Chapter 16

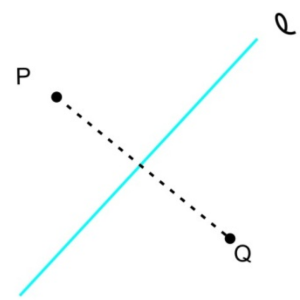
SYMMETRY

(i) Line Symmetry (ii) Point Symmetry

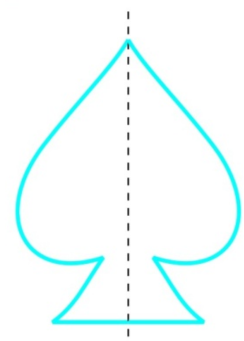
(iii) Rotational symmetry

LINE SYMMETRY

(a) Two points P and Q are symmetric about a line l if and only if line l is the perpendicular bisector of line segment PQ.



(b) A geometrical figure is called symmetric about a straight line if the figure can be divided into to two parts by the line such that one part is an image of the other in the line.
The dotted line is called the line of symmetry or the axis of symmetry.

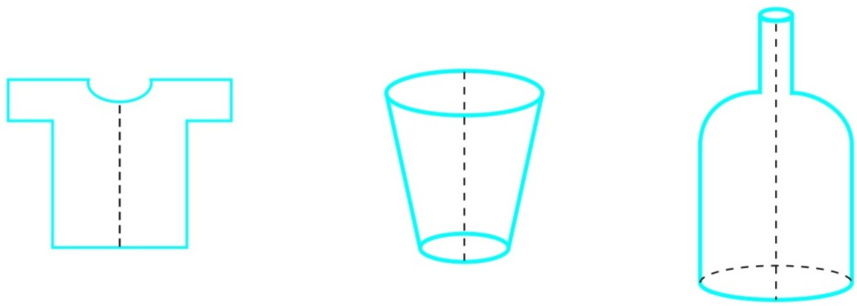


Explanation:-

Draw a curve on a folded paper which is placed on a carbon paper as shown in the figure.

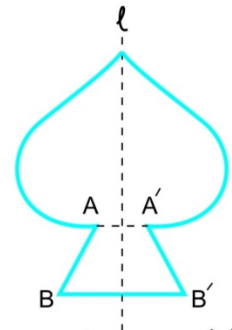
Now unfold the paper. The curves on both the sides of the dotted lined are congruent and form a closed curve. The one part of the figure is the reflection of the other in the line.

For example: Following figures are symmetric about the dotted line.



Properties of Line Symmetry:-

The figure (a) is symmetric about the straight line *l*.

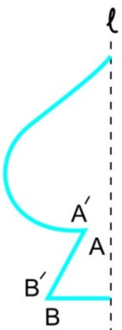


(a)

(i) When the paper is folded at the line of symmetry *l*, the parts of the figure on both sides of the line are exactly overlap.

(ii) Symmetric points are equidistance from the line of symmetry. As symmetric points A and A' are equidistance from the line *l*.

(iii) A line segment joining symmetric points is perpendicular to the line of symmetry. As the line segments AA' and BB' are perpendicular to the line of symmetry *l*.

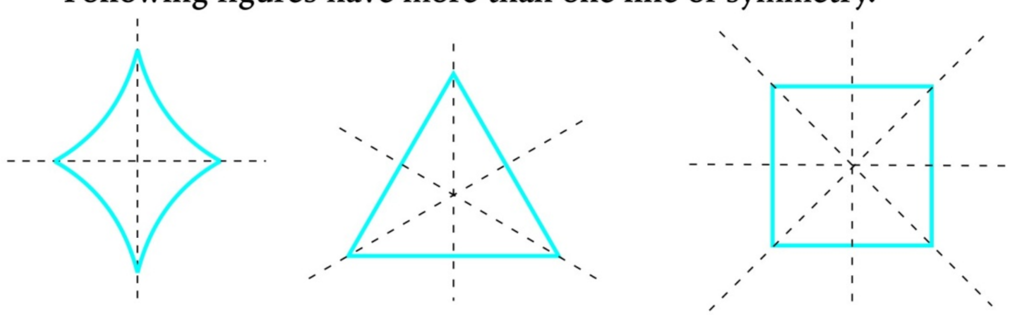


(b)

Lines of Symmetry:-

A figure may have more than one line of symmetry.

Following figures have more than one line of symmetry.



Note: Number of lines of symmetry of a regular polygon is equal to the number of sides of the regular polygon.

For example : The following figures have rotational symmetry of order 1, 2, 3 and 4 respectively.

(1) (2) (3) (4)

EXERCISE D-4

State "the number of lines of symmetry" and "the order of rotational symmetry" of the following letters and figure. Also find which of the following has point symmetry.

(1) **A**

(2) **D**

(3) **E**

(4) **I**

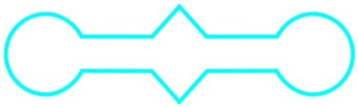
(5) **N**

(6) **S**

(7) **X**

(8) **Z**

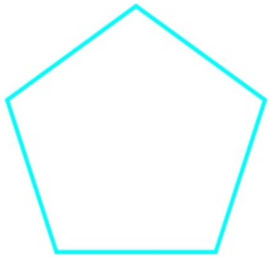
(9)



(10)

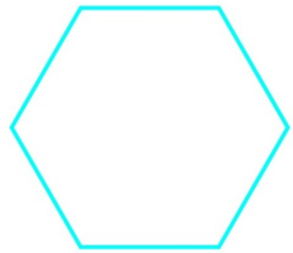
(11)

Regular pentagon



(12)

Regular hexagon



(13)

(14)

(15)

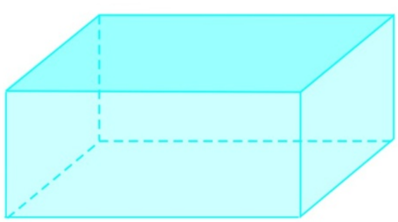
(16)

For each of the following solids, state

(a) the number of planes of symmetry and also write the name of the plane of symmetry.

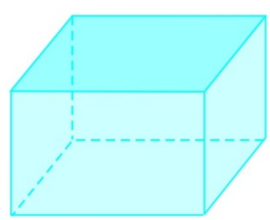
(b) the number of axes of rotational symmetry.

(17)



Cuboid

(18)



Cube

(6) How many different letters in the word PAKISTAN have exactly one line of symmetry?

(a) 3 (b) 4 (c) 6 (d) 7

(7) How many letters in the word TWISHON have more than one line of symmetry?

(a) 1 (b) 2 (c) 3 (d) 4

(8) How many letters in the word ASHTEN have rotational symmetry of order 2.

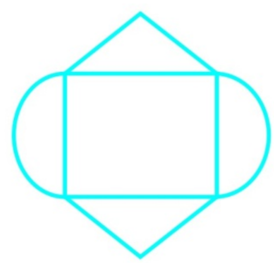
(a) 2 (b) 3 (c) 4 (d) 0

(9) What is the order of rotational symmetry?

(a) 1 (b) 2
(c) 3 (d) 6

(10) What is the order of rotational symmetry?

(a) 0 (b) 2
(c) 3 (d) 4



(11) How many lines of symmetry can be drawn on the figure?

(a) 0 (b) 1
(c) 2 (d) 3

(12) How many lines of symmetry can be drawn on the figure?

(a) 0 (b) 1
(c) 2 (d) 3

Chapter 17

LOCUS

Definition 1:

"Locus is the path traced out by a moving point satisfying certain condition."

It is not a powerful definition of locus because a locus may con sist of a number of disconnected parts such as hyperbola. The powerful definition of locus.

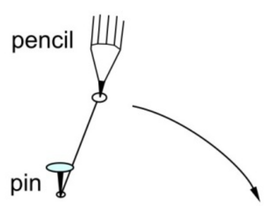
Definition 2:

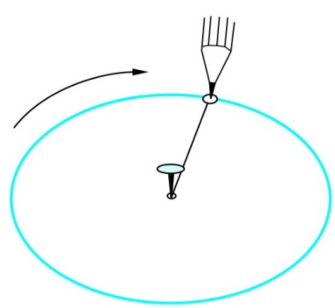
"Locus of a point is the set of all points satisfying certain given condition."

Explanation of Def. 1 (Path of Moving point):

Locus can be explained by the path of a moving point satisfying a give n condition. For example

(i) Circle: Two rings are attached to the ends of a thread. A pin is fixed at point A. Pin p asses through a ring and a pencil to the other ring. The thread is pulled taut by the pencil point.



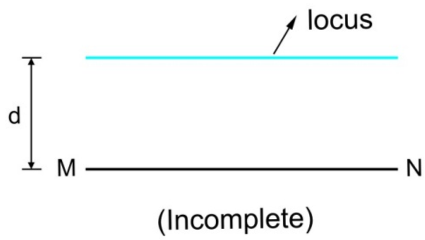


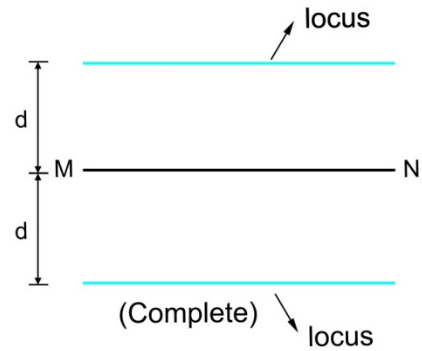
Move pencil point so as to keep the thread tight. The path trace d out by the moving point is a circle.

"A circle is the locus of a point which moves in a plane so that it is always at a constant distance from a fixed point."

(2) At a Distance d from a Fixed Line:

Suppose that $\overline{MN}$ is a straight line. All the points which are at a distance d from $\overline{MN}$ form two straight lines.

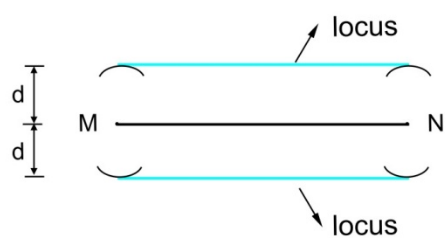




Construction:

Draw a straight line $\overline{MN}$ of given length. Draw two small arcs centre M and radius d. Similarly draw two arcs centre at N and radius d.

