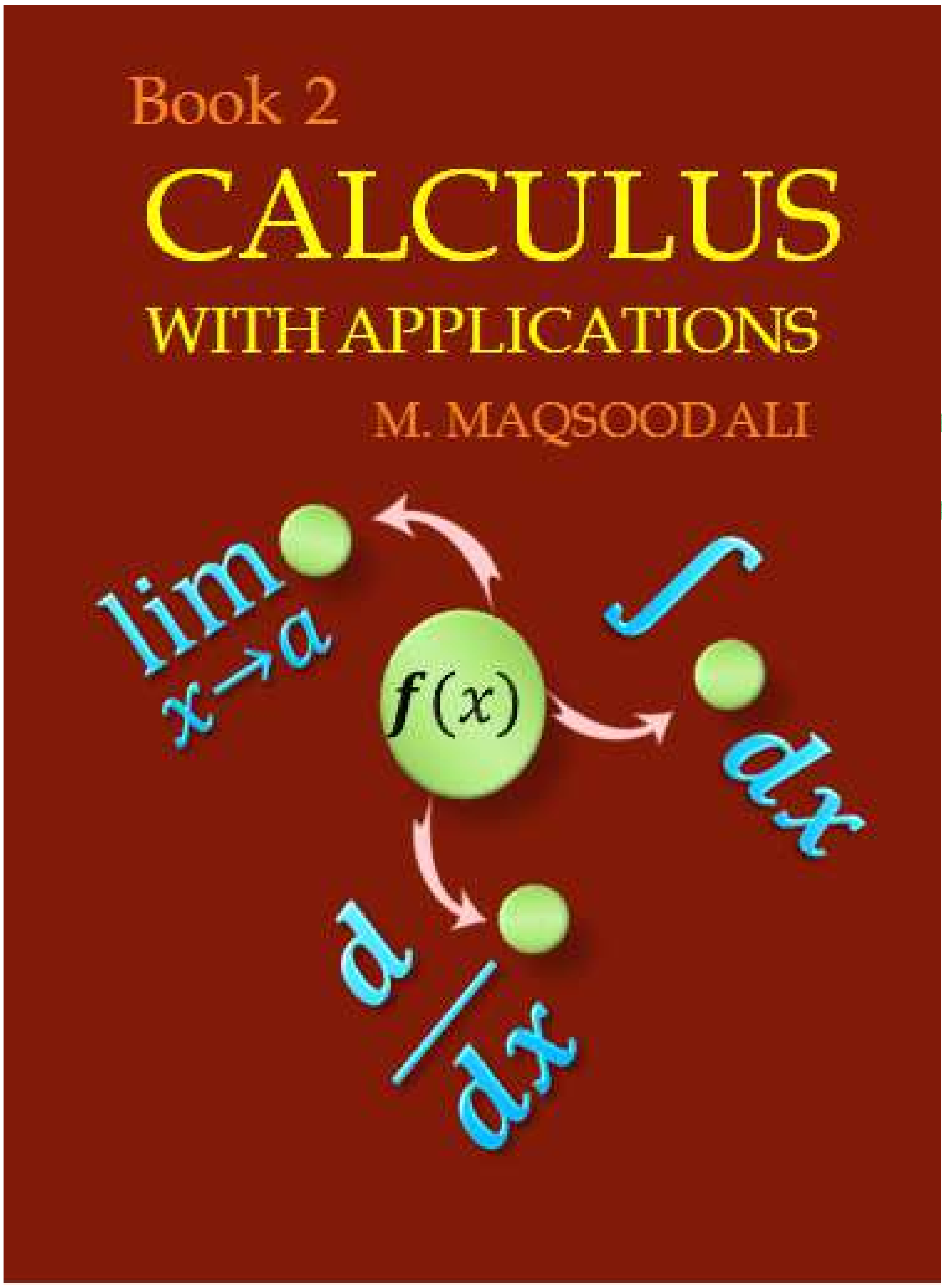
Book 2
CALCULUS
WITH APPLICATIONS
M. MAQSOOD ALI
$\lim_{x \to a}$
$f(x)$
$\int dx$
$\frac{d}{dx}$

