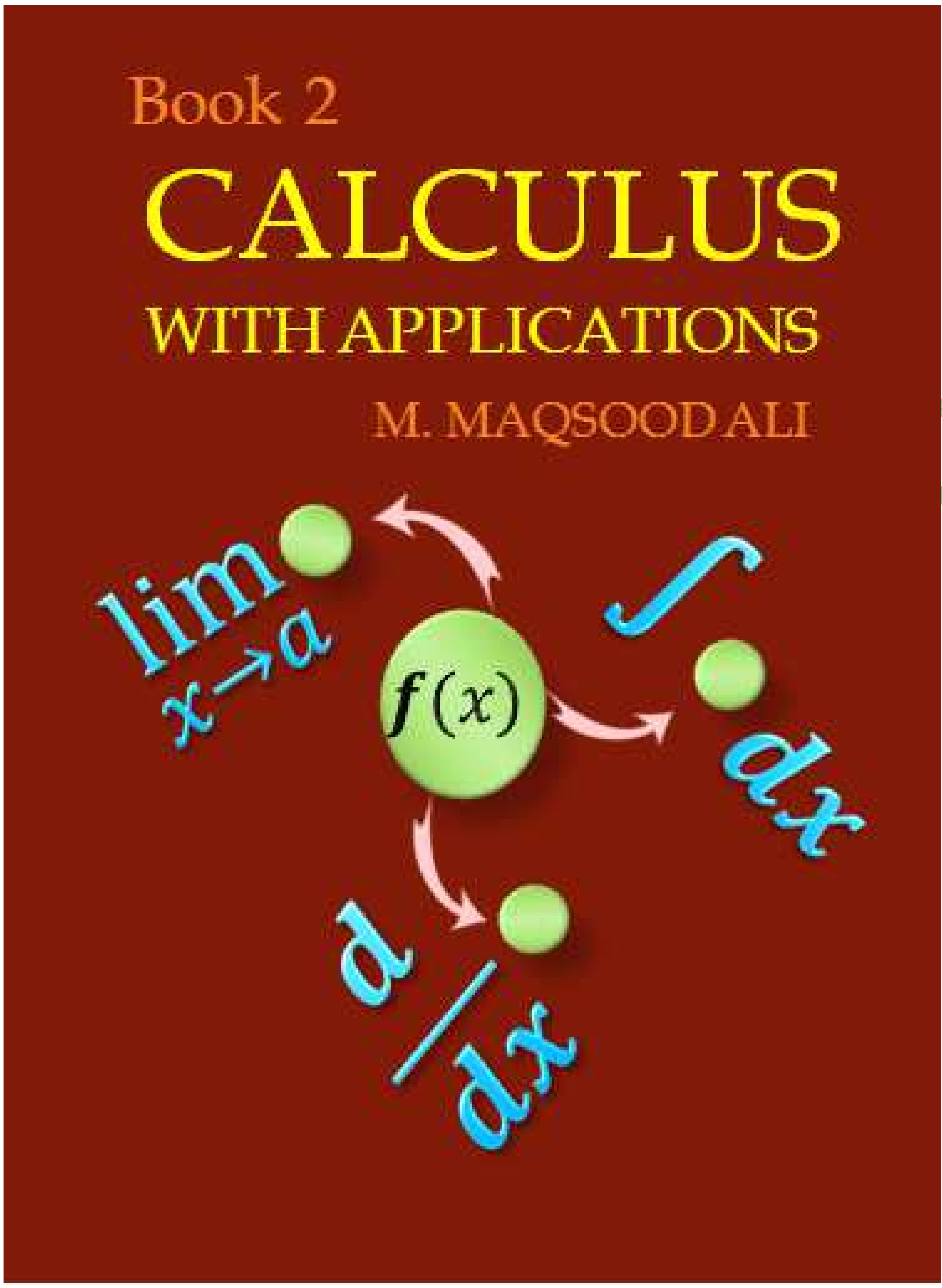
Book 2
CALCULUS
WITH APPLICATIONS
M. MAQSOOD ALI
$\lim_{x\to a}$
$f(x)$
$\int dx$
$\frac{d}{dx}$

Business and Economics

MARGINAL COST, REVENUE AND PROFIT:

In the field of business and economics marginal cost, marginal revenue and marginal profit are very important to maximize the profit.

Marginal cost:

If $C(q)$ is the total cost function for producing q units, then $C'(q)$ is called marginal cost. The marginal cost

$$C'(q) = \frac{d}{dq}C(q)$$

is the rate of change of the total production cost with respect to the number of units produced.