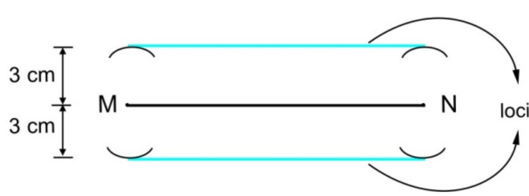
Draw common tangent on the arcs above the line $\overline{MN}$ and aother common tangent on the arcs below the line $\overline{MN}$.



Example: Draw locus of a point which is at a distance 3cm from the line $\overline{MN}$. $\overline{MN}$ is a line of length 5 cm.

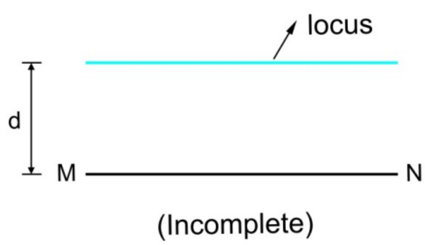
Solution:-

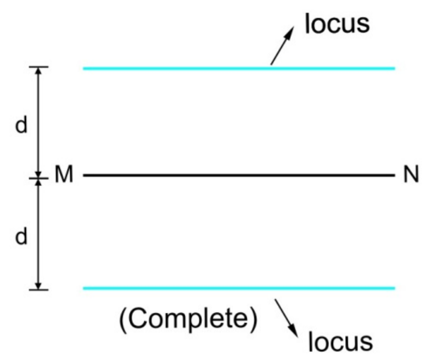
Draw arcs of radius 3 cm centre at M and then N. Draw the common tangents on the arcs above $\overline{MN}$ and then on the arcs below $\overline{MN}$.



(2) At a Distance d from a Fixed Line:

Suppose that $\overline{MN}$ is a straight line. All the points which are at a distance d from $\overline{MN}$ form two straight lines.

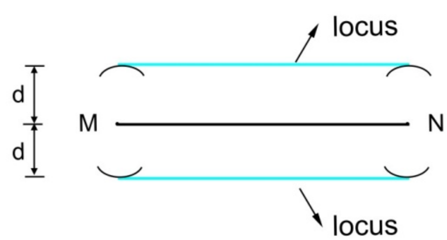




Construction:

Draw a straight line $\overline{MN}$ of given length. Draw two small arcs centre M and radius d. Similarly draw two arcs centre at N and radius d.

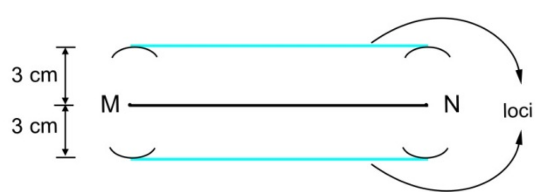
Draw common tangent on the arcs above the line $\overline{MN}$ and aother common tangent on the arcs below the line $\overline{MN}$.



Example: Draw locus of a point which is at a distance 3cm from the line $\overline{MN}$. $\overline{MN}$ is a line of length 5 cm.

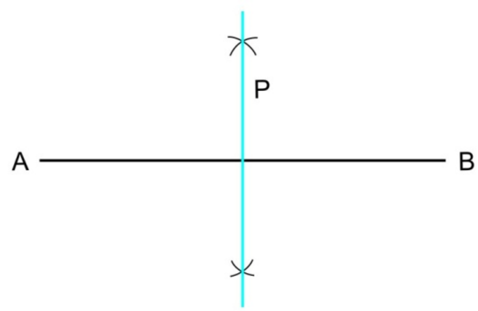
Solution:-

Draw arcs of radius 3 cm centre at M and then N. Draw the common tangents on the arcs above $\overline{MN}$ and then on the arcs below $\overline{MN}$.

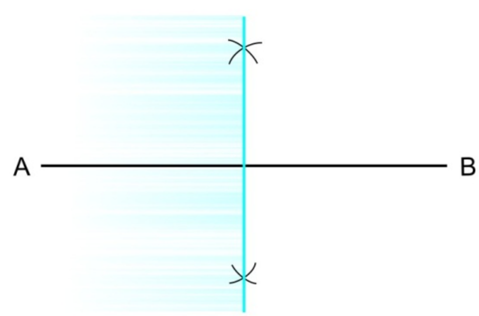


(2) AX $\leq$ BX or AX $\geq$ BX:

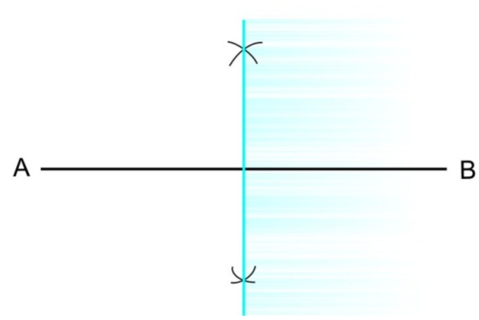
(i) Locus of a point P which is equidistance from two fixed points A and B is perpendicular besector of $\overline{AB}$, shown in the figure.



(ii) If AX $\leq$ XB, X lies on or the left side of perpendicular bisector of AB.

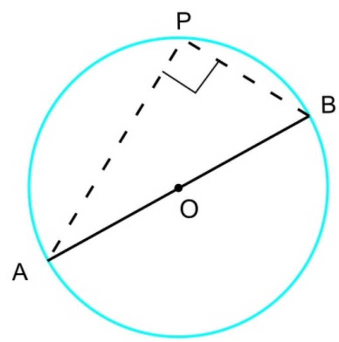


(iii) If AX $\geq$ XB, X lies on or the right side of perpendicular bisector of $\overline{AB}$.

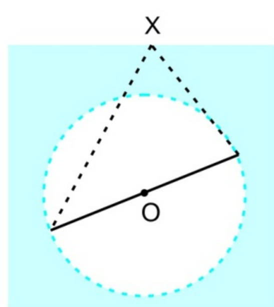


(3) $A\hat{X}B < 90^\circ$ or $A\hat{X}B > 90^\circ$

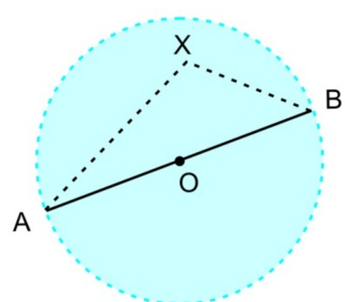
(i) Locus of a point P such that $A\hat{P}B = 90^\circ$ where A and B are two fixed points, is a circle of diameter AB.



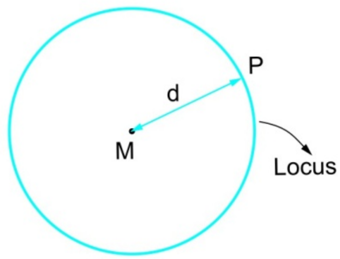
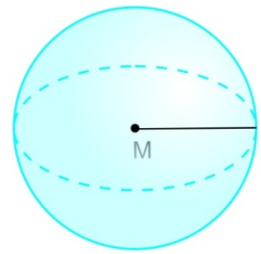
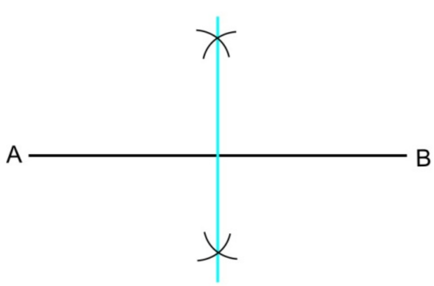
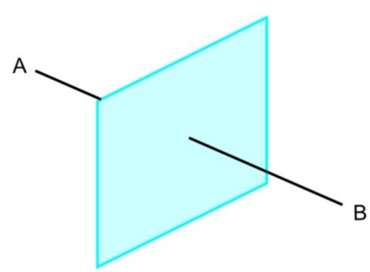
(ii) If $A\hat{X}B < 90^\circ$, the point X is outside the circle.



(Iii) If $A\hat{X}B > 90^\circ$, the point X is inside the circle.



Comparison of two Dimensions and Three Dimensions:

In Two Dimensions	In Three Dimensions
(1) Locus of a point P which is at a distance d from a fixed point M is a circle. 	(1) Locus of a point P which is at a distance d from a fixed point is a sphere. 
(2) Locus of a point P equidistance from a line $\overline{AB}$ are two straight lines parallel to and equidistance from $\overline{AB}$. 	(2) Locus of a point P equidistance from a line $\overline{AB}$ is a cylinder. 
(3) Locus of a point P equidistance from two fixed points A and B is a straight line. 	(3) Locus of a point equidistance from two points A and B is a plane. Line $\overline{AB}$ is normal to the plane. 

EXERCISE D-5

(1) **Construct a triangle ABC in which AB = 6cm, BC = 5cm** and AC = 4 cm.

On the same diagram draw

(i) the locus of points which are 4cm from B.

(ii) the locus of points which are equidistance from the lines BA and BC.

Mark with the letter M the point inside the triangle ABC which is 4cm from B and equidistance from BA and BC.

(2) **Construct a triangle ABC in which AB = 7cm, BC = 5cm** and $C\hat{A}B = 45^{\circ}$.

On the same diagram, draw

(i) the locus of points which are 5cm from A.

(ii) the locus of points 2.5cm from the line AB.

Mark with the letter M the point inside the triangle ABC which is 5 cm from A and 2.5 cm from the line AB.

(3) Draw equadrilateral ABCD with AB = 8 cm, AD = 5.7cm, CD = 2.7cm and $A\hat{B}C = 60^{\circ}$ where $\overline{CD}$ is parallel to $\overline{AB}$.

On the same diagram, draw the locus of points,

(i) equidistance from $\overline{AB}$ and $\overline{BC}$.

(ii) 4.5cm from A.

(4) **Construct quadrilateral ABCD with AB = 6 cm, AD = 5cm,** $D\hat{A}B = 70^{\circ}$, $A\hat{B}C = 90^{\circ}$ and $C\hat{D}A = 140^{\circ}$.

On the same diagram, draw the locus of points

(a) (i) 4.8cm from C.

(ii) equidistance from $\overline{AB}$ and $\overline{AD}$.

(iii) equidistance from A and D.

