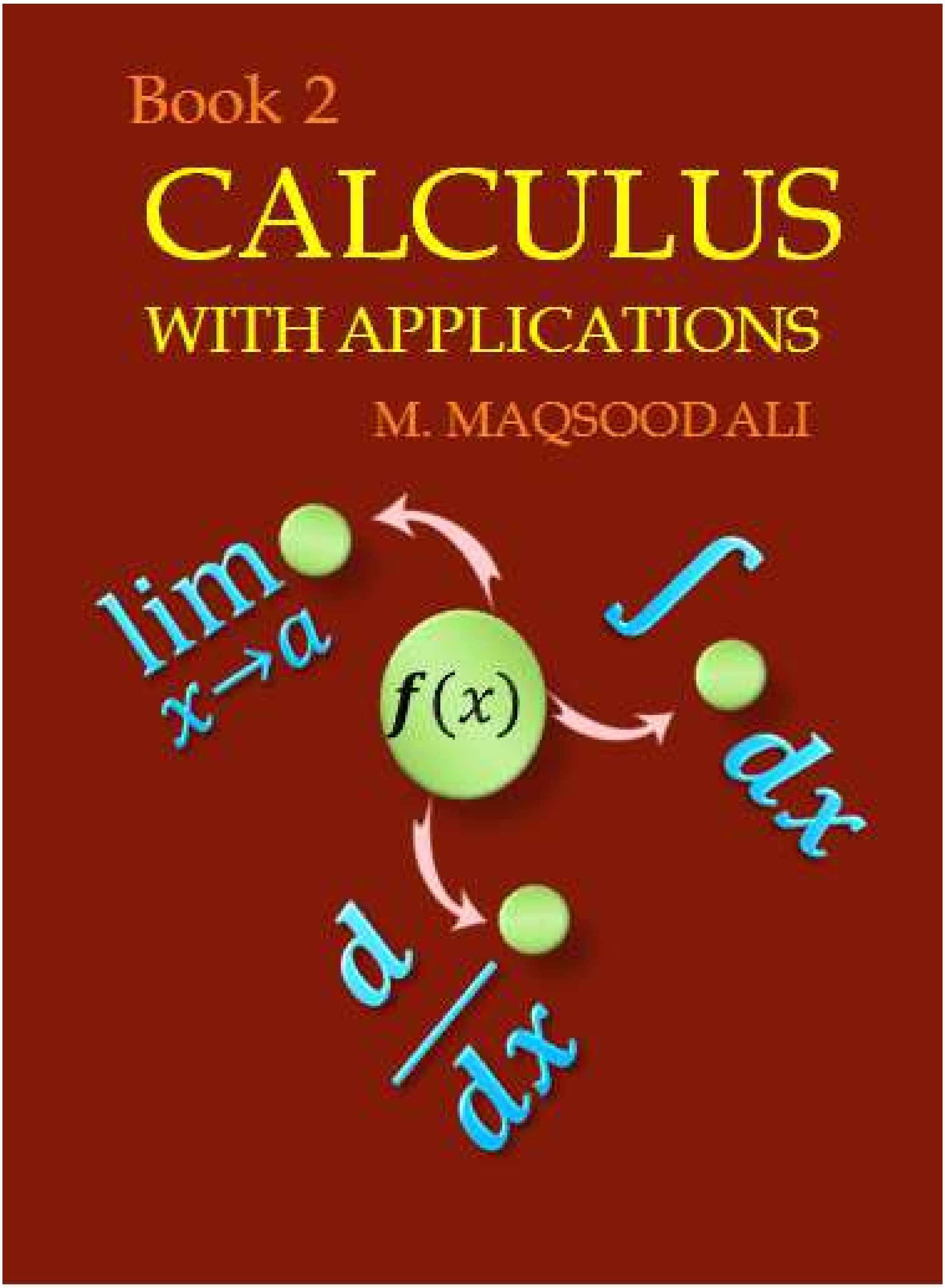
Book 2
CALCULUS
WITH APPLICATIONS
M. MAQSOOD ALI
lim
x→a
f(x)
∫
dx
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Chapter 10

INTEGRATIONS

There are two types of integration.

i) Indefinite integration ii) Definite integration

It is necessary to know before study the application of integration that what is the difference between these integrations.