Example 6.7: **Business And Economics**

A man imports oil. The cost (in dollars) to import the oil as a function of volume of oil x in liters is given below.

$$C(x) = \begin{cases} \frac{1}{3}x + 800 & for \quad 0 < x \leq 9000 \\ \frac{1}{3}x + \frac{800}{0.001x - 8} & for \quad 9000 < x \leq 30000 \end{cases}$$

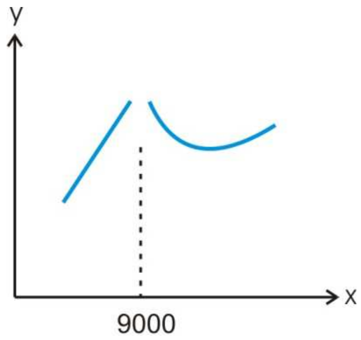

Figure 6.5 a

(a) Discuss the continuity at $x = 9000$.

(b) If $C(x)$ is continuous then draw a small solid circle on the graph (**figure 6.5a**) at $x = 9000$ otherwise a circle.

(c) Calculate the cost to import 6000 liters and 18000 liters oil.

(d) Explain the cost function $C(x)$.

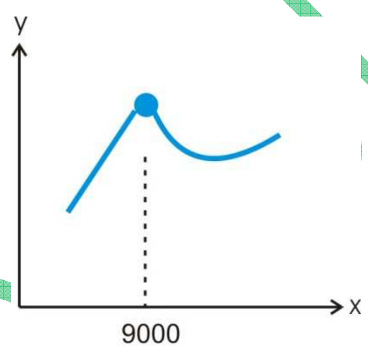

Figure 6.5 b

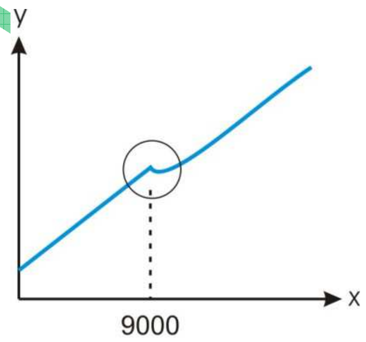

Figure 6.5 c

Solution

(a) The value of the function at $x = 9000$:

$$f(9000) = \frac{1}{3} \times 9000 + 800 = 3800$$

Left hand limit at 9000 :

$$\lim_{x \to 9000^-} f(x) = \lim_{x \to 9000^-} \left(\frac{1}{3}x + 800\right)$$

$$= \frac{1}{3} \times 9000 + 800$$
$$= 3800$$

Right hand limit at 9000:

$$\lim_{x \to 9000^+} f(x) = \lim_{x \to 9000^+} \frac{1}{3}x + \frac{800}{0.001x - 8}$$

$$= \frac{1}{3} \times 9000 + \frac{800}{2}$$
$$= 3800$$

The left hand limit is equal to the right hand limit of the function at $x = 9000$, so that limit of the function exists

$$\lim_{x \to 9000} f(x) = 3800$$

The limit of the function is equal to value of the function at $x = 9000$

$$\lim_{x \to 9000} f(x) = f(9000)$$

Therefore, the function $f(x)$ is continuous at $x = 9000$.

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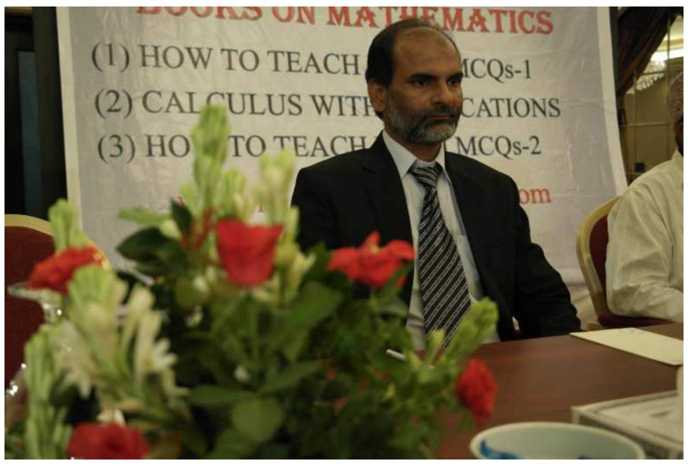