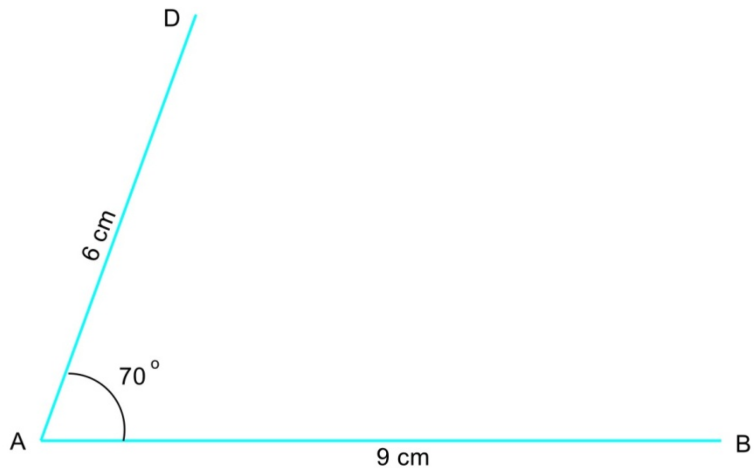
(b) Mark with the letters M and N respectively

(i) the point equidistance from $\overline{AB}$ and $\overline{AD}$ and also from A and D.

(ii) the point equidistance from $\overline{AB}$ and $\overline{AD}$ and 4.8cm from C.

(9) Complete the parallelogram ABCD.

Given $AB = 9cm$, $AD = 6cm$ and $D\hat{A}B = 70^{o}$



On the same diagram:

(a) draw the locus of points

(i) 7.5 cm from C

(ii) nearer to B from A

(b) Draw the locus of point X such that $A\hat{X}B = 90^{o}$

(c) Shade the region point Z lies if $CZ \leq 7.5$, $BZ \leq AZ$ and $A\hat{Z}X \geq 90^{o}$

(10) Complete the pentagon ABCDE.

D lies on the perpendicular bisector of $\overline{AB}$ and CD = 4 cm.

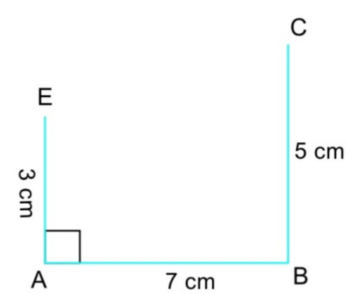
ABCDE is a park.

Shaded the area of marble floor, if it is

(i) nearer to $\overline{BC}$ than $\overline{CD}$.

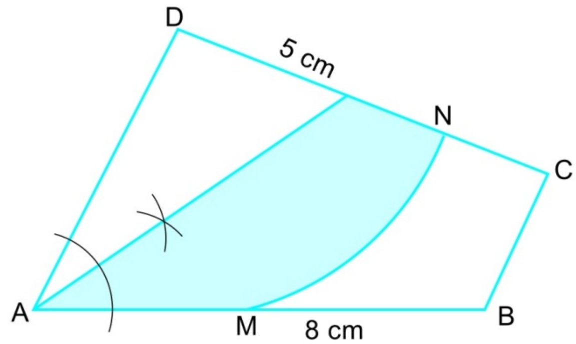
(ii) 5.5cm or less from D.

(iii) mark M which is 5.5cm from D and $\overline{CM}$ is the gate of the park



M.C.Q'S D-7

(1) ABCD is a quadrilateral. The moving point P lies in the shaded region. MN is the arc of an circle centre D. A line bisects angle DAB.



Which of the following is true?

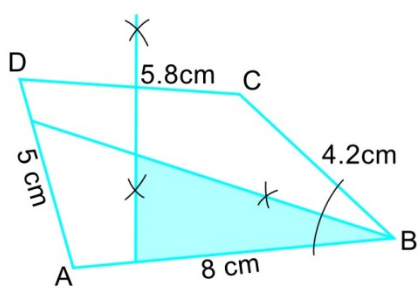
(a) P is nearer to D than C and P is nearer to $\overline{AB}$ than $\overline{AD}$.

(b) P is nearer to A than B and less than 5 cm from D.

(c) P is nearer to AD than AB and P is nearer to A than B.

(d) P is less than 5cm from D and P is nearer to $\overline{AB}$ than $\overline{AD}$.

(2) ABCD is a quadrilateral. The moving point P lies in the shaded region.



Which of the following is true?

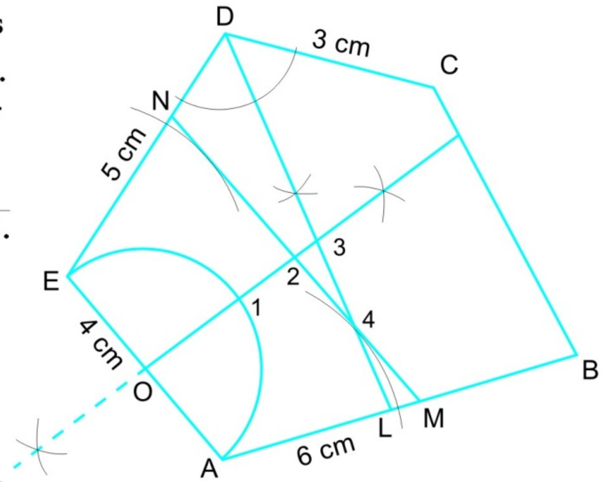
(a) P is nearer to $\overline{AB}$ than $\overline{CD}$ and nearer to B than A.

(b) P is nearer to $\overline{AB}$ than $\overline{BC}$ and nearer to C than D.

(c) P is nearer to A than C and nearer to $\overline{AB}$ than $\overline{BC}$.

(d) P is nearer to B than D and nearer to A than D.

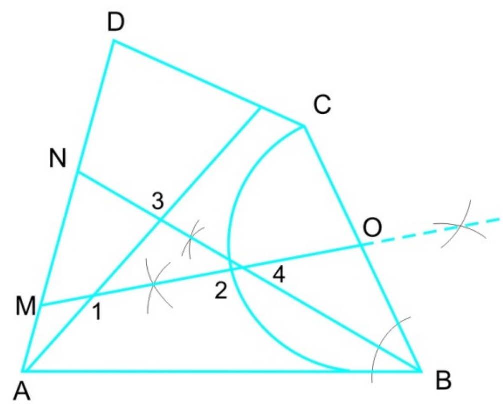
(6) ABCDE is a pentagon. AE is the diameter of circle, centre at O. What is the position of point P if $A\hat{P}B = 90^\circ$. Given $\overline{DL}$ bisects angle $E\hat{D}C$ and $\overline{MN}$ parallel to $\overline{AE}$.



(a) 1
(b) 2
(3) 3
(4) 4

(7) $\overline{BC}$ is the diameter of the circle centre at O. $\overline{OM}$ perpendicular bisector of $\overline{BC}$ and $\overline{BN}$ is locus of the points equidistance from $\overline{AB}$ and $\overline{BC}$.

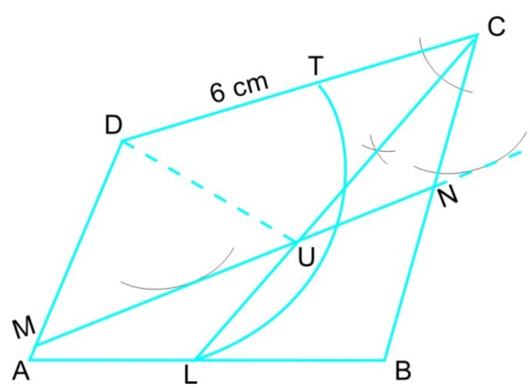
What is the position of point P if $B\hat{P}C > 90^\circ$

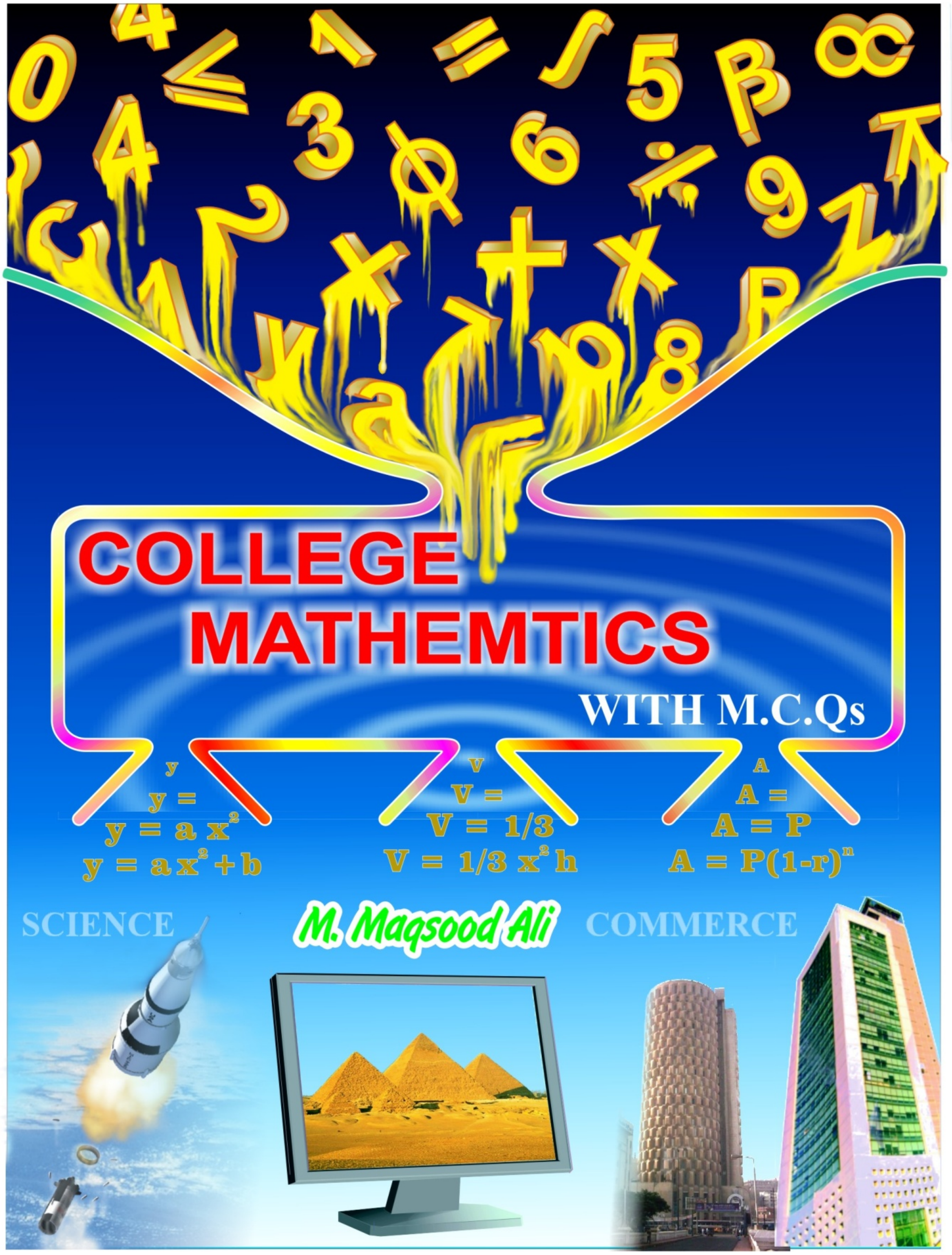


(a) 1
(b) 2
(c) 3
(d) 4

(8) ABCD is a quadilateral $\overline{CL}$ bisects angle $D\hat{C}B$, $\overline{MN}$ parallel to $\overline{DC}$ and LT an arc of a circle centre at D. If area of ΔDUC is 6cm^2, then which triangle has the same area?

