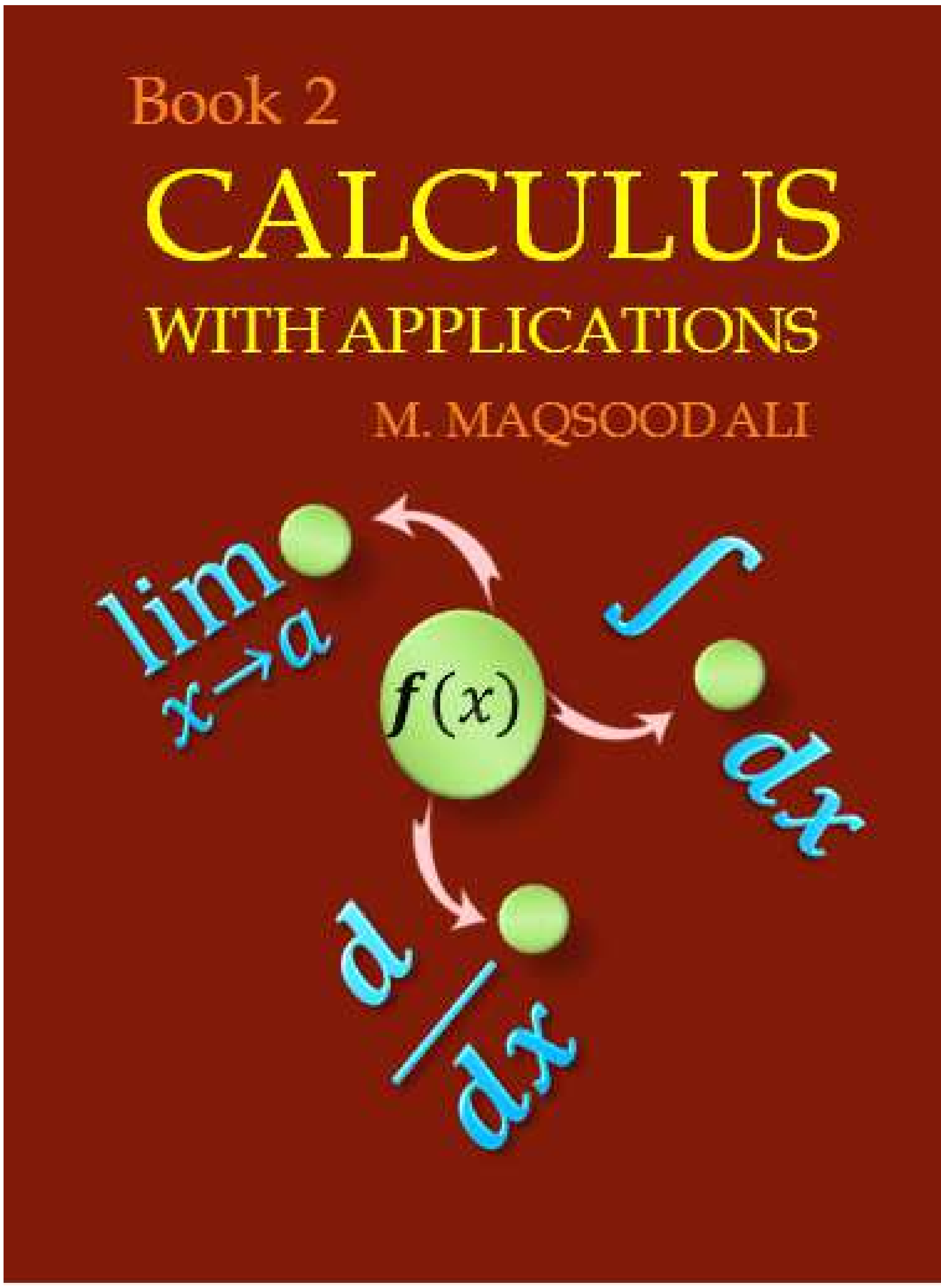
Book 2
CALCULUS
WITH APPLICATIONS
M. MAQSOOD ALI
lim
x→a
f(x)
∫
dx
d
dx

POLYNOMIAL FUNCTIONS OF DEGREE TWO:

A function of the form

$$f(x) = ax^2 + bx + c$$

Is called a polynomial function of degree 2, where a, b, c are real numbers.

