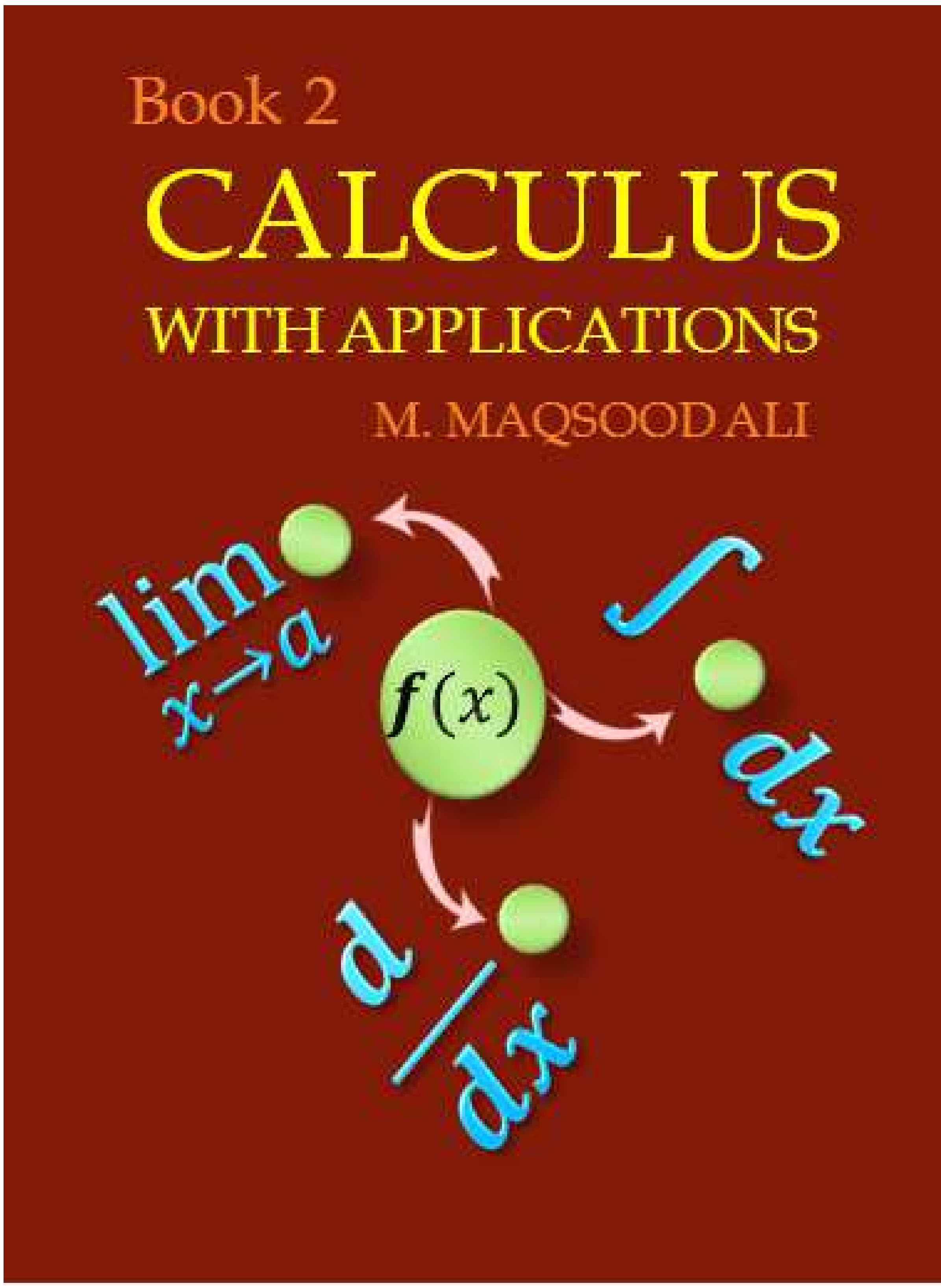
Book 2
CALCULUS
WITH APPLICATIONS
M. MAQSOOD ALI
lim x→a
f(x)
∫ dx
d/dx

Chapter 9

TAYLOR'S AND MACLAURIN'S SERIES

The values of all real valued functions at any real number can be calculated by a scientific calculator. The buttons on scientific calculator give an idea that almost all real valued functions are formed by $x^{m/n}$, ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and four operations $+$, $-$, $\times$, $\div$ except the functions involving $log_b x$, b^x and trigonometric functions, where m, n and b are real numbers. These functions are also represented by a polynomial or a series of x. An approximate value of e^x, $sinx$ or $cosx$ can be found for a real value x by pressing a button on a calculator. All the values of these trigonometric functions are not fed in the calculator because there are infinite values, so a program is fed in a calculator or a computer which are based on polynomials. Taylor's polynomials of degree n for e^x and $cosx$ centred at zero are given below, where n is a non-negative integer.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

$$cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!}$$

Extending Taylor's polynomials obtain a power series that represents the function exactly.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n}}{(2n)!}$$

Truncation error occurs when the series of a function $f(x)$ is truncated to finite number of terms, which is a polynomial, to calculate the value of the function for a real value $x = a$. The round off error arises as the power of x increases of the polynomial.