(a) DLC
(b) DSC
(c) DMU
(d) DMC




COLLEGE
MATHEMTICS
WITH M.C.Qs
y
y =
$y = a x^2$
$y = ax^2 + b$
V
V =
V = 1/3
$V = 1/3\ x^2 h$
A
A =
A = P
$A = P(1-r)^n$
SCIENCE
M. Maqsood Ali
COMMERCE

Chapter 18

GEOMETRICAL TRANSFORMATION

In this chapter we study Translation, Reflection, Rotation, Stretch, Enlargement and Shear. These transformations change the position , shape and size of a plane figure according to the given condition.

In "Computer Graphics" we watch different position shapes and size of an object on the screen of the monitor. The computer graphics are the applications of above transformations.

The coordinates of the vertices of the image of an object can be found using.

(i) geometrical instruments (ii) transformation matrices

This chapter is divided into two sections A and B.

SECTION "A"

The centre of transformation is (0, 0) and the invariant line is x-axis or y-axis. These conditions are used in the following topics as given below.

(1) TRANSLATION:

(2) REFLECTION:

Line of reflection is x-axis, y-axis, the line $y = x$ or $y = -x$.

(3) ROTATION:

Centre at origin.

(4) STRETCH:

Invariant line is x-axis or y-axis. Centre of stretch (0, 0).

(5) ENLARGEMENT:

Centre of enlargement (0, 0).

(6) SHEARS:

Invariant line is x-axis or y-axis. Centr of share (0, 0).

Section A is divided into two types.

Type 1: Translation, Reflection and Rotation

The shape and size of the object and image are same.

TRANSLATION

A transformation called translation moves all the points of a plane figure through the same distance and same direction.

Explanation:

A(a, b), B(c, d) and C(e, f) are the vertices of a triangle ABC. The triangle $A'B'C'$ is the image of ΔABC under a translation $\begin{pmatrix} p \\ q \end{pmatrix}$. The vertices of triangle A'B'C' can be found as

$$\begin{array}{ccc} A & B & C \end{array} \qquad\qquad\qquad \begin{array}{ccc} A' & B' & C' \end{array}$$

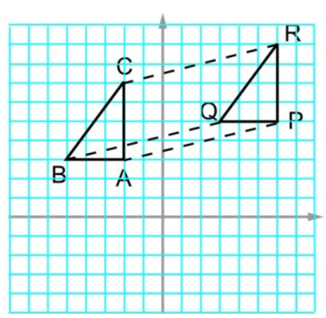
$$\begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix} + \begin{pmatrix} p & p & p \\ q & q & q \end{pmatrix} = \begin{pmatrix} a+p & c+p & e+p \\ b+q & d+q & f+q \end{pmatrix}$$

Example: A(−2, 3), B(−5, 3) and C(−2, 7) are the vertices of a triangle ABC. Triangle PQR is the image of triangle ABC under a translation $\begin{pmatrix} 8 \\ 2 \end{pmatrix}$. Draw the triangle ABC and PQR on a graph paper.

Solution:-

$$\begin{array}{ccc} A & B & C \end{array} \qquad\qquad\qquad \begin{array}{ccc} P & Q & R \end{array}$$

$$\begin{pmatrix} -2 & -5 & -2 \\ 3 & 3 & 7 \end{pmatrix} + \begin{pmatrix} 8 & 8 & 8 \\ 2 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 3 & 6 \\ 5 & 5 & 9 \end{pmatrix}$$



ROTATION

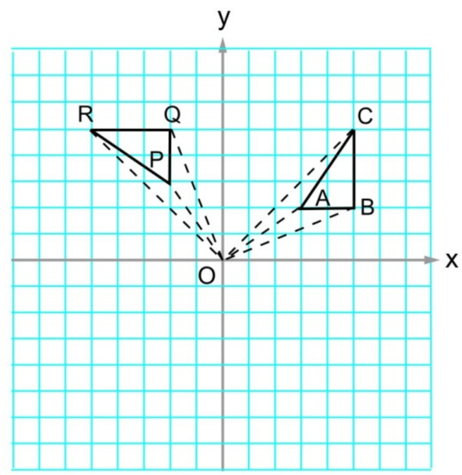
A transformation called rotation which rotates all points of a plane figure about a point through a given angle.

Note:-

(i) The rotation may be clockwise or anti-clockwise direction.
(ii) The points of plane figure rotate about a point called centre of rotation.
(iii) The angle of rotation will be 90^o, 270^o or 180^o in this chapter.
(iv) Rotation 90^o clockwise and rotation 270^o anticlockwise are same.
(v) Rotation 180^o clockwise and 180^o anticlockwise are same.

Explanation (Geometrically):

A(3, 2), B(5, 2) and C(5, 5) are the vertices of a triangle ABC. Triangle PQR is the image of triangle ABC under a rotation 90^o, anticlockwise and centre at origin.



(i) Join all the vertices of ΔABC to the origin by dahsed lines.
(ii) Draw lines $\overline{OP}$, $\overline{OQ}$ and $\overline{OR}$ making angle 90^o in anticlockwise direction with the lines $\overline{OA}$, $\overline{OB}$ and $\overline{OC}$ respectively.
(iii) $|\overline{OP}| = |\overline{OA}|$, $|\overline{OQ}| = |\overline{OB}|$ and $|\overline{OR}| = |\overline{OC}|$.
(iv) Join the points P, Q and R.

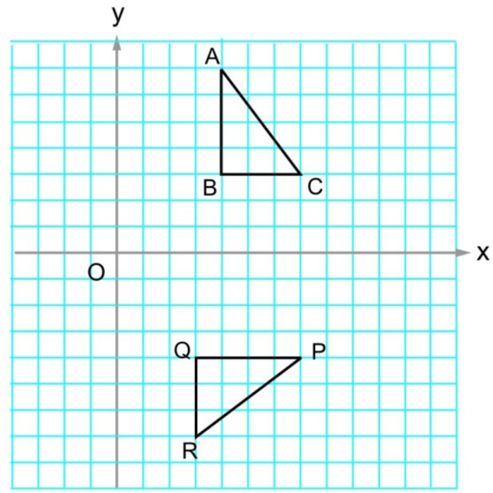
Transformation Matrix:

S.No.	Matrix	Angle	Direction	Centre
1	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	90^o	anti-clockwise	origin
2	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	90^o 270^o	clockwise anticlockwise	origin
3	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	180^o	clockwise or anticlockwise	origin

Example: A(4, 7), B(4, 3) and C(7, 3) are the vertices of a triangle ABC. Triangle PQR is the image under a rotation 90 ° clockwise, centre at origin. Draw the triangle ABC and PQR on a graphpaper.

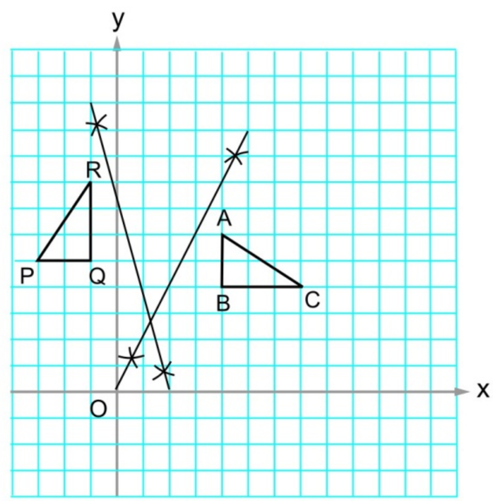
Solution:-

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 4 & 4 & 7 \\ 7 & 3 & 3 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{pmatrix} 7 & 3 & 3 \\ -4 & -4 & -7 \end{pmatrix} \end{array}$$



HOW TO FIND CENTRE OF ROTATION:

Triangle ABC is mapped onto a triangle PQR under a rotation. We can find the centre of rotation by the following method.



(i) Join any two vertices of the object to the corresponding vertices of the image. Join B to Q and C to R. So we get the line segments $\overline{BQ}$ and $\overline{CR}$.

(ii) Draw perpendicular bisectors of line segments $\overline{BQ}$ and $\overline{CR}$.

(iii) The point of intersection of perpendicular bisectors is the centre of rotation. M(1, 2) is the centre of rotation.

Angle of Rotation and Direction:

(iv) Join a vertex of the object and corresponding vertex of the image to the centre of rotation. We join B and Q to the centre of rotation (1, 2).

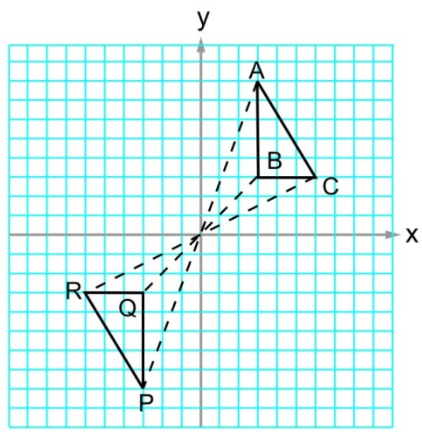
(v) Angle $B\hat{M}Q = 90°$ and direction is anticlockwise.

(3) Rotation:

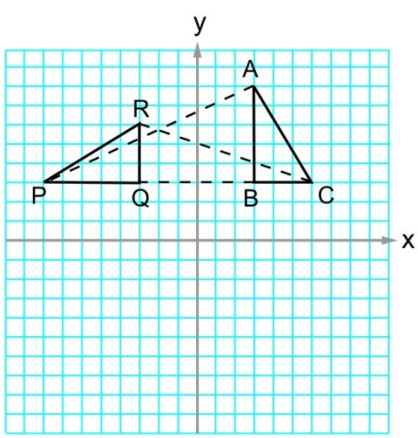
(i) The line segments $\overline{AP}$, $\overline{BQ}$ and $\overline{CR}$ are not parallel.