The right hand limit of f at 6:

$$\lim_{x\to 6^+} f(x) = \lim_{x\to 6^+}(6000 + 500\lfloor x \rfloor)$$
$$= 6000 + 500 \times 6$$
$$= 9000$$

Since the left hand limit of f is not equal to the right hand limit at $x = 6$

$$\lim_{x\to 6^-} f(x) \neq \lim_{x\to 6^+} f(x)$$

The limit of the function does not exist at $x = 6$.
Therefore the function f is discontinuous at $x = 6$.
Since $\lim_{x\to 6^+} f(x) = f(6)$ but $\lim_{x\to 6^-} f(x) \neq f(6)$.
The function $f(x)$ is continuous from the right, so the solid circle will be on right side and circle on left side, **Figure 6.6b.**

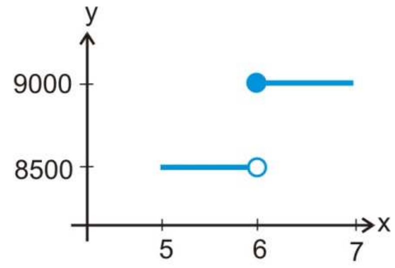

Figure 6.6 b

Example 6.9:

Physical science

In an experiment the temperature of a solid body is decreasing. The temperature of the solid body as a function of time t given below

$$f(t) = \begin{cases} 300\,cost + 50 \quad for \quad 0 < x \leq \pi/3 \\ 100 + 200\,\dfrac{1 - \cos\left(t - \frac{\pi}{3}\right)}{\left(t - \frac{\pi}{3}\right)^2} \; for\ \pi/3 < t \leq \pi/2 \end{cases}$$

(a) Discuss the continuity at $t = \pi/3$.
(b) A part of the graph is shown in the **figure 6.67a**. Draw a small solid circle on the graph at $t = \pi/3$ if the function is continuous otherwise a circle.

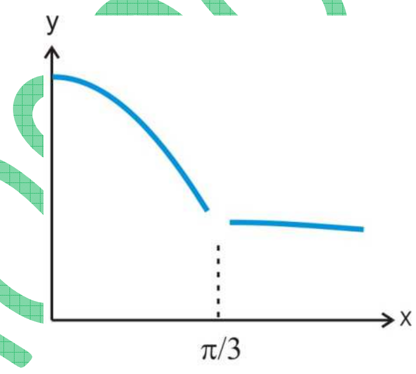

Figure 6.7 a

Solution:
The value of the function at $= \pi/3$:
$f(\pi/3) = 300cos\ \pi/3 + 50 = 200$

Left hand limit of the function at $x = \pi/3$:

$$\lim_{x\to \pi/3^-} f(x) = \lim_{x\to \pi/3^-}(300cost + 50)$$

$$= 300\cos \pi/3 + 50 = 200$$

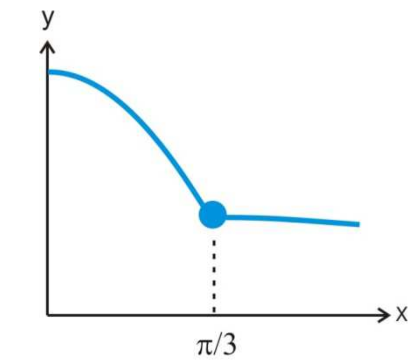

Figure 6.7 b

Right hand limit of the function at $x\ \pi/3$:

$$\lim_{x\to\pi/3^+} f(x) = \lim_{x\to\pi/3^+}\left\{100 + 200\frac{1-\cos(t-\pi/3)}{(t-\pi/3)^2}\right\}$$