Numerical Analysis, a branch of mathematics is studied to learn how mathematical problems computerized to solve them. Almost all the topics of Numerical Analysis related to calculate the values of the functions for a real value of x are based on polynomials. Following topics from Numerical Analysis explain how polynomial forms.

- Curve fitting
- Interpolating polynomials
- Lagrangian polynomials
- Divided differences

The polynomials are formed using the above methods following a pattern and is used for computer programming. The following polynomials do not follow a pattern.

- $2x^5 - 6x^2 + 9$
- $6x^8 - 2x^2 + 3x$
- $x^7 + 5x^4 + 2x + 6$

Above polynomials can be represented by the polynomials which follow a pattern using the above methods. Taylor's series expresses most functions as a power series based on derivatives of the functions. Taylor's series is expanded about a point $x = a \in \mathbb{R}$, when $a = 0$ the series is called Maclaurin's series.

Polynomials and Value of the functions:

The accuracy of the value of a function $f(x)$ depends on number of terms of its polynomial.

$$f(x) \cong P_n(x)$$

More terms of polynomial gives more accuracy in the value of the function.

Taylor's series of $sinx$ is

$$cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

This series gives exact value of $cosx$.

Taylor's polynomial for $cosx$ is

$$cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

The function $cosx$ and its Taylor's polynomial of degree 1, 4 and 6 are given below.

$$cosx = P_1(x) = 1 - \frac{x^2}{2!}$$

$$cosx = P_4(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$cosx = P_6(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

The graphs of $cosx$ and its polynomials between 0 and 2π are given below.