(ii) Two line segments intersect at a point when the rotation is 90 °.

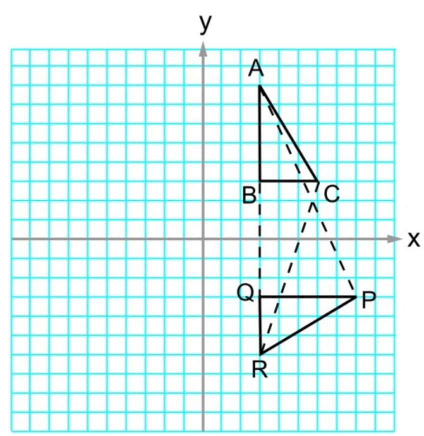
(iii) All three line segments intersect at a point when the rotation is 180°.



rotation 180°



rotation 90 ° anticlockwise

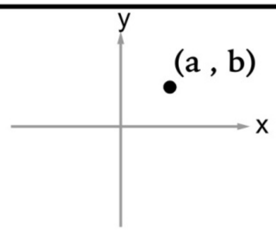
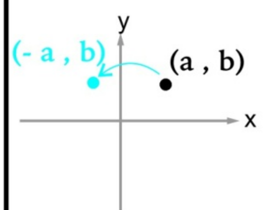
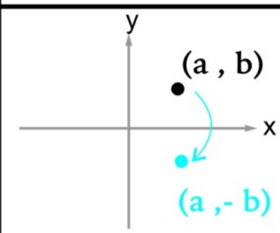
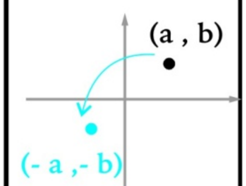


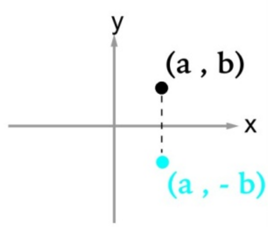
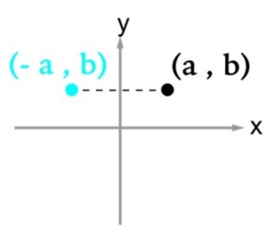
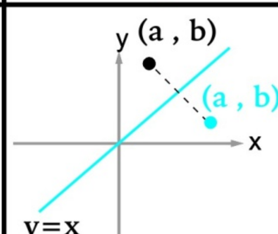
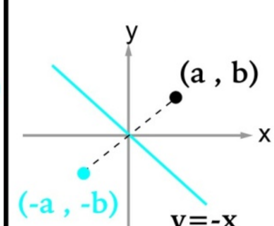
rotation 90 ° clockwise

HOW TO LEARN TRANSFORMATION MATRIX

Unit matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is converted into transformation matrix as given below. Suppose that a > 0 and b > 0.

(1) If a and b interchange, 1 and 0 interchange their positions in matrix.
(2) If a changes the sign, the sign of first row will change.
(3) If b changes the sign, the sign of second row will change.

ROTATION			
	90º anticlockwise	***90º clockwise or 270º anticlockwise***	***180º***
 Unit matrix of order 2 $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	 Transformation matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	 Transformation matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	 Transformation matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

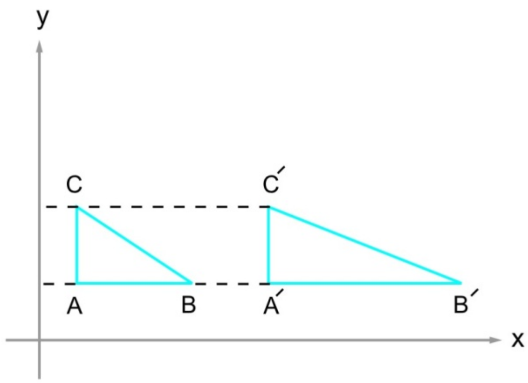
REFLECTION IN			
x-axis	***y-axis***	***the line y = x***	***the line y = x***
 Transformation matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	 Transformation matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	 Transformation matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	 Transformation matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

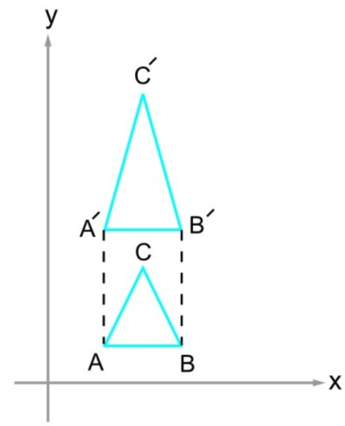
Type 2: Stretch, Enlargement and Shear.

The shape or size or both of the object and image are not same.

STRETCH

This transformation enlarge the figure k (Stretch factor) times the original size in a direction perpendicular to invariant line. Thus the direction of stretch is always perpendicular to the invarient line.





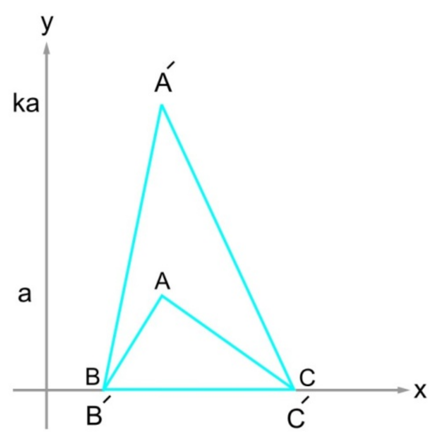
Explanation:

ΔABC is made by an elastic and thumb pins ΔA'B'C' is the image of ΔABC under a stretch scale factor k and invariant line x-axis.

(i) Vertex C is at a distance c from invariant line. So vertex C' is at a distance kc from that line.

(ii) The vertices lie on the invariant line do not move because the distance of vertices B and C from invariant are zero. So $k(0) = 0$.

(iii) Point of intersection of invariant line and y-axis is the centre of stretch.



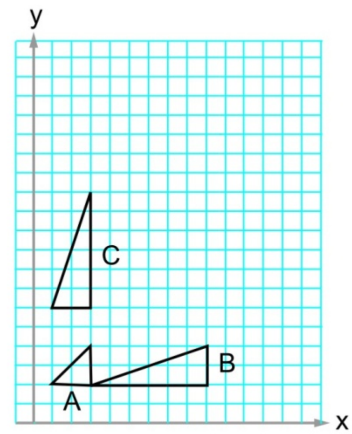
$$\text{Stretch factors} = \frac{ka}{a} = k$$

Area:

Area of image = k × Area of object

Examples: (1, 2), (3, 2) and (3, 4) are the vertices of a triangle A. Find the coordinates of the image of ΔA, under a stretch scale factor 3 and the invariant line (i) x-axis (ii) y-axis. Also draw the triangles on a graph paper.

Solution:-



(i) $\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}1 & 3 & 3\\ 2 & 2 & 4\end{pmatrix} = \begin{pmatrix}1 & 3 & 3\\ 6 & 6 & 12\end{pmatrix}$

(ii) $\begin{pmatrix}3 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 3 & 3\\ 2 & 2 & 4\end{pmatrix} = \begin{pmatrix}3 & 9 & 9\\ 2 & 2 & 4\end{pmatrix}$

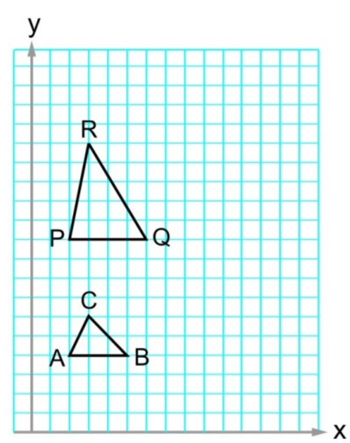
Example: ΔABC is mapped onto a ΔPQR under a stretch. Find (i) invariant line (ii) centre of stretch (iii) stretch factor

Solution:-

(i) x-axis is the invariant line.

(ii) centre of stretch (0, 0)

(iii) Stretch factor = 15/6 = 2.5



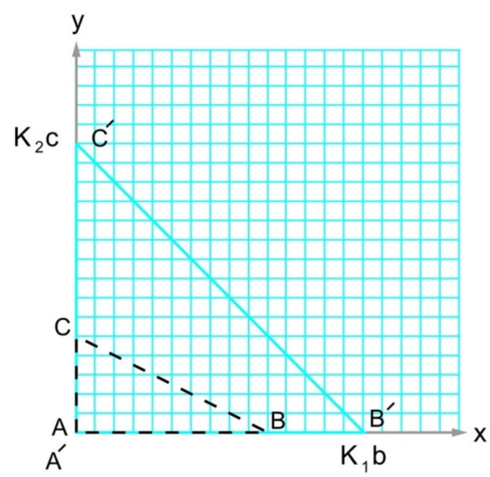
DOUBLE STRETCH

Double stretch enlarge the figure k_1 and k_2 times the original size in a direction perpendicular to y-axis and x-axis respectively, where y-axis and x-axis are invariant lines.

The point of intersection of invariant line is the centre of stretch.

Explanation:

ΔABC is made by an elastic and thumb pins. $\Delta A'B'C'$ is the image of ΔABC under a stretch defined above.



(i) Vertices B and C are at a distance b and c from the invariant lines respectively. So vertices B' and C' are at a distance k_1b and k_2c from the invariant lines respectively.

(ii) The vertex A, lies on the centre of stretch, does not move.

Transformation Matrix:

Stretch matrix	Stretch factor	Invariant Line	Direction of Stretch
$\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$	k_1 k_2	y-axis x-axis	perpendicular to y-axis perpendiclular to x-axis

Area:

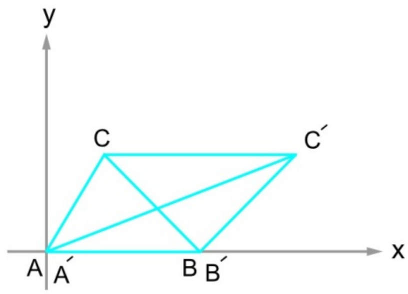
Area of image $= k_1k_2 \times$ area of object

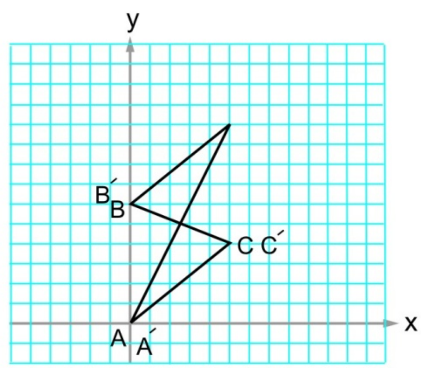
SHEAR

The transformation changes the shape of the object in this manner that area remains unchanged.