FORMING FUNCTIONS USING GIVEN CONDITIONS:

Example 2.14:

Business and Economics

A firm produces shampoo at a cost of 80 cents per bottle. According to the market survey reports consumes will buy $(900 - 5x)$ bottle per day, where x is the price of a bottle. Find the profit as a function of price per bottle.

(a) What is the price of shampoo bottle for no profit and then explain the answer?

(b) Draw the graph for maximum profit.

Solution:

Price of the bottle $= x$

Cost per bottle$=$ 80 cents

Therefore, profit per bottle$= x - 80$

Number of bottles sell out $= 900 - 5x$

Profit =(number of bottle sold)(profit per bottle)

The profit function is

$$P(x) = (900 - 5x)(x - 80)$$
$$P(x) = -5x^2 + 1300x - 72000$$

(a) There is no profit, so that

$$P(x) = 0$$
$$(900 - 5x)(x - 80) = 0$$

Either $900 - 5x = 0$ or $x - 80 = 0$

$x = 180$, $x = 80$

If the price of the bottle $x = 80$ cents, which is equal to cost price , so there is not profit.

If the price of the bottle $x = 180$ cents then there is no sell, so the profit is zero.

(b) The graph for maximum profit is **figure 2.21**. The maximum profit is 12500 cents, when the price of a bottle is 130 cents.

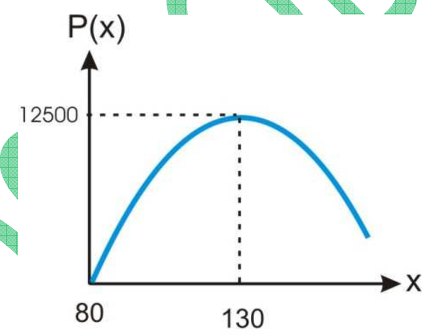

Figure 2.21

Example 2.15:

Physical Science:

A bullet is fired vertically upward with a velocity 280 meter per second at a height 50 metres.

(a) Find the height of the bullet as a function of time.

(b) By drawing the graph find the maximum height of the bullet.

Solution:

(a) Acceleration due to gravity

$$a = -g \quad , \quad \textbf{figure 2.22}$$

$$\frac{dv}{dt} = -g$$

$$v = -g\int dt$$

$$v = -gt + A \quad \rightarrow (1)$$

$v = 280\ {}^{m}/_{s}$ at $t = 0$, putting in equation (1)

$$A = 280$$

Putting in equation (1)

$$v = -gt + 280 \quad \rightarrow (2) \quad , \textbf{figure 2.23}$$

is the velocity of the bullet at time t.

Let h be the height of the bullet at time t from the ground.

$$\frac{dh}{dt} = -\mathrm{g}t + 280$$

$$\int dh = \int(-\mathrm{g}t + 280)dt$$

$$h = -\frac{1}{2}\mathrm{g}t^2 + 280t + B \quad \rightarrow (3)$$

$h = 50$ metres $at\ t = 0$, putting in equation (3)

$$50 = 0 - 0 + B$$

$$B = 50$$

Putting in equation (3)

$$h = -\frac{1}{2}\mathrm{g}t^2 + 280t + 50$$

is the height of the bullet at time t.

(b) By the graph, **figure 2.24**, the maximum height of the bullet is 4050 metres at time t=28.57 seconds.