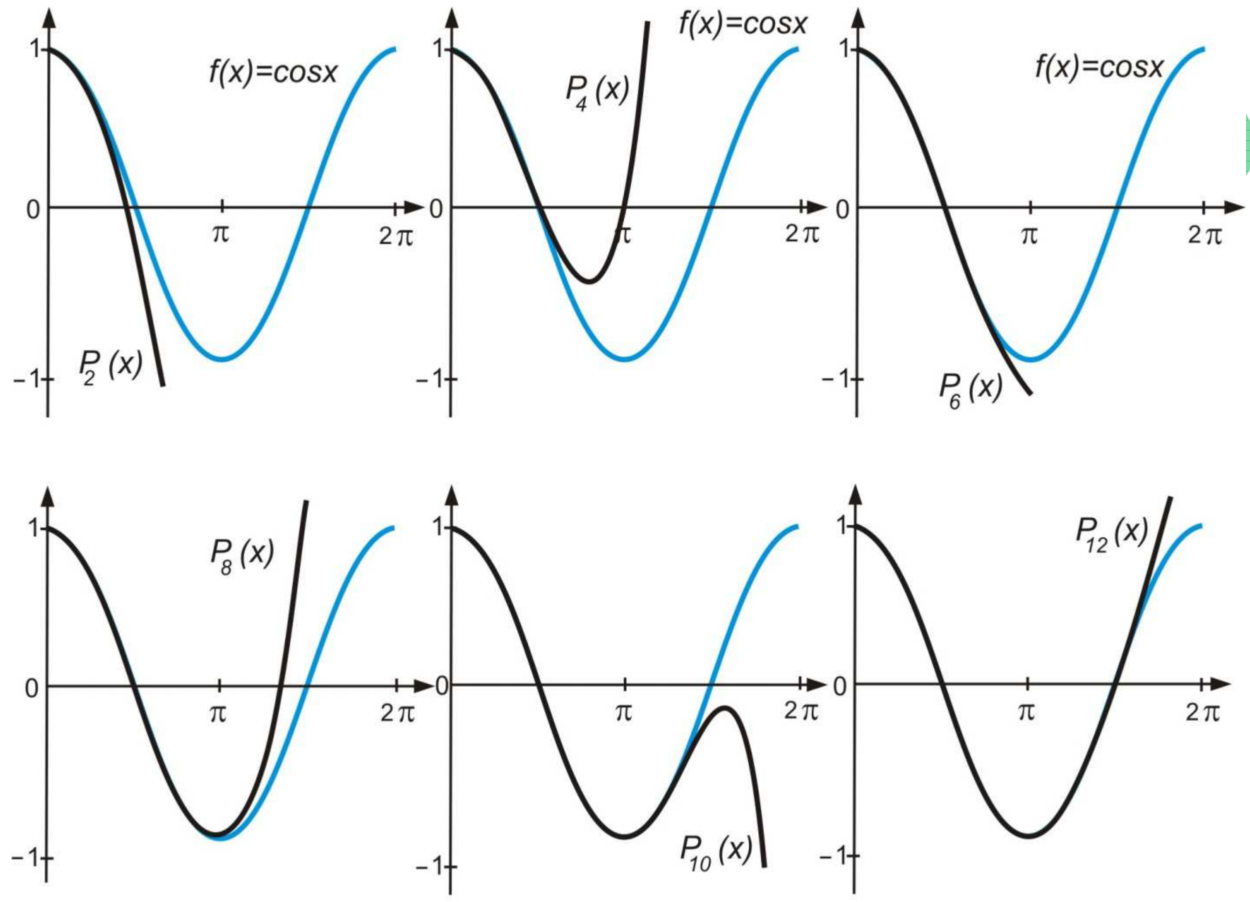

Figure 9.1

All graphs are drawn from $(0,0)$ but $P_{12}(x)$ is the nearest graph to the graph of $cosx$ between 0 and 2π.

TAYLOR'S SERIES

If $f(x)$ is a function and all derivatives of $f(x)$ exist at $x = a$, then

$$f(x) = f(a + (x - a))$$

$$= f(a) + f'(a).(x-a) + \frac{1}{2!}f''(a).(x-a)^2 + \frac{1}{3!}f'''(a).(x-a)^3 + \cdots + \frac{1}{n!}f^{(n)}(a).(x-a)^n + R_n(x)$$

when $R_n(x) \to 0$ as $n \to \infty$

where $R_n(x) = \frac{1}{(n+1)!}f^{n+1}(c)(x-a)^{n+1} \quad for \quad c \in (a, b)$

Proof:

The following is the proof for $a < b$. The proof for $a > b$ is left for the reader.

$P_n(x)$ is a Taylor's polynomial of degree n.

$$P_n(x) = f(a) + f'(a).(x-a) + \frac{1}{2!}f''(a).(x-a)^2 + \frac{1}{3!}f'''(a).(x-a)^3 + \cdots + \frac{1}{n!}f^{(n)}(a).(x-a)^n \quad \to (1)$$

In general $P_n(x)$ is equal to $f(x)$ for some values of $x \in \mathbb{R}$, but not for all values, as shown in the following figures 9.2 (a), (b).

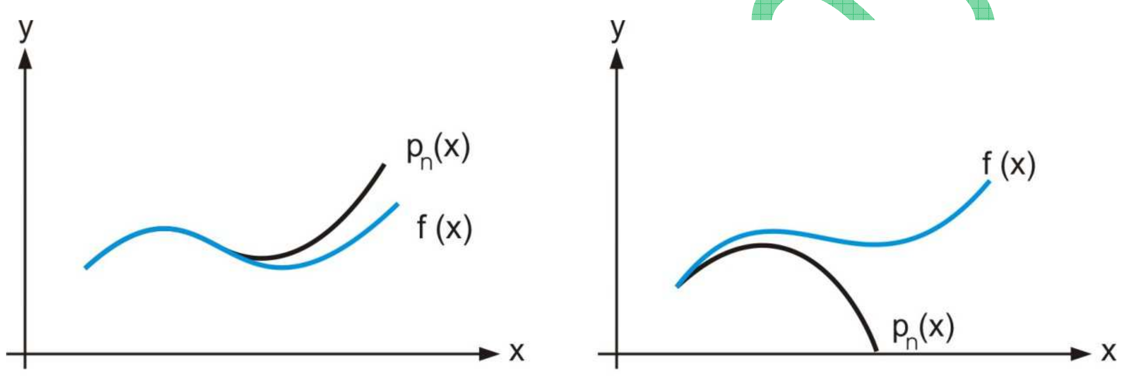

Figure 9.2 (a), (b)