The direction of shear is always parallel to the invariant line.

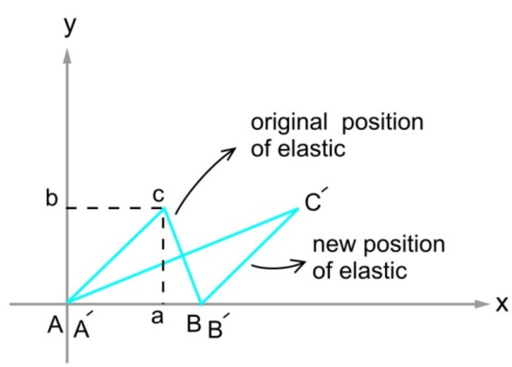
$\Delta A'B'C'$ is the image of ΔABC under a shear.





Explanation:

ΔABC is made by an elastic and thumb pins. $\Delta A'B'C'$ is the image of ΔABC under a share, scale factor k and invariant line x-axis.



(i) The vertices A and B lie on the invariant line. So these vertices do not move.

(ii) The coordinates of C are (a, b). So the coordinates of C are (a + kb, b).

(iii) Point of intersection of invariant line and y-axis is the centre of shearing.

(iv) Shear factor $= \dfrac{(a + kb) - a}{b}$.

Transformation Matrix:

Shear matrix	Shear factor	Invariant line	Direction of shear
$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$	k	x-axis	parallel to x-axis
$\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$	k	y-axis	parallel to y-axis

How does it Work:

It multiplies "the distance of a vertex from invariant line" to the "scale factor" and add abscissa or ordinate (according to the invariant line).

So the vertex nearer to the invariant line moves less than that vertex which is farther from invariant line.

Invariant line	Share factor	A vertex of the object	Corresponding vertex of the image
x-axis	k	(a, b)	(a + kb, b)
y-axis	k	(a, b)	(a, b + ka)

Explanation:

A(1, 2), B(1, 5) and C(4, 2) are the vertices of a triangle ABC. Triangles PQR and LMN are the images of ΔABC under a share, scale factor 2.

(i) x-axis is the invariant line:

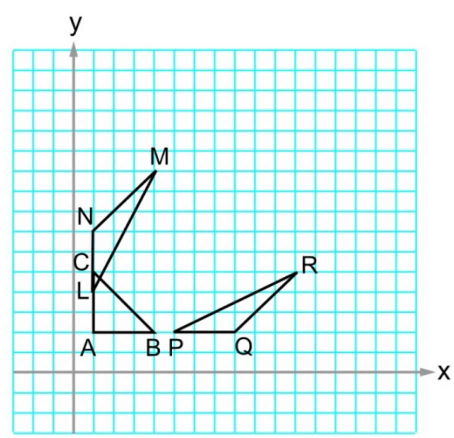
$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 1 & 1 & 4 \\ 2 & 5 & 2 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{pmatrix} 5 & 11 & 8 \\ 2 & 5 & 2 \end{pmatrix} \end{array}$$

A travels = 5 – 1 = 4

B travels = 11 – 1 = 10

C travels = 8 – 4 = 4

Since vertex C is farther from invariant line (x-axis) then vertices A and B. So vertex C travels more than A or B.



(ii) y-axis is the invariant line:

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 1 & 4 & 1 \\ 2 & 2 & 5 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{pmatrix} 1 & 4 & 1 \\ 4 & 10 & 7 \end{pmatrix} \end{array}$$

HOW TO LEARN TRANSFORMATION MATRICES

You will note that in Reflection and Rotation if the abscissa of the point change the sign the 1st row multiplye by (-1) and if the ordinate change the sign the 2nd row multiply by (-1).

In stretch and shear if the direction is parallel to x-axis multiply 1^{st} row by k and if the direction is parallel to y-axis multiply 2nd row by k.

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is unit matrix and k scale factor.

TYPE - 1

Transformation		Interchange the Position	Multiply (-1) to the Row	Transformation Matrix
REFLECTION	(a) x-axis	-	2^{nd}	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
	(b) y-axis	-	1^{st}	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
	(c) $y = x$	0 and 1	-	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
	(d) $y = -x$	0 and 1	1^{st} and 2^{nd}	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
ROTATION	(a) 90^{o} anticlockwise	0 and 1	1^{st}	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
	(b) 90^{o} clockwise	0 and 1	2^{nd}	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
	(c) 180^{o}	-	1^{st} and 2^{nd}	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
STRETCH	(a) Parallel to x-axis	-	multiply 1^{st} row by k	$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$
	(b) Parallel to y-axis	-	multiply 2^{nd} row by k	$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$

S H E A R	(a) Parallel to x-axis	-	Replace 0 with k in 1^{st} row.	$\begin{pmatrix}1 & k\\0 & 1\end{pmatrix}$
	(b) Parallel to y-axis	-	Replace 0 with k in 2^{nd} row	$\begin{pmatrix}1 & 0\\k & 1\end{pmatrix}$

EXERCISE D-6

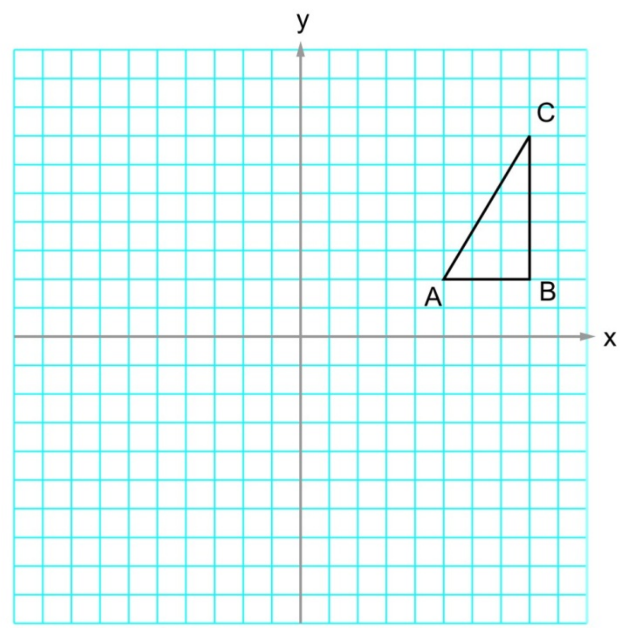
(1) Find the image of triangle ABC under the translation:

(i) $\begin{pmatrix}3\\2\end{pmatrix}$

(ii) $\begin{pmatrix}-6\\8\end{pmatrix}$

(iii) $\begin{pmatrix}-5\\-1\end{pmatrix}$

(iv) $\begin{pmatrix}4\\-2\end{pmatrix}$



(2) (4, 2), (8, 2) and (8, 5) are the vertices of a triangle P. The following translations

(i) $\begin{pmatrix}2\\4\end{pmatrix}$

(ii) $\begin{pmatrix}-15\\6\end{pmatrix}$

(iii) $\begin{pmatrix}-10\\-6\end{pmatrix}$

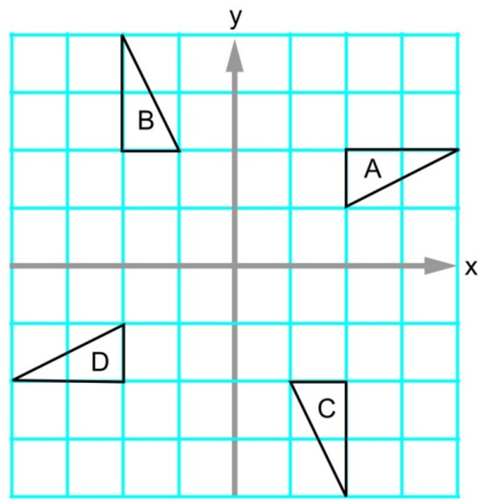
maps ΔP on to triangles Q, R and S respectively. Draw the traingles P, Q, R and S on the same graph paper, using 1 unit = 1cm.

(8) Describe fully the transformations which maps ΔA onto

(i) ΔB

(ii) ΔC

(iii) ΔD



(9) (2, 2), (4, 2), (4, 3) and (3, 3) are the vertices of a trapezium P. The following rotations centre at origin.

(i) 90^o clockwise (ii) 90^o anticlockwise (iii) 180^o

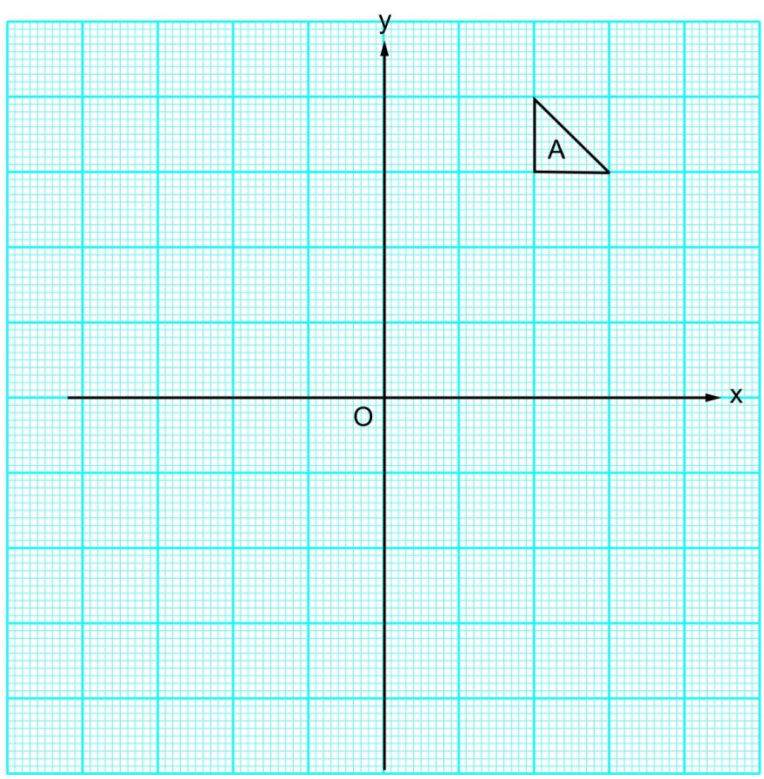
map trapezium P onto trapeziums Q, R and S respectively. Draw the trapeziums P, Q, R and S on the same graph paper, using 1unit = 1cm