In other words it is approximately the cost of producing one additional unit.

Total cost of producing q units $= C(q)$

The cost of producing the $(q+1)th$ unit$= C'(q)$

So that marginal cost is approximately the cost of only one unit that is $(q+1)th$.

Example 2.19:

Suppose that the total cost of producing q units is

$$C(q) = -0.02q^2 + 8q + 100$$

(a) Find the marginal cost function.

(b) Find the total cost for $q = 0,1,70,71,99,100$ units and marginal cost of $(q+1)th$ unit when $q = 0,70,99$. Compare the actual cost and the marginal cost of $(q+1)th$.

(c) Find the average cost of 1, 71, 100 units.

(d) Draw the graph of total cost function $C(q)$ and margined cost function.

Solution:

The total cost of producing q units is

$$C(q) = -0.02q^2 + 8q + 100$$

Figure 2.33.

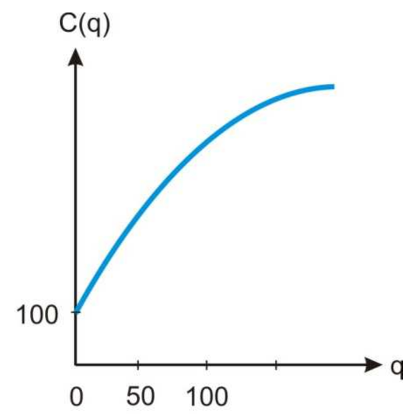

Figure 2.33

(a) Marginal cost function $= \frac{d}{dq}C(q)$

$$C'(q) = -0.04q + 8$$

(b) Actual cost of $1st$ unit$= C(1) - C(0)$

$$= 107.98 - 100 = 7.98$$

Marginal cost of $1st$ unit$= C'(1) = C'(0+1)$

$$= -0.04(0) + 8 = 8.00$$

Actual cost of $71st$ unit$= C(71) - C(70)$

$$= 567.18 - 562 = 5.18$$

Marginal cost of $71st$ unit= $C'(71) = C'(70+1)$
$= -0.04(70) + 8 = 5.20$

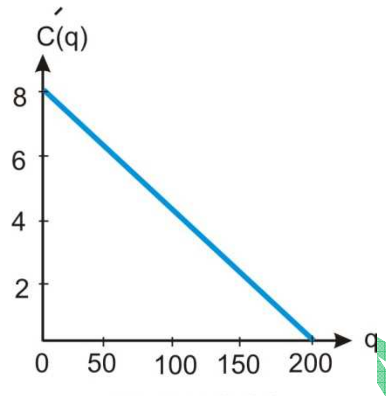

Figure 2.34

Actual cost of $100th$ unit= $C(100) - C(99)$
$= 700 - 695.98 = 4.02$
Marginal cost of $100th$ unit= $C'(100) = C'(99+1)$
$= -0.04(99) + 8 = 4.04$

(c)

q	$C(q)$	Average= $C(q)/q$
1	107.98	107.98
71	567.18	7.99
100	700	7.00

(d) The graph of total cost function $C(q)$ and marginal cost function C′(q) are **figures 2.33** and figure **2.34**.

MARGINAL REVENUE:

If $R(q)$ is the total revenue function for selling q units, then $R'(q)$ is called the marginal revenue function.

The marginal revenue

$$R'(q) = \frac{d}{dq}R(q)$$

is the rate of change of the total revenue with respect to the number of unit sold.

It is approximately the revenue of selling one additional unit.

Total revenue of selling q unit= $R(q)$
The revenue of selling the $(q+1)th$ unit = $R'(q)$
So that marginal revenue is the approximately the revenue of only one unit that is $(q+1)th$.

Example 2.20: **Business and Economics**

The demand of an item according to the marketing studies is

$$q = 500 - 25p$$

where q is number of units demanded each month and p price of one unit.

(a) Find the revenue function and marginal revenue function.
(b) Compare the revenue and marginal revenue for $1st$, $51st$ and $121st$ unit.

(c) Find the average revenue of 1, 51, 121 units.