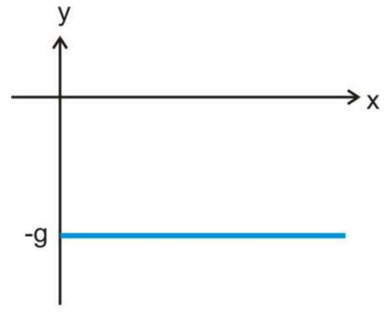

Figure 2.22

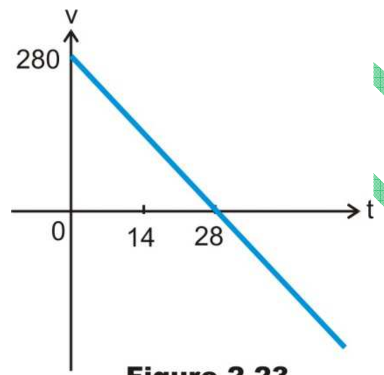

Figure 2.23

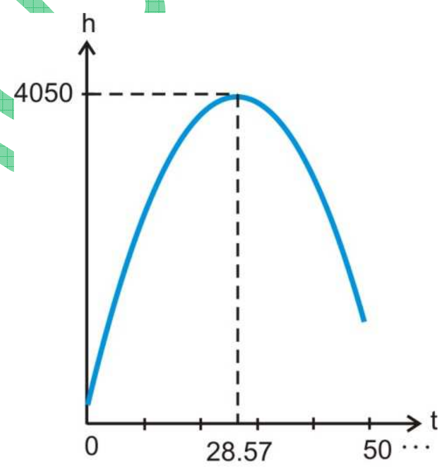

Figure 2.24

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Solution:

The graph of $C(x)$ against x is

figure 2.27.

The graph is a parabola. The cost function must be quadratic. The cost function is

$$C(x) = a\,x^2 + bx + c \qquad \rightarrow (1)$$

Putting there values $x = 100, 500,\ 800$ and $C(x) = 3000,\ 7000$ and 14200 respectively in (1), we have

$$3000 = 10000a + 100b + c \rightarrow (2)$$
$$7000 = 250000a + 500b + c \rightarrow (3)$$
$$14200 = 640000a + 800b + c \rightarrow (4)$$

Solving equations $(2),\ (3)$, and (4) simultaneously

$$a = 0.02, \quad b = -2, \quad c = 3000$$

The cost function is

$$C(x) = 0.02\,x^2 - 2x + 3000$$

Figure 2.27

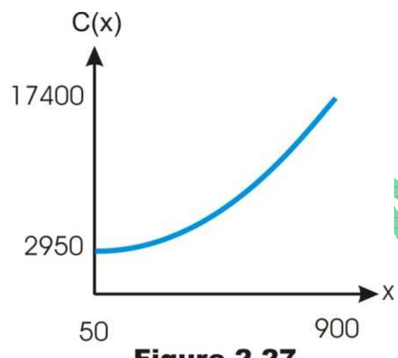


Figure 2.27

Note:

Study "INTERPOLATION" a topic from Numerical Analysis to form polynomial functions of degree n, where n is a natural number, for a computer program.

RATE OF CHANGE

A car is moving 80 meters in 7 seconds, 33 meters in 2 second, 69 meters in 5 seconds, 183 meters in 12 seconds. It is not easy to estimate that in which interval of time the car is moving fast. But the change in distance with respect to time (which is the speed) of the car 41 km/h, 59 km/h, 50 km/h and 55 km/h, gives a clear concept that when the car is moving fast. A man who does not know about the speed of the car, he can say that the speed of the car

120km/h is very slow or 10 km/h very fast. A man who does not know about the run rate of cricket, he can say 3 run per over is very fast. In a foot ball match, rate of change 3 goal per hour is useless for the audience. So that rate of change is playing an important role in many fields.

It is very important to know about the rate of change according to the field.

- A common man should know about the speed (rate of change in displacement with respect to time) of a car, a plane , a train etc.