(10) Rotate figure A centre at (2, 1)

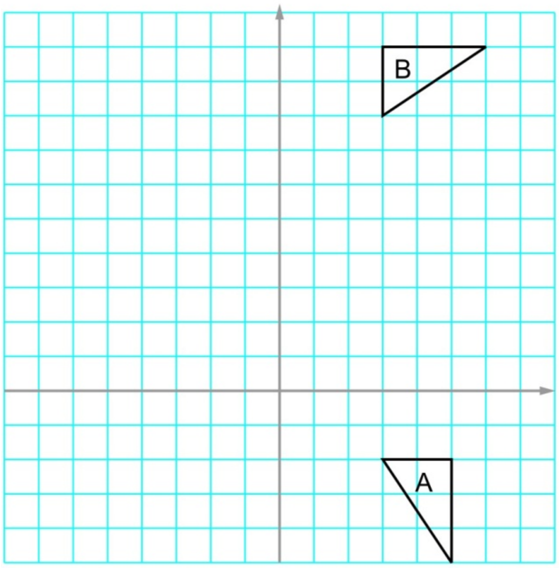
(i) 90^o clockwise

(ii) 90^o anticlockwise

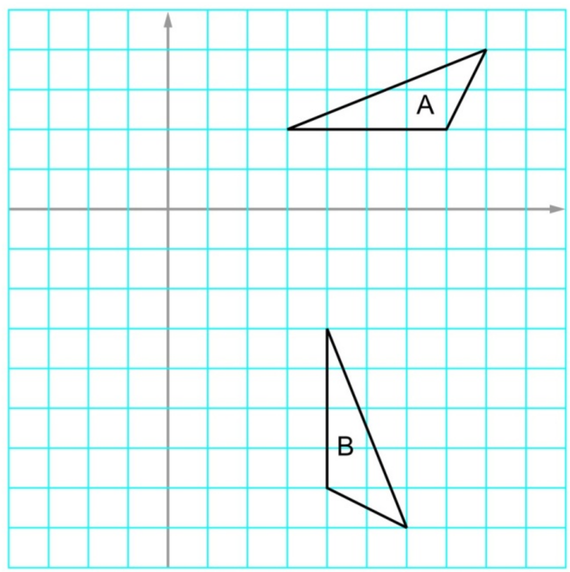
(iii) 180^o



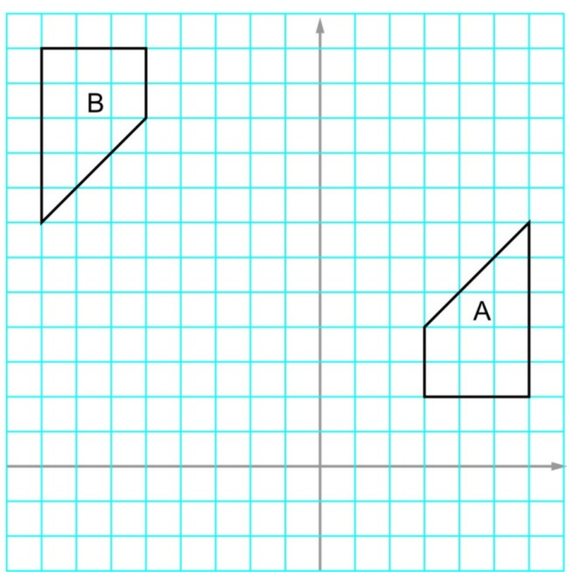
(11) A rotation maps ΔA onto ΔB. Find the centre of rotation and diection of rotation.



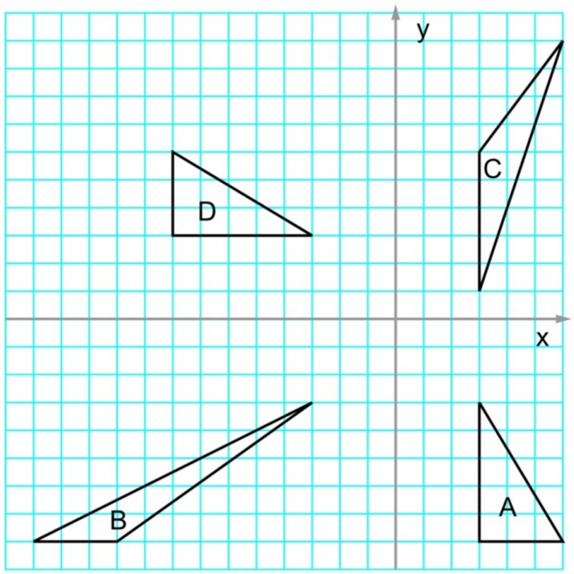
(12) A rotation maps ΔA onto ΔB. Find the centre of rotation and direction of rotation.



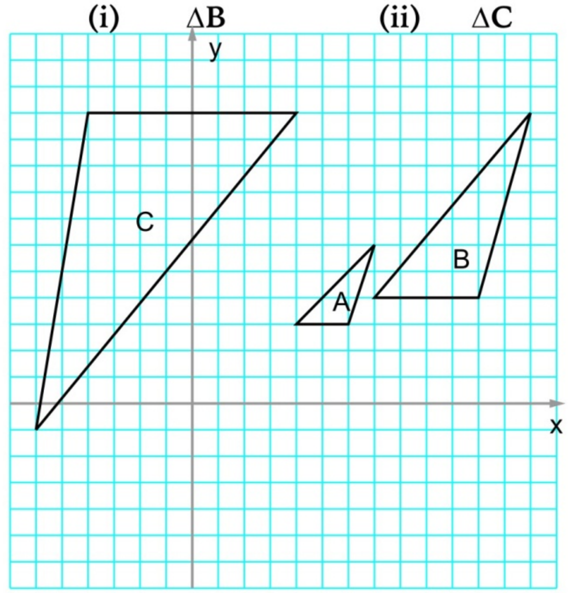
(13) A rotation maps ΔA onto ΔB. Find the centre of rotation and direction of rotation.



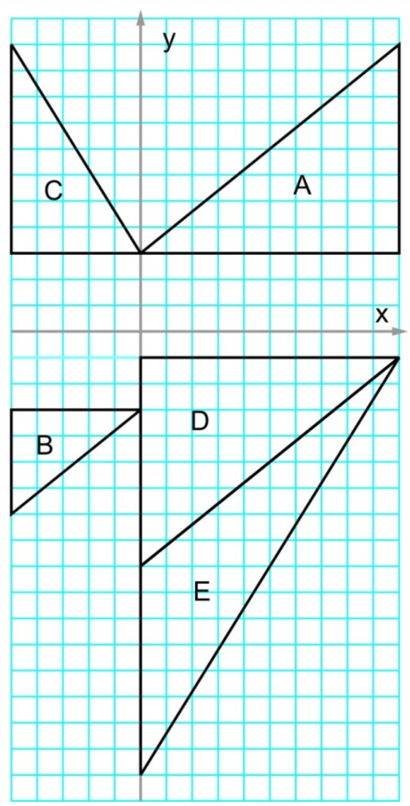
(19) Find the shear factor and invariant line such that ΔA maps onto
(i) ΔB (ii) ΔC (iii) ΔD



(20) Find the coordinates of centre of enlargement and scale factor such that ΔA maps onto
(i) ΔB (ii) ΔC



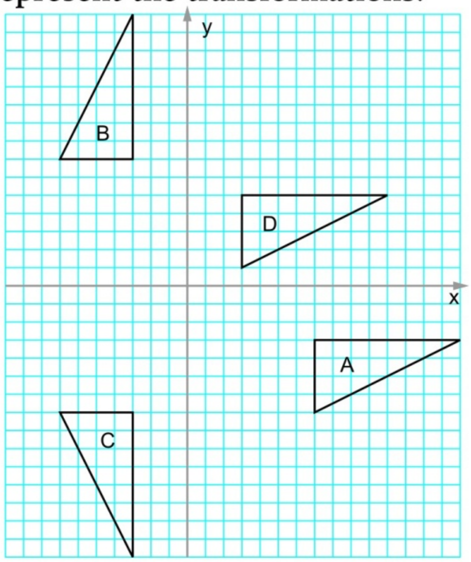
(21) The enlargement maps
(i) ΔA onto ΔB (ii) ΔB onto ΔC
(iii) ΔA onto ΔD (iv) ΔD onto ΔE.
Find the scale factor and coordinates of centres of enlargement in each case.



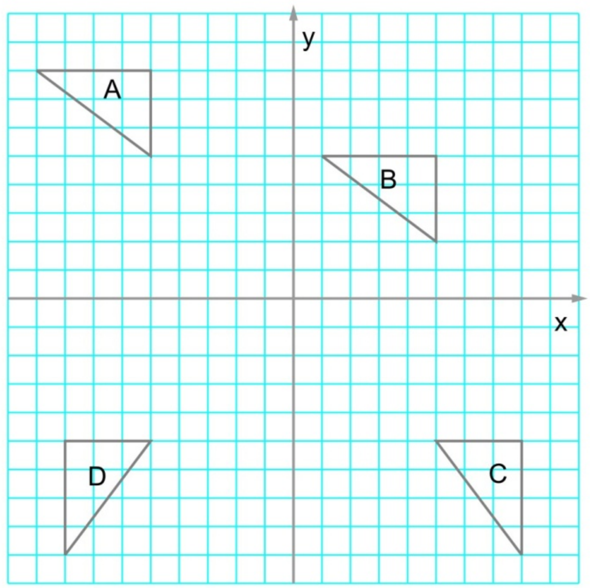
(22) (3, 2), (3, 4), (6, 4) and (8, 2) are the vertices of a trapezium A. An enlargement maps trapezium A onto trapezium B, centre at $(2, -2)$ and scale factor 2. Draw the trapeziums A and B on the same graph paper, using 1 unit = 1cm.

(27) Describefully the transformations which map ΔA onto (i) ΔB (ii) ΔC (iii) ΔD.

