$i^{37} = i$$

**Note:**  $i^{37} = i^{36} \cdot i = (i^2)^{18} \cdot i = (-1)^{18} \cdot i = i$

The answer is (c).

**MCQ-20:**

$$i^{23} = ?$$

- (a) 1                      (b) -1                      (c)  $i$                       (d)  $-i$

**Solution:**

$23 - 1 = 22$  is not exactly divisible by 4, but divisible by 2.

$$i^{23} = -i$$

**Note:**  $i^{23} = i^{22} \cdot i = (i^2)^{11} \cdot i = (-1)^{11} \cdot i = -i$

The answer is (d).

**EXERCISE-2.5**

(1)  $i^5 = ?$

- (a) -1                      (b) 1                      (c)  $i$                       (d)  $-i$

(2)  $2i^7 = ?$

- (a)  $-2i$                       (b)  $2i$                       (c)  $-128i$                       (d)  $-2$

(3)  $(2i)^6 = ?$

- (a)  $-64$                       (b)  $64i$                       (c) 64                      (d)  $-64i$

(4)  $i^{13} + i^{19} = ?$

- (a)  $i$                       (b)  $-i$                       (c) 0                      (d) 1

(5)  $i^{12} + i^{10} = ?$

- (a)  $i$                       (b)  $-i$                       (c) 0                      (d) 1

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