$U_n(x)$ is another polynomial such that

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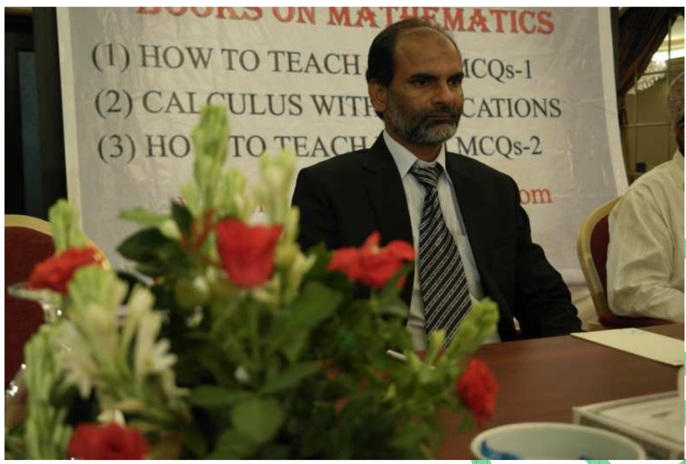
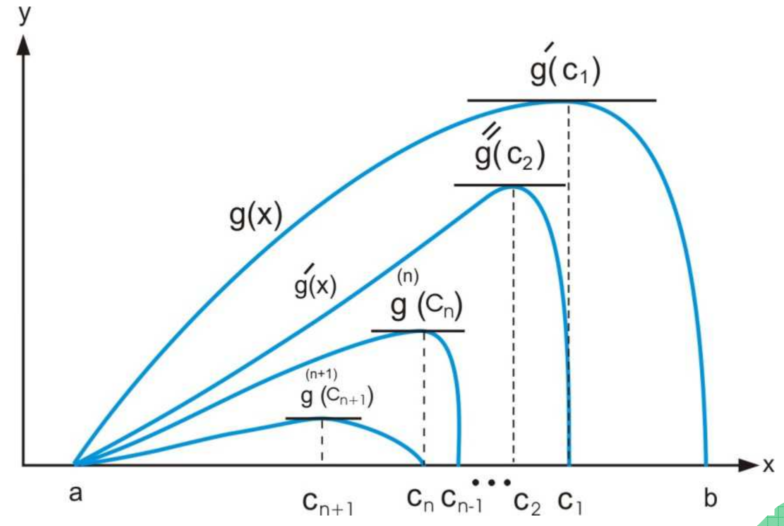


Figure 9.4



The difference of $f(x)$ and $U_n(x)$ is a function $g(x)$ such that

$$g(x) = f(x) - U_n(x) \qquad \to (6)$$

$$g(x) = f(x) - f(a) - f'(a).(x-a) - \frac{1}{2!}f''(a).(x-a)^2 - \frac{1}{3!}f'''(a).(x-a)^3 - \cdots - \frac{1}{n!}f^{(n)}(a).(x-a)^n - M(x-a)^{n+1} \qquad \to (7)$$

$$g'(x) = f'(x) - f'(a) - f''(a).(x-a) - \frac{1}{2!}f'''(a).(x-a)^2 - \cdots - \frac{1}{(n-1)!}f^{(n)}(a).(x-a)^{n-1} - M(n+1)(x-a)^n \qquad \to (8)$$

$$g''(x) = f''(x) - f''(a) - f'''(a).(x-a) - \cdots - \frac{1}{(n-2)!}f^{(n)}(a).(x-a)^{n-2} - M(n+1)n(x-a)^{n-1} \qquad \to (9)$$

$$\vdots$$

$$g^{(n)}(x) = f^{(n)}(x) - f^{(n)}(a) - M(n+1)!\,(x-a) \qquad \to (10)$$

$$g^{(n+1)}(x) = f^{(n+1)}(x) - M(n+1)! \qquad \to (11)$$

since $g(a) = g(b) = 0$ by (7) and (6) respectively and $g(x)$ continuous on $[a,b]$ and derivable on (a,b), by Rolle's theorem

$$g'(c_1) = 0 \qquad \text{for some} \qquad c_1 \in (a,b)$$

since $g'(a) = g'(c_1) = 0$ and $g'(x)$ continuous on $[a,b]$ and derivable on (a,b), by Rolle's theorem

$$g''(c_2) = 0 \qquad \text{for some} \qquad c_2 \in (a,c_1)$$

$$\vdots$$

$$g^{(n)}(c_n) = 0 \qquad \text{for some} \qquad c_n \in (a,c_{n-1})$$

$$g^{(n+1)}(c_{n+1}) = 0 \qquad \text{for some} \qquad c_{n+1} \in (a,c_n)$$

Putting $x = c_{n+1}$ in (11)

$$g^{(n+1)}(c_{n+1}) = f^{(n+1)}(c_{n+1}) - M\,(n+1)!$$
$$0 = f^{(n+1)}(c_{n+1}) - M\,(n+1)!$$
$$M = \frac{1}{(n+1)!}\,f^{(n+1)}(c_{n+1})$$

Putting $c_{n+1} = c \in (a,b)$

$$M = \frac{1}{(n+1)!}\,f^{(n+1)}(c) \qquad \rightarrow (12)$$

Putting in (5)

$$f(b) = P_n(b) + \frac{1}{(n+1)!}f^{(n+1)}(c)\,(b-a)^{n+1} \qquad , \qquad c \in (a,b)$$

Hence the theorem is proved.