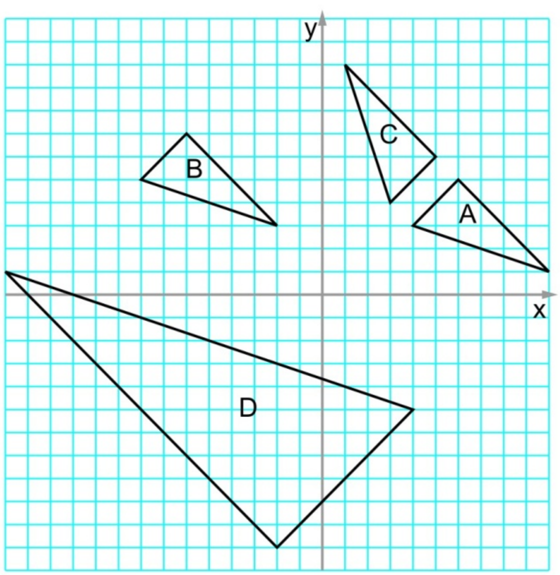
Also find the matrices which represent the transformations.



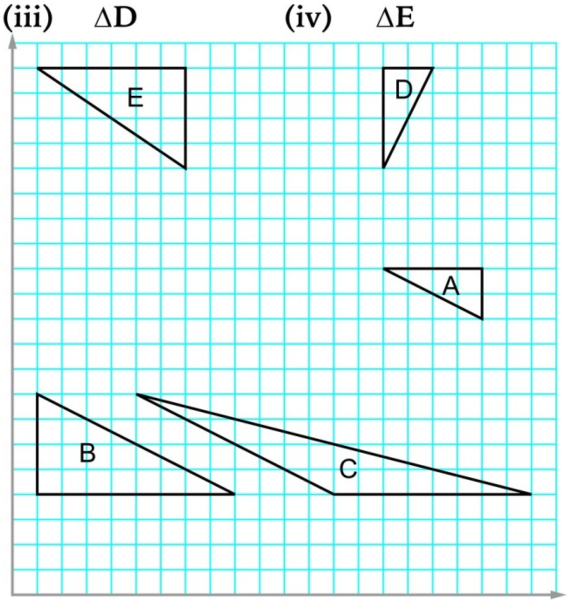
(28) Describefully the transformations which map ΔA onto (i) ΔB (ii) ΔC (iii) ΔD



(29) Describefully the transformations which map ΔA onto (i) ΔB (ii) ΔC (iii) ΔD

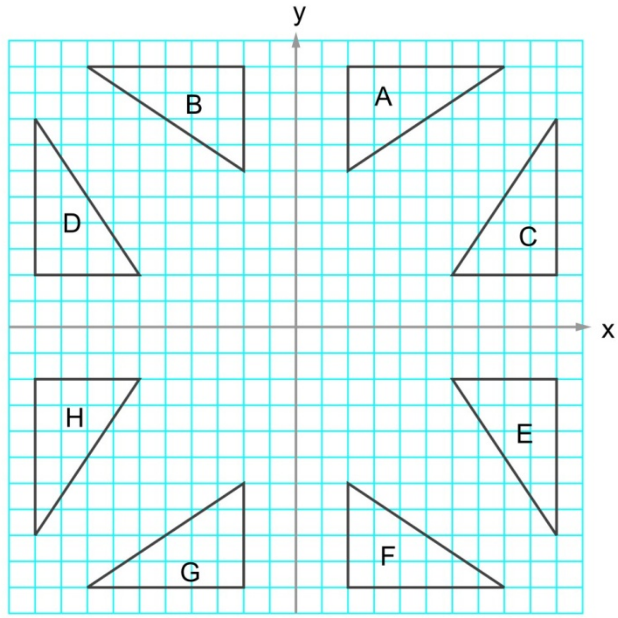


(30) Describefully the transformations which map ΔA onto (i) ΔB (ii) ΔC (iii) ΔD (iv) ΔE



M.C.Q's D-8

The following triangles are drawn for M.C.Qs. (1- 7).



(1) Identify the transformation which maps ΔA onto ΔB.
- (a) Translation
- (b) Rotation 90° anticlockwise
- (c) Reflection about y-axis
- (d) Rotation 90° clockwise

(2) What is the transformation which maps ΔA onto ΔC.
- (a) Rotation 90° clockwise
- (b) Reflection about y-axis
- (c) Translation
- (d) Rotation 180°

(3) Identify the transformation which maps ΔA onto ΔD.
- (a) Reflection about the line $y = x$
- (b) Translation
- (c) Rotation 90° clockwise
- (d) Rotation 90° anticlockwise

(4) Identify the transformation which maps ΔA to ΔE.
- (a) Translation
- (b) Rotation 90° clockwise
- (c) Rotation 90° anticlockwise
- (d) Reflection about a line.

(5) Identify the transformation which maps ΔA onto ΔF.
- (a) Reflection about x-axis
- (b) Rotation 90° clockwise
- (c) Translation
- (d) Rotation 180°

(6) Identify the transformation which maps ΔA onto ΔG.

(a) Reflection about $y = -x$ (b) Reflection about $y = x$

(c) Rotation 180° (d) Translation

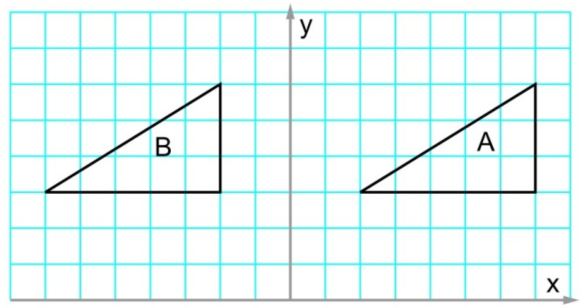
(7) Identify the traslation which maps ΔA onto ΔH.

(a) Translation (b) Rotation 180°

(c) Rotation 90° anticlockwise (d) Reflection about $y = -x$

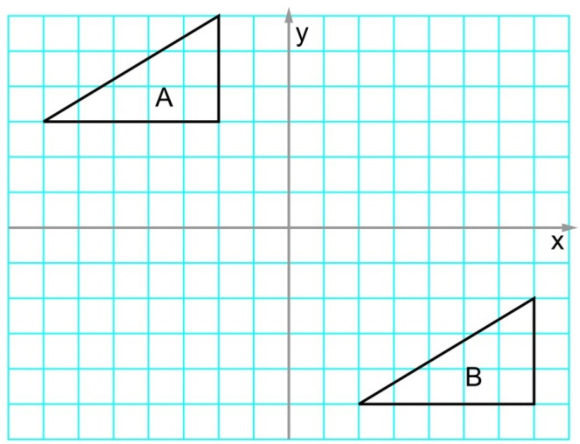
(8) ΔA maps onto ΔB. Identify the transformation.

(a) Rotation
(b) Reflection
(c) Translation
(d) Stretch



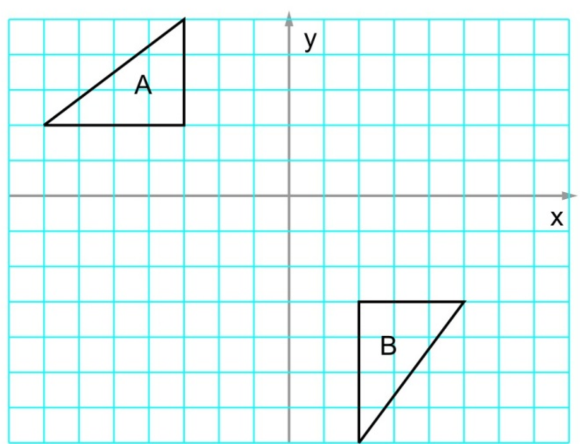
(9) ΔA maps onto ΔB. Identify the transformation.

(a) Rotation
(b) Reflection
(c) Translation
(d) Stretch



(10) ΔA maps onto ΔB. Identify the transformation.

(a) Rotation
(b) Reflection
(c) Translation
(d) Stretch



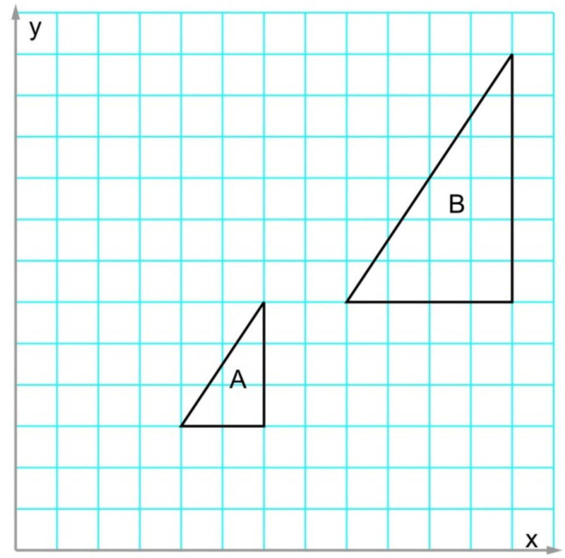
(19) The enlargement maps ΔA onto ΔB, what is the scale factor?

(a) 2

(b) -2

(c) $1/2$

(d) $-1/2$



(20) Point A' is the image of point A(3, 8) under a rotation 90° clockwise centre (0, 0). What are the coordinate of A'?

(a) $(-3, -8)$ (b) $(8, -3)$ (c) $(-8, 3)$ (d) $(8, 3)$

(21) Point P is the image of point A(3, 2) under an enlargement with centre (0, 0), scale factor 3. What are the coordinate of P?

(a) $(6, -9)$ (b) $(-9, -6)$ (c) $(6, 9)$ (d) $(9, 6)$

(22) Point P is the reflection of point A $(-3, 2)$ in the line $y = x$. What are the coordinates of P?

(a) $(-2, -3)$ (b) $(-3, -2)$ (c) $(3, -2)$ (d) $(2, -3)$

(23) Point P is the reflection of Point A $(5, -8)$ in the line $y = 0$, What are the coordinates of P?

(a) $(8, -5)$ (b) $(-5, -8)$ (c) $(5, 8)$ (d) $(8, 5)$

(24) Point A$(6, -2)$ is mapped onto point P under a stretch wityh stretch factor 2 and invariant line x-axis. What are the coordinates of P?

(a) $(6, -4)$ (b) $(3, -1)$ (c) $(12, -2)$ (d) $(2, -6)$

(25) Point A (2, 5) is mapped onto point P under a shear with shear factor -2 and invariant line y-axis. What are the coordinates of P?

(a) $(2, 7)$ (b) $(2, 1)$ (c) $(2, -5)$ (d) $(5, -4)$

(26) Point P (6, 2) is the image of point A (2, 5) under a translatio n. What is the coloumn matrix of the translation?

(i) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ (iii) $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ (iv) $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

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