$$= \lim_{x\to\pi/3^+}\left\{100 + 200\frac{2\,sin^2\left(\frac{t-\pi/3}{2}\right)}{(t-\pi/3)^2}\right\}$$

$$= 100 + 400\left\{\lim_{x\to\pi/3^+}\frac{\sin\left(\frac{t-\frac{\pi}{3}}{2}\right)}{2\left(\frac{t-\frac{\pi}{3}}{2}\right)}\right\}^2$$

$$= 100 + 100(1)^2$$
$$= 200$$

The left hand limit is equal to the right hand limit, so that limit of the function exists at $x = \pi/3$.

$$\lim_{x\to\pi/3^-} f(x) = \lim_{x\to\pi/3^+} f(x)$$

Therefore $\lim_{x\to\pi/3} f(x) = 200$

The limit of the function is equal to the value of the function at $x = \pi/3$.

$$\lim_{x\to\pi/3} f(x) = f(\pi/3)$$

Therefore, the function is continuous at $t = \pi/3$.

(b) The function is continuous at $t = \pi/3$ so draw a solid circle on the graph, as shown in **figure 6.67b.**

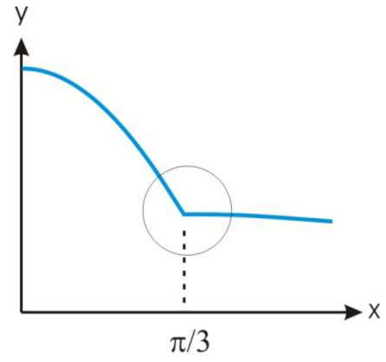

Figure 6.7 c

Physical science

Example 6.10:

A solid of revolution is created revolving the region bounded by the graph $f(x)$ and x-axis between $x = 0$ and $x = 7$ about x-axis, figure 6.8b, where

$$f(x) = \begin{cases} -\frac{5}{4}x + h & for \quad 0 \le x \le 4 \\ k(x-7)^2 + 10 & for \quad 4 < x \le 7 \end{cases}$$

Figure 6.8a

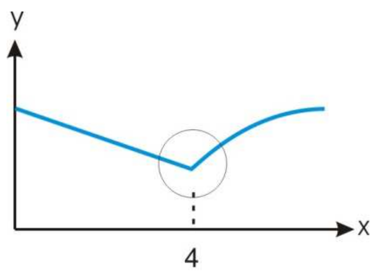

Figure 6.8 *a*

Find the values of h and k, if the function is continuous at $x = 4$. The diameter of left circular side of the solid is 20 cm.

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Solution:
Value of the function at $t = \pi/3$:

$$f(\pi/3) = 105$$

Limit of the function at $\pi/3$:

$$\lim_{t\to\pi/3} f(t) = \lim_{t\to\pi/3} \frac{100 \sin\left(t - \frac{\pi}{3}\right)}{\left(t - \frac{\pi}{3}\right)}$$

$$= 100 \lim_{(t-\pi/3)\to 0} \frac{\sin\left(t - \frac{\pi}{3}\right)}{\left(t - \frac{\pi}{3}\right)}$$

$$= 100(1)$$

$$= 100$$

Since

$$f\left(\frac{\pi}{3}\right) \neq \lim_{t\to\pi/3} f(t)$$

So that the function f(t) is discontinuous at $t = \pi/3$.
(b) Since the function is discontinuous at $t = \pi/3$, so that a circle is drawn between the lines on the graph and a solid circle at point $\left(\frac{\pi}{3}, 105\right)$.

EXERCISE 6

(1) $f(x) = |x| + 2$

(a) Discuss the continuity of the function $f(x)$ at $x = 0$.

(b) An incomplete graph of $f(x)$ is shown in the figure 6.9. Draw a small solid circle on it if the function is continuous at $x = 0$ otherwise a circle.