Replacing b by x

$$f(x) = P_n(x) + \frac{1}{(n+1)!}\,f^{(n+1)}(c)(x-a)^{n+1}$$

MACLAURIN'S SERIES

Let $f(x)$ be a function, if $f(x)$ has $(n+1)$ derivatives at $x = 0$, the following is the Maclaurin's series.

$$f(x) = f(0) + f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots + R_n(x)$$

For equality $R_n(x) \rightarrow 0$ as $n \rightarrow \infty$

where $$R_n(x) = \frac{1}{(n+1)!}\,f^{(n+1)}(c).\,(x)^{n+1}$$

General Form of the Polynomial and n:

Macluarin's polynomial for $sinx$ is

$$P_n(x) = sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots \pm nth\ term$$

There is a confusion for n, when the polynomial is written in general form, such that

$$P_n(x) = sinx = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

it is wrong because if $n = 5$, then

$$P_5(x) = sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}$$

which is wrong, because

$$P_5(x) = sinx = f(0) + \frac{x}{1!} f^{(1)}(0) + \frac{x^2}{2!} f^{(2)}(0)$$
$$+ \frac{x^3}{3!} f^{(3)}(0) + \frac{x^4}{4!} f^{(4)}(0) + \frac{x^5}{5!} f^{(5)}(0)$$
$$= x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

The solution of this problem. Write down the general term of the polynomial

$$\text{General term} = \frac{(-1)^k x^{2k+1}}{(2k+1)!}\ ,\ k = 0,1,2.3, \cdots$$

$$So \quad n = 2k + 1 \quad \Rightarrow \quad k = \frac{n-1}{2}$$

The general form of the polynomial is

$$P_n(x) = sinx = \sum_{k=0}^{\frac{n-1}{2}} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

which is true.
For example, if $n = 5$

$$P_5(x) = sinx = \sum_{k=0}^{2} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$
$$= x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

which is exactly true.

EXAMPLES FOR TAYLOR AND MACLAURIN POLYNOMIAL:

Example 9.1:

Write down the first n terms, remainder and the general form of order n of Maclaurin polynomial to the function

$$f(x) = sinx.$$

(a) Using Maclaurin polynomial find and approximate value of $\sin \pi/4$, correct to five decimal places, for $n = 5$.

(b) (i) Find remainder and approximate value of $sin\pi/6$, for $n = 3$.

(ii) Find the value of c such that $sin\,\pi/6 = 0.5$.

(c) Prove that $f(x) = P_5(x)$, correct to one decimal places for all $x \in (0, \frac{\pi}{2})$.

Solution:

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$sinx$	0
1	$cosx$	1
2	$-sinx$	0
3	$-cosx$	-1
4	$sinx$	0
5	$cosx$	1
6	$-sinx$	0
⋮	⋮	⋮
n	$\pm sins$ or $\pm cosx$	

$$f(x) = P_n(x) + R_n(x)$$
$$= f(0) + \frac{x}{1!}f^{(1)}(0) + \frac{x^2}{2!}f^{(2)}(0) + \frac{x^3}{3!}f^{(3)}(0)$$
$$+ \cdots + \frac{x^n}{n!}f^{(n)}(0) + \frac{x^{n+1}}{(n+1)!}f^{(n+1)}(c)$$

$$sinx = 0 + \frac{x}{1!}(1) + 0 + \frac{x^3}{3!}(-1) + 0 + \frac{x^5}{5!}(-1) + \cdots$$
$$+ \frac{x^{n+1}}{(n+1)!}f^{n+1}(c)$$

where

$$P_n(x) = P_{2k+1}(x) = \sum_{k=0}^{\frac{n-1}{2}} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \quad \text{and}$$

$$R_n(x) = \frac{x^{n+1}}{(n+1)!} f^{(n+1)}(c)$$

(a) $n = 5 \;\Rightarrow\; 2k+1 = 5 \;\Rightarrow\; k = 2$

$$P_5(x) = \sum_{k=0}^{2} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

Figure 9.5

so

$$f\left(\frac{\pi}{4}\right) \cong P_3\left(\frac{\pi}{4}\right)$$

$$\sin \pi/4 \cong \frac{\pi}{4} - \frac{\left(\frac{\pi}{4}\right)^3}{3!} + \frac{\left(\frac{\pi}{4}\right)^5}{5!}$$

$$= 0.785398 - 0.080796 + 0.002490$$

$$= 0.707142$$

$$= 0.70714 \quad \text{(correct to five decimal)}$$

(b) (i) $n = 3 \;\Rightarrow\; 2k+1 = 3 \;\Rightarrow\; k = 1$

$$P_3(x) = \sum_{k=0}^{1} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$= x - \frac{x^3}{3!}$$