<table>
<tr><th>INDEFINITE INTEGRATION</th><th>DEFINITE INTEGRATION</th></tr>
<tr><td>

Function is not given, it is to find out by the given rate of change and initial values.

To find curve or function:

The rate of change of a continuous function f(x) with respect to independent variable x is given that is $P(x)$. The indefinite integration is used to find out the function $f(x)$, such as

$$\frac{d}{dx}f(x) = P(x)$$

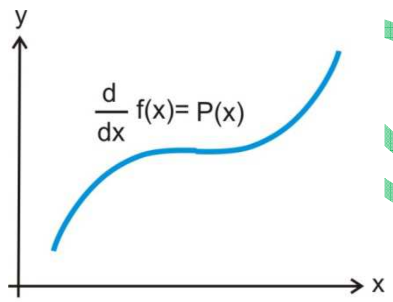

Figure 10.1

Using indefinite integration the function $f(x)$ can be found, such as

$$f(x) = \int P(x)\,dx$$
$$= F(x) + C$$

Where C is a constant of integration.

</td><td>

Function is given, which represents a curve, the area under the curve is to find out between two vertical lines.

To find area under the curve:

The function $f(x)$ is given which is continuous on $[a, b]$. The graph of $f(x)$ is a curve. The definite integration is used to find the area under the curve between two vertical lines $x = a$ and $x = b$.

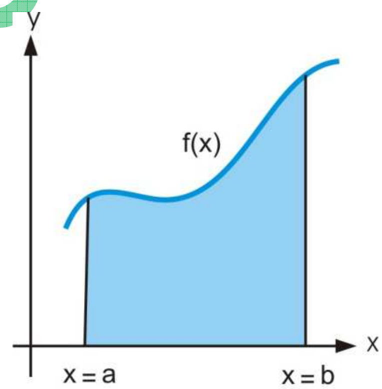

Figure 10.3 a

$$Area = \int_a^b f(x)dx$$
$$= [Q(x)]_b^a$$
$$= Q(b) - Q(a)$$

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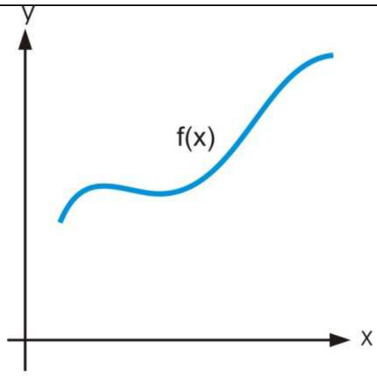

Figure 10.2

Constant term C:

After integrating a term C is added, which is called constant of integration.

Reason:

Suppose that

$f(x) = 4x^3 - 21x^2 + 43x + 10 \quad \rightarrow (1)$

Firstly, the rate of change of $f(x)$ w.r.t. x.

Derivative of $f(x)$ w.r.t. x

$\frac{d}{dx} f(x) = P(x) = 12x^2 - 42x + 43 \ \rightarrow (2)$

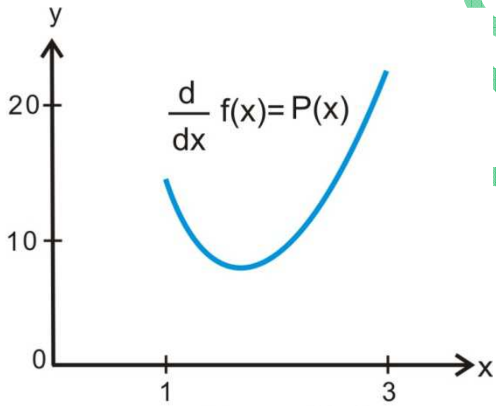

Figure 10.4

Graph of $\frac{d}{dx} f(x)$ is shown in figure 8.4.