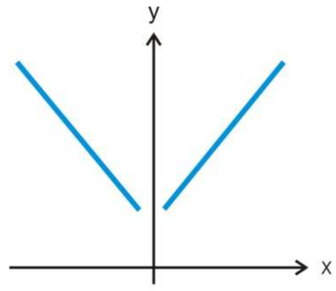

Figure 6.9

(2) $f(x) = \lfloor x \rfloor$, the greatest integer function

(a) Discuss the continuity of the function $f(x)$ at $x = 2$ and $x = 2.5$.

(b) The graph of the function is shown in the figure 6.10. Draw the small circles and solid circle on the graph at $x = 2$ according to the discontinuity and continuity respectively and cross the extra lines.

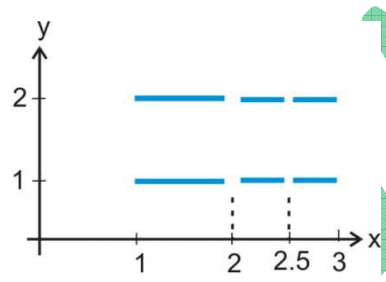

Figure 6.10

(3) $f(x) = \lceil x \rceil$, least integer function.

(a) Discuss the continuity of the least integer function $f(x)$ at $x = 2$ and $x = 2.5$.

(b) The graph of the function is shown in the figure 6.11. Draw the circles and solid circle on the graph at $x = 2$ according to the discontinuity and continuity and cross the extra lines.

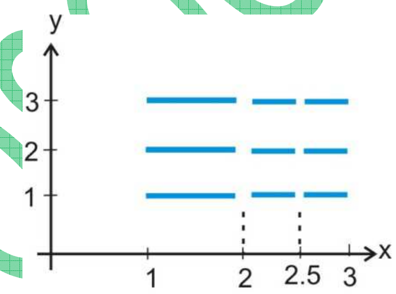

Figure 6.11

(4) Show that $\cos x$ is continuous for all values of $x \in \mathbb{R}$.

(5) A man imports oil. The cost to import the oil as a function of volume of oil x in titers is given below.

$$C(x) = \begin{cases} \frac{1}{5}x + 600 & \text{for} \quad 0 < x \le 6000 \\ \frac{5}{24}x + \frac{600}{0.001x - 4.75} & \text{for} \quad 6000 < x < 12000 \end{cases}$$

(a) Discuss the continuity at $x = 6000$.

(b) If $C(x)$ is continuous then draw a small solid circle on the graph at $x = 6000$ otherwise a circle, figure 6.11A.

(c) Calculate the cost to import 5000 liters and 9000 liters oil.

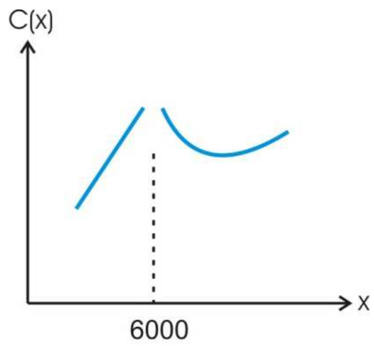

Figure 6.11A

(6) The volume of the water in a tank from 01:00 a.m. to 11:00 a.m. is $f(t)$ liters , function of time t , is given below

$$f(t) = 2000\sin\left(\frac{\pi}{12}t\right)$$

(a) Discuss the continuity for all values of $t \in [1, 11]$.

(b) What is the volume of the water in the tank at $t = 3, 6, 9$ hours.

(7) In a labotary a solid metallic body is heated by a heater and then after a particular temperature the heater is removed and the body is getting cool. A student forms a function $f(t)$,temparatuer of the body in ℃ , at time t hours ,given below

$$f(t) = \begin{cases} 200\,sin\,t + 50 & for \quad 0 < t \le \pi/6 \\ 50 + 100\,\dfrac{six(t - \frac{\pi}{6})}{t - \frac{\pi}{6}} & for \quad \pi/6 < t \le 2\pi \end{cases}$$