Figure 9.6

Now

$$f(\pi/6) = P_3(\pi/6) + R_3(\pi/6)$$

$$\sin\frac{\pi}{6} = \frac{\pi}{6} - \frac{\left(\frac{\pi}{6}\right)^3}{3!} + \frac{\left(\frac{\pi}{6}\right)^4}{4!} f^{(4)}(c)$$

$$= 0.499674 + 0.003132\, \sin c$$

Approximate value of $\sin\frac{\pi}{6} = 0.499674$

Remainder $= 0.003132\, \sin c$

(ii) Exact value of $\sin\left(\frac{\pi}{6}\right)$ is 0.5 (i.e. $\sin\left(\frac{\pi}{6}\right) = 0.5$), so

$$0.5 = 0499674 + 0.003132\, \sin c$$

$$c = 0.10427 \in \left(0, \frac{\pi}{6}\right)$$

(c) $n = 5 \;\Rightarrow\; 2k+1 = 5 \;\Rightarrow k = 2$

$$P_5(x) = \sum_{k=0}^{2} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

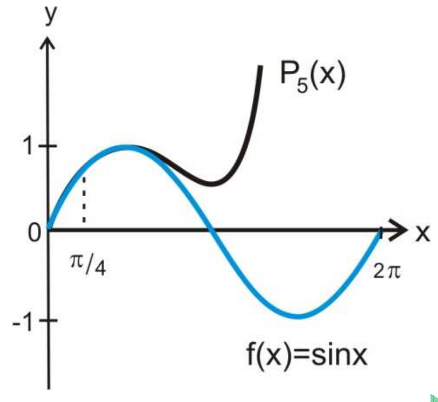


Figure 9.5

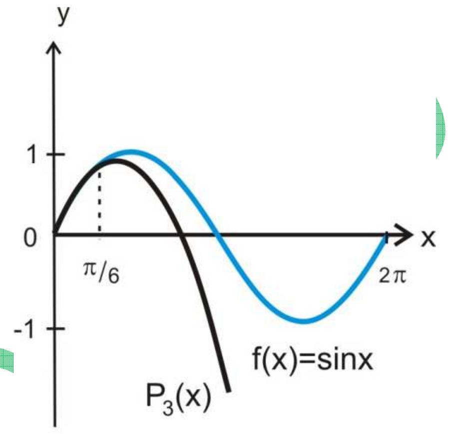


Figure 9.6

Since

$$f(\pi/2) = P_5(\pi/2) + R_5(\pi/2)$$

$$sin\frac{\pi}{2} = \frac{\pi}{2} - \frac{\left(\frac{\pi}{2}\right)^3}{3!} + \frac{\left(\frac{\pi}{2}\right)^5}{5!} + \frac{\left(\frac{\pi}{2}\right)^6}{6!} f^{(6)}(c)$$

$$= 1.004 - 0.02\, sinc \qquad \rightarrow (1)$$

since $0 < sinc < 1$ for all $c \in \left(0, \frac{\pi}{2}\right)$, so

$R_5\left(\frac{\pi}{2}\right) = -0.02\, sinc \;\rightarrow 0$, correct to one decimal place.

since $|R_5(x)| < \left|R_5\left(\frac{\pi}{2}\right)\right|$ for all $x \in \left(0, \frac{\pi}{2}\right)$

so $R_5(x) \rightarrow 0$,correct to one decimal place, $\forall\, x \in \left(0, \frac{\pi}{2}\right)$.

Thus $f(x) = P_n(x)$, correct to one decimal, $\forall\, x \in \left(0, \frac{\pi}{2}\right)$.

Example 9.2:

Write down the first n terms, remainder in term of c and the general form of the Maclaurin polynomial for the following function

$$f(x) = e^x \qquad \text{for} \quad -\infty < x < \infty.$$

(a) Find the approximate value and remainder in term of c when $x = 3$ and $n = 4$.

(b) The exact value of e^3 is 20.0855, correct to four decimal places, find the remainder.

(c) Prove that $c \in (0,3)$.