Since indefinite integration is anti derivative, so the function f(x) can get back integrating equation (2).

$$\int df(x) \int (12x^2 - 42x + 43)dx$$

$$f(x) = 4x^3 - 21x^2 + 43x \quad \rightarrow (3)$$

Comparing (1) and (3), a term is missing which is constant term 10, so a constant term C is added in (3).

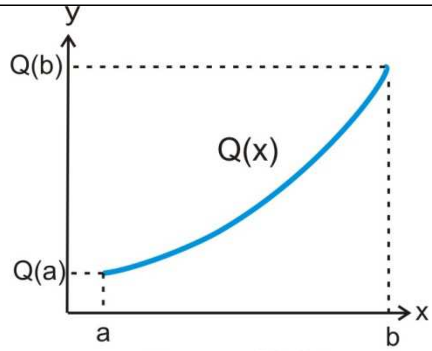

Figure 10.3 b

No constant term C:

After integrating no constant term C is added.

Reason:

Suppose that

$$P(x) = 12x^2 - 42x + 43$$

is a curve.

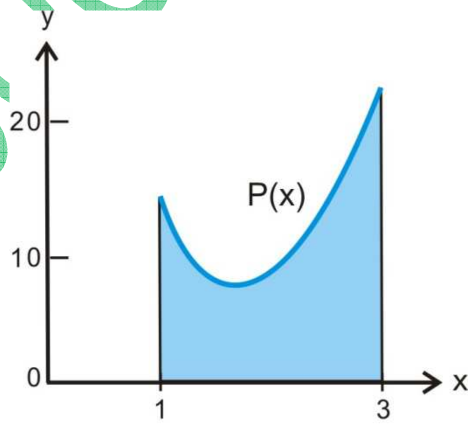

Figure 10.6 a

Area under the curve between the vertical lines $x = 1$ and $x = 3$.

Figure 10.6a

$$Area = \int_a^b P(x)dx$$

$$Area = \int_1^3 (12x^2 - 42x + 43\,)dx$$

$$= [4x^3 - 21x^2 + 43x]_1^3$$

The point is to be noted that there is no

$$f(x) = 4x^3 - 21x^2 + +43x + C \quad \rightarrow (4)$$

The value of C is calculated by initial values, which is a given point on the curve $f(x)$.

Initial condition $f(x) = 36$ at $x = 1$.

Putting these values in (4), we have

$$C = 10$$

So that

$$f(x) = 4x^3 - 21x^2 + 43x + 10$$

which is the required function same as (1).

The graph of the function $f(x)$ is shown in figure 10.5.

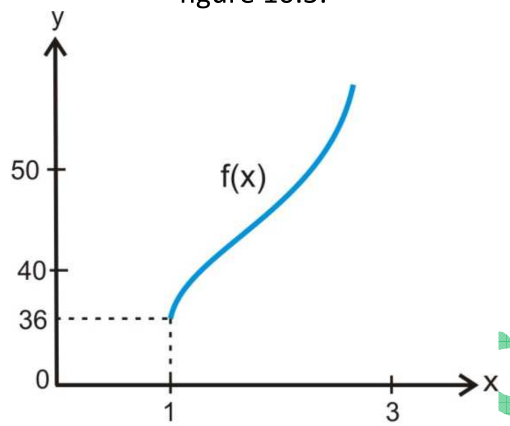

Figure *10.5*

Initial Value:

Initial value is a point (a, b) on the curve $f(x)$

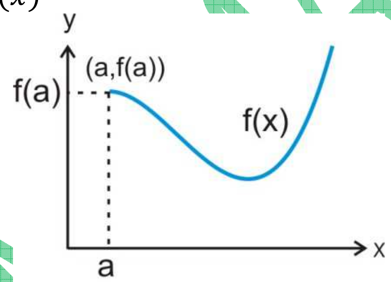

Figure *10.7*

a: abscissa (value on x-axis)

b: ordinate (value on y-axis)

Initial value is used to find the value of integral constant C.