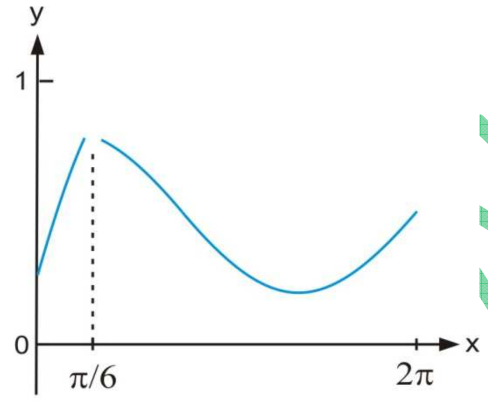

Figure *6.11B*

(a) Discuss the continuity of the function at $t = \pi/6$ hours.

(b) The graph of the function is shown in the figure 6.11B. Draw the circles and solid circle on the graph at $t = \pi/6$ according to the discontinuity and continuity respectively.

(c) What is the temperature of the body at time $t = 15$ minutes and $t = 1$ hour 20 minutes.

(8) A man deposits \$ 8000 in a bank. The interest is added at the beginning of each month is given below.

$$f(x) = 8000 + 70\lfloor x \rfloor$$

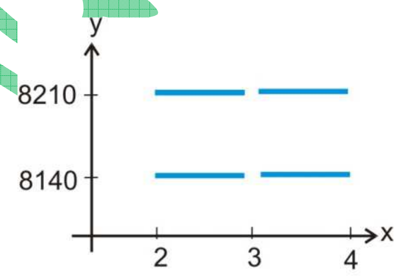

Figure 6.12

(a) Discuss the continuity at $x = 3$.

(b) The graph of the function is shown in the figure 6.12. Draw the circles and solid circle on the graph at $x = 3$ according to the discontinuity and continuity and cross the extra lines.

(9) The bus fare in a city is ¢ 10 fixed and ¢ 2 per kilometer. The fare as a function of distance x travel in kilometer is given below

$$f(x) = 10 + [x]$$

(a) Discuss the continuity at $x = 2$.

(b) The graph of the function is shown in the figure 6.13. Draw the circles and solid circle on the graph at $x = 2$ according to the discontinuity and continuity and cross the extra lines.

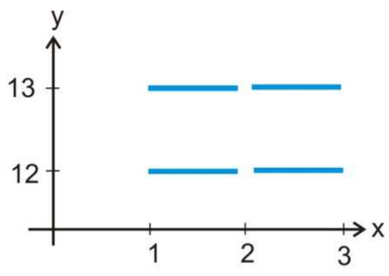

Figure 6.13

(10) A businessman gets 12% profit on the amount which he invests. He invest the amount which is its own and some time taking loan from the bank. The loan is represented by negative sing. The total amount after adding profit as a function of total amount invested is given below.

$$f(x) = x + 0.12|x|$$

Discuss the continuity at

(a) $x = 200$ (b) $x = -300$

(11) A solid is created by revolving the region bounded by the graph

$$f(x) = \begin{cases} 2 & for \quad 0 \le x \le 3 \\ 1 + \dfrac{\sin(x-3)}{x-3} & for \quad 3 < x \le 8 \end{cases}$$

and x-axis about x-axis. Discuss the continuity of f at $x = 3$. Draw a solid circle on the curve, figure 6.14, at $x = 3$ if the function is continuous otherwise a small circle,

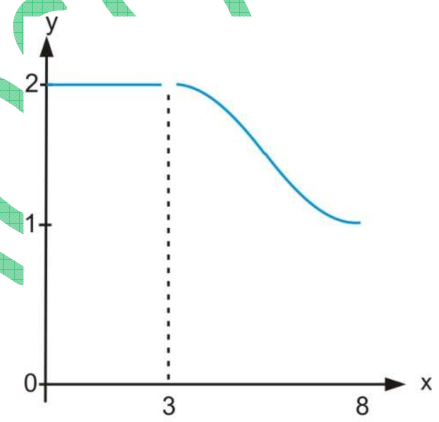

Figure 6.14