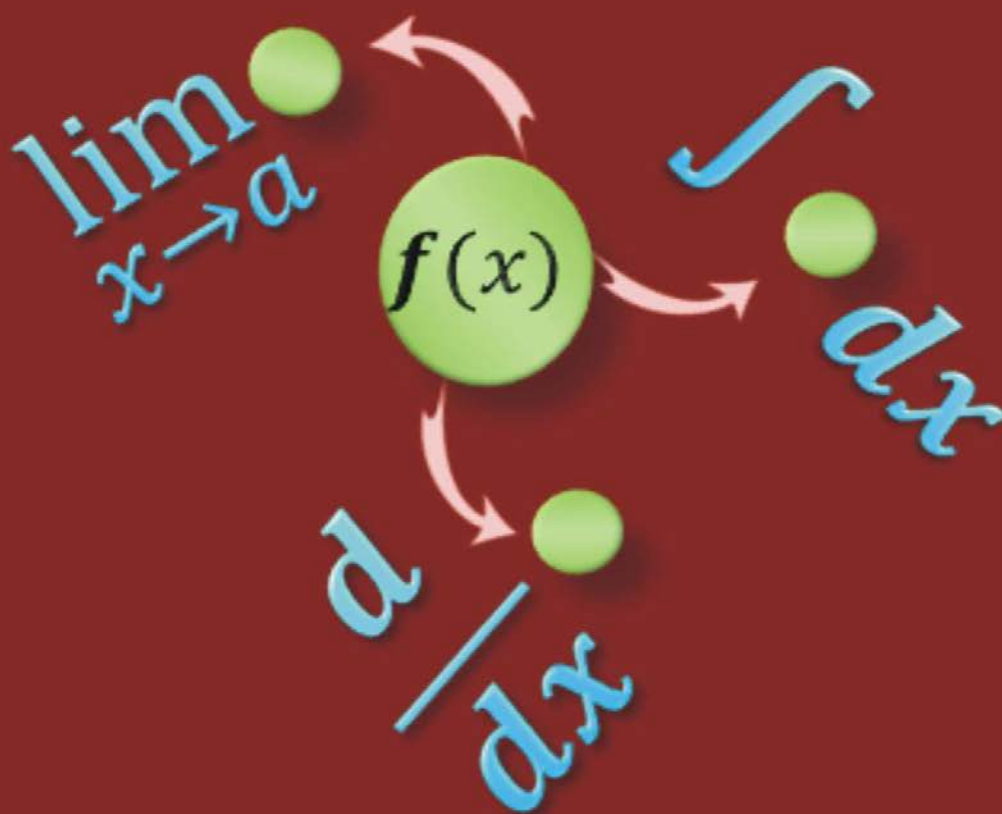


Book 2

CALCULUS

WITH APPLICATIONS

M. MAQSOOD ALI



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BOOK-2

CALCULUS

WITH APPLICATIONS

For Class B.Sc.

BY

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