Graph of the function:

The rate of change in $f(x)$ is given but the function $f(x)$ is not. The graph of the term is missing, so integral constant C will not be added.

$$= \{4 \times 3^3 - 21 \times 3^2 + 43 \times 3\}$$
$$-\{4 \times 1^3 - 21 \times 1^2 + 43 \times 1$$
$$= 48 - 26$$
$$= 22$$

Figure 10.6b

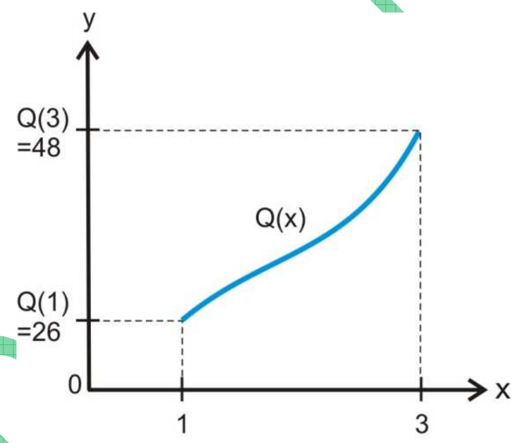

Figure *10.6 b*

Boundry condition:

Area under the curve $f(x)$ between two vertical lines $x = a$ and $x = b$.

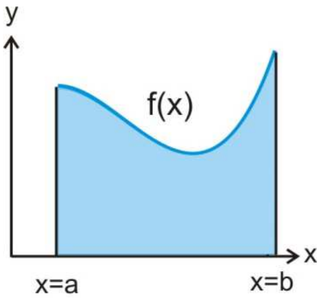

Figure *10.8*

The boundry conditions $x = a$ and $x = b$ are vertical lines not a point on the curve $f(x)$ and both the lines intersect x-axis.

Graph of the function:

The curve $f(x)$ is given, so the graph of the function can be drawn before integration.

function $f(x)$ can be drawn after finding the function by integrating $\frac{d}{dx}f(x)$.

For example, the rate of change in $f(x)$ with respect to x is give.

$$\frac{d}{dx}f(x) = P(x)$$

The graph of $\frac{d}{dx}f(x)$ is drawn, figure 10.1 but the graph of $f(x)$ can not be drawn.

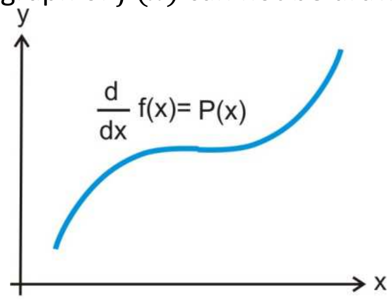

Figure 10.1

To draw the the graph of $f(x)$, we will have to integrate

$$\frac{d}{dx}f(x) = P(x)$$

$$f(x) = \int P(x)\,dx$$

$$= F(x) + C$$

Now the graph of $f(x)$ can be drawn, as shown in figure 10.2

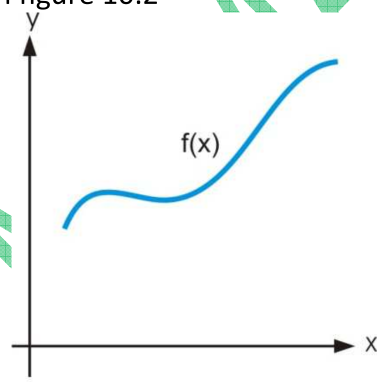

Figure 10.2

For example, The function $f(x)$ is given which is continuous on $[a, b]$. To find the area under the curve between two vertical lines $x = a$ and $x = b$. The graph of the function can be drawn as shown in figure 10.3a.

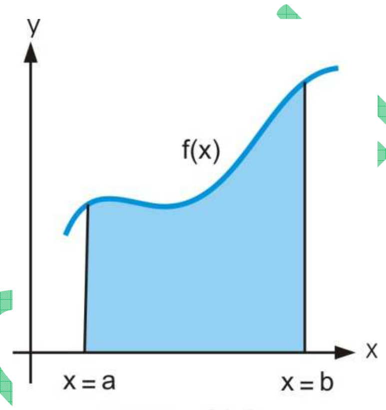

Figure 10.3 a