Solution:

k	$f^{(k)}(x)$	$f^{(k)}(0)$
0	e^x	1
1	e^x	1
2	e^x	1
3	e^x	1
4	e^x	1
5	e^x	1
$\vdots$	$\vdots$	$\vdots$
n	e^x	
$n+1$	e^x	

Maclaurin's series for e^x is

$$f(x) = f(0) + \frac{x}{1!}f^{(1)}(0) + \frac{x^2}{2!}f^{(2)}(0) + \frac{x^3}{3!}f^{(3)}(0) + \frac{x^4}{4!}f^{(4)}(0) + \frac{x^5}{5!}f^{(5)}(0) + \cdots + \frac{x^n}{n!}f^{(n)}(0) + \frac{x^{n+1}}{(n+1)!}f^{(n+1)}(c)$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots + \frac{x^n}{n!} + \frac{x^{n+1}}{(n+1)!}e^c$$

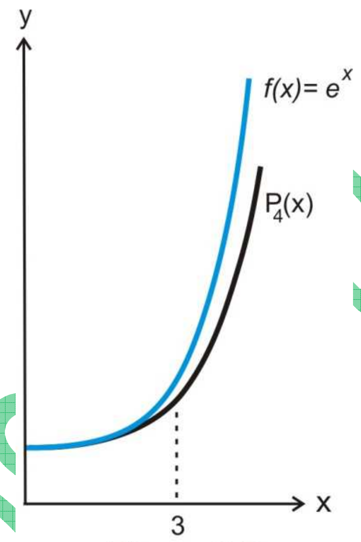


Figure 9.7

$$\text{General term} = \frac{x^k}{k!} \quad , \quad k = 0,1,2,3.\cdots$$

So $n = k \quad \Rightarrow \quad k = n$

The general form of the polynomial is

$$P_n(x) = e^x \cong \sum_{k=0}^{n} \frac{x^k}{k!}$$

(a) For $n = 4$

$$P_4(x) = e^x \cong \sum_{k=0}^{4} \frac{x^k}{k!}$$

So that

$$e^x = P_4(x) + R_4(x)$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}e^c$$

For $x = 3$

$$e^3 = 1 + \frac{3}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!}e^c$$

$$= 16.375 + 2.025e^c$$

Approximate value of e^3:

$$e^3 \cong P_4(3) = 16.375$$

Figure 9.7

Remainder in term of c:

$$R_4(3) = 0.025e^c$$

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where Maclaurin polynomial is

$$P_n(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots + \frac{x^n}{n!}$$
$$= \sum_{k=0}^{n} \frac{x^n}{n!}$$

and the remainder

$$R_n(x) = \frac{x^{n+1}}{(n+1)!}\, e^c$$

(a)

$$P_6(x) = \sum_{k=0}^{6} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!}$$

$$e^x \cong P_6(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!}$$

$$e^3 \cong P_6(3) = 1 + \frac{3}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!} + \frac{3^6}{6!}$$

$$= 19.4125$$

(b)

$$e^2 \cong P_6(2) = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \frac{2^5}{5!} + \frac{2^6}{6!}$$

$$= 7.355556$$

The remainder is

$$R_6(2) = f(2) - P_6(2)$$

$$= 7.389056 - 7.355556$$

$$= 0.0335$$

Figure 9.8

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$$(a)\quad P_4(2)=\frac{1}{5}-\frac{2}{25}+\frac{2^2}{125}-\frac{2^3}{625}+\frac{2^4}{3125}$$
$$=0.14432$$
$$R_4(2)=-\frac{2^5}{(c+5)^6}=-\frac{32}{(c+5)^6}$$

Now

$$R_4(2)=f(2)-P_4(2)$$
$$=0.142857-0.14432$$
$$=-0.001463$$

Figure 9.9

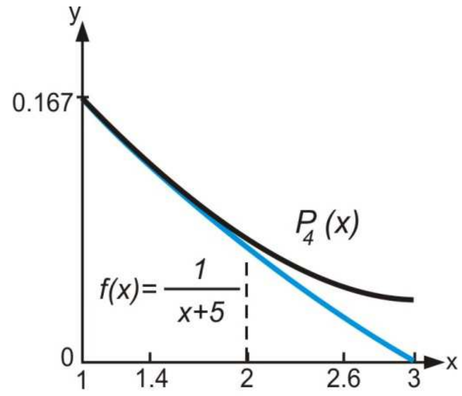

Figure 9.9

(b) For the value of c

$$R_4(2)=-\frac{32}{(c+5)^6}$$
$$-0.001463=-\frac{32}{(c+5)^6}$$
$$c=0.288318\in(0,2)$$

Example 9.5:

Find the general form of Taylor polynomial about 1 for the function

$$f(x)=lnx$$

Solution:

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	lnx	0
1	$x^{-1}=\frac{1}{x}$	1
2	$-x^{-2}=\frac{-1}{x^2}$	-1
3	$2x^{-3}=\frac{2!}{x^3}$	$2!$
4	$-6x^{-4}=\frac{-3!}{x^4}$	$-3!$
5	$24x^{-5}=\frac{4!}{x^5}$	$4!$

$$f(x)=f(1)+\frac{(x-1)}{1!}f^{(1)}(1)+\frac{(x-1)^2}{2!}f^{(2)}(1)$$
$$+\frac{(x-1)^3}{3!}f^{(3)}(1)+\frac{(x-1)^4}{4!}f^{(4)}(1)+\frac{(x-1)^5}{5!}f^{(5)}(1)$$

$$lnx = 0 + (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5 + \cdots$$

$$= (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5 + \cdots$$

The general term is

$$\frac{(-1)^k}{k+1}(x-1)^{k+1} \quad , \quad k = 0, 1, 2, 3, \cdots$$

Since $n = k+1 \quad \Rightarrow \quad k = n-1$

General expression of the polynomial is

$$P_n(x) = \sum_{k=0}^{n-1} \frac{(-1)^k}{k+1}(x-1)^{k+1}$$

Example 9.6:

Find the general form of Taylor polynomial and remainder about a for the function

$$f(x) = lnx$$

(a) Find the approximate value and remainder (in term of c) of $ln6$ using Taylor's polynomial for $n = 4$ about 1 and 5.

(b) $ln6 = 1.7917595$, correct to six decimal places, find the approximate value and remainder of $ln6$ by Taylor's polynomial for n=4 about 10 and prove that $c \in (6,10)$.

Solution:

n	$f^{(n)}(x)$	$f^{(n)}(a)$
0	lnx	lna
1	$x^{-1} = \frac{1}{x}$	$\frac{1}{a}$
2	$-x^{-2} = \frac{-1}{x^2}$	$\frac{-1}{a^2}$
3	$2x^{-3} = \frac{2!}{x^3}$	$\frac{2!}{a^3}$
4	$-6x^{-4} = \frac{-3!}{x^4}$	$\frac{-3!}{a^4}$
5	$24x^{-5} = \frac{4!}{x^5}$	$\frac{4!}{a^5}$

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The remainder is

$$R_4(6) = \frac{(-1)^4(6-1)^5}{5c^5} = \frac{5^5}{5c^5}$$

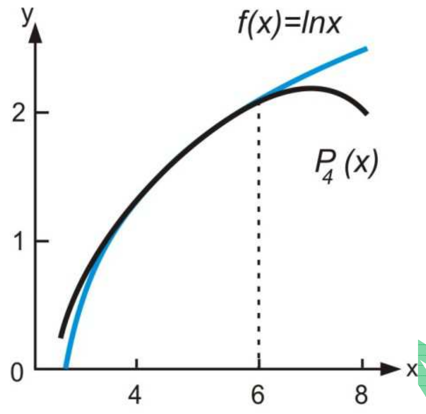

Figure 9.11

$For\ n = 4, about\ a = 5\ and\ x = 6:$

$$P_4(6) = ln5 + \sum_{k=0}^{3} \frac{(-1)^k(6-5)^{k+1}}{(k+1)a^{k+1}}$$

$$= 1.6094379 + \frac{1}{5} - \frac{1^2}{2\times 5^2} + \frac{1^3}{3\times 5^3} - \frac{1^4}{4\times 5^4}$$

$$= 1.7917046$$

Figure 9.11

The remainder is

$$R_4(6) = \frac{(-1)^4(6-5)^5}{5c^5} = \frac{1}{5c^5}$$

(b) $For\ n = 4, about\ a = 10\ and\ x = 6:$

$$P_4(6) = ln10 + \sum_{k=0}^{3} \frac{(-1)^k(6-10)^{k+1}}{(k+1)10^{k+1}}$$

$$= 2.302585 + \frac{(-4)}{10} - \frac{(-4)^2}{2\times 10^2} + \frac{(-4)^3}{3\times 10^3} - \frac{(-4)^4}{4\times 10^4}$$

$$= 1.7996517$$

Figure 9.12

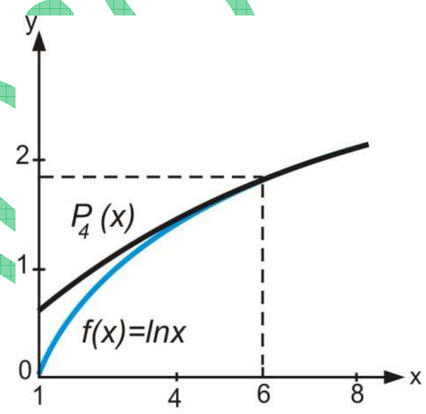

Figure 9.12

The remainder is

$$R_4(6) = \frac{(-1)^4(6-10)^5}{5c^5} = -\frac{1024}{5c^5}$$

Since

$$R_4(6) = ln6 - P_4(6)$$
$$= 1.7917595 - 1.7996517$$
$$= -0.0078922$$

For the value of c

$$-\frac{1024}{5c^5} = -0.0078922$$
$$c = 7.635 \in (6,10)$$