Solution:

(a) The demand is

$$q = 500 - 25p \qquad \rightarrow (1)$$

The total revenue function R is

$$R = pq \qquad \rightarrow (2)$$

According to equation (1)

$$p = 20 - 0.04\,q \qquad \rightarrow (3)$$

The total revenue function $R(q)$ by (2) is

$$R(q) = 20q - 0.04\,q^2 \qquad \rightarrow (4)$$

The marginal revenue function is

$$MR = R'(q) = 20 - 0.08\,q \qquad \rightarrow (5)$$

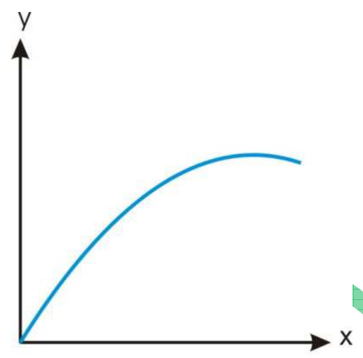

Figure 2.35

(b)

Actual revenue of $1st$ unit$= R(1) - R(0)$

$$= 19.96 - 0 = 19.96$$

Marginal revenue of $1st$ unit$= R'(1) = R'(0+1)$

$$= 20 - 0.08(0) = 20$$

Actual revenue of $51st$ unit$= R(51) - R(50)$

$$= 915.96 - 900 = 15.96$$

Marginal revenue of $51st$ unit$= R'(51) = R'(50+1)$

$$= 20 - 0.08(50) = 16$$

Actual revenue of $121st$ unit$= R(121) - R(120)$

$$= 1834.36 - 1824 = 10.36$$

Marginal revenue of $121st$ unit$= R'(121) = R'(120+1)$

$$= 20 - 0.08(120) = 10.40$$

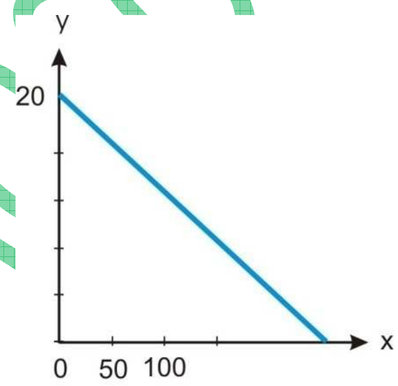

Figure 2.36

(c)

q	$R(q)$	Average$= R(q)/q$
1	19.96	19.96
51	915.96	17.96
121	1834.36	15.16

Marginal profit (MP):

If $R'(q)$ and $C'(q)$ are marginal revenue and marginal cost functions, then marginal profit function $MP = P'(q)$ is

$$P'(q) = R'(q) - C'(q)$$

The marginal cost, marginal revenue and marginal profit functions are used to maximize the profit.

Example 2.21: **Business and Economics**

The total cost and total revenue functions of q unit are

$$R(q) = -0.04q^2 + 20q$$
$$C(q) = 0.03q^2 + 2q + 200$$

(a) Find the total profit function and marginal profit function.
(b) Find the number of units where the profit is maximum.
(c) Find the maximum profit.
(d) Draw $R(q)$ and $C(q)$ and $P(q)$.

Solution:

(a) Total profit function is

$$P(q) = R(q) - C(q)$$
$$P(q) = -0.07q^2 + 18q - 200$$

Marginal profit function is

$$P'(q) = -0.14q + 18$$

(b) For critical points

$$P'(q) = 0$$
$$q = 129$$

The second derivative of $P(q)$ is

$$P''(q) = -0.14$$

Second derivative test

$$P''(129) = -0.14 < 0$$

Therefore, profit is maximum when the number of units are 129.

(c) Maximum profit:

$$P(129) = 957.13 \text{ dollars}$$

(d) Graphs:

Figure 2.37

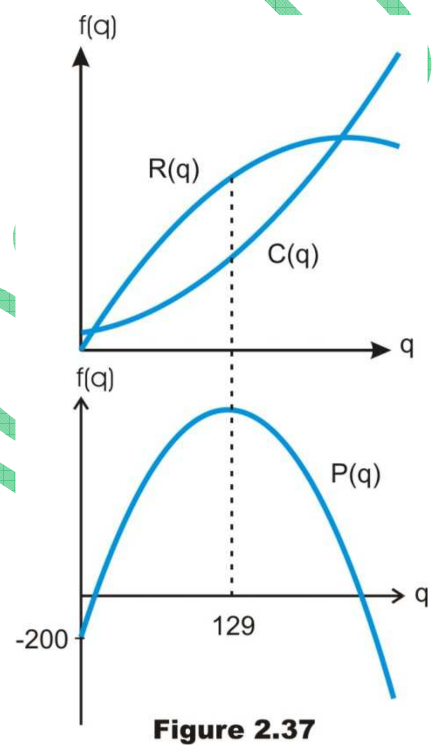

Figure 2.37