- A physical scientist should know about the speed of light, sound , electromagnetic wave etc.
- A scientist in the field of chemistry should know the rate of reaction of elements.
- A economist should know about rate of change in cost (marginal cost), rate of change in revenue (marginal revenue),rate of change in profit (marginal profit).
- A biologist should know the rate of change in micro-organism (for example bacteria).
- A should know the rate of change in radioactive element C_{14} to estimate the age of fossils.

  In many situations and experiments the collected in formations is the rate of change, which is integrated to find the required function.

Graphs of $\frac{d}{dx}f(x)$ and $f(x)$:

Compare the graphs of rate of change in $f(x)$ and the function $f(x)$ shown in the

figure 2.30.

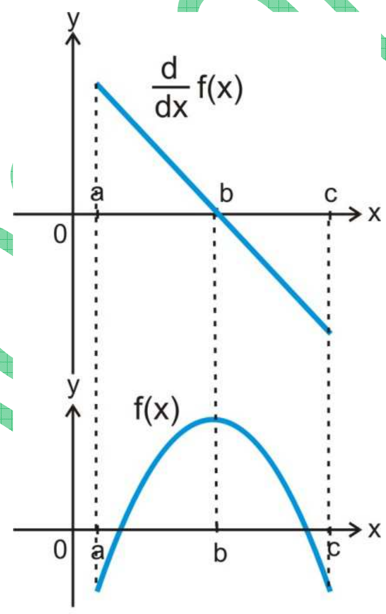


Figure 2.30

Relation between the graphs are given below.

* $\frac{d}{dx}f(x) > 0$ on the interval $[a, b)$, so the value of the function $f(x)$ is increasing on this interval.
* $\frac{d}{dx}f(b) = 0$, rate of change at $x = b$ is zero, so $(b, f(b))$ is a relative maximum (or relative minimum in any other example) point on the curve.
* $\frac{d}{dx}f(x) < 0$ on the interval $(b, c]$, so the value of the function $f(x)$ is decreasing on this interval.

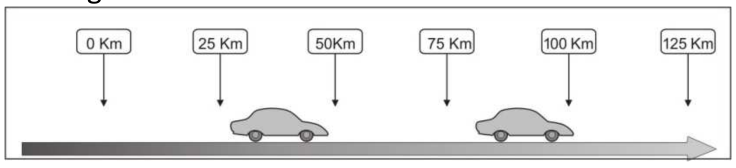
FORMING DIFFERENTIAL EQUATIONS OF FIRST ORDER:

In many experiments scientist cannot form a function using the data. They form a differential equation from the data. The differential equation of order one is simultaneously rate of change. Integral calculus tells that how a function can be form by a differential equation. How a differential equation can be formed?

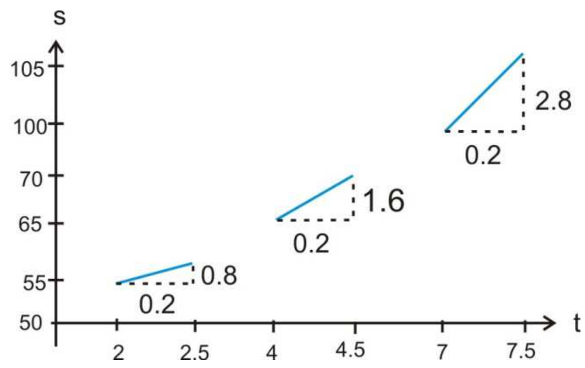
Example 2.18:

Physical Science:

A car is running on a straight road. A student write(note) its distance from the origin. The car is at a distance 50km from the origin when he observes the car and start the clock.



Time intervals(t) minute	2.0 - 2.5	4.0-- 4.5	7.0 - 7.5
Distance intervals	54.25-56.25	66.25-70.25	99.25-106.25



t	2	4	7	$\cdots$	t
Δt	0.2	0.2	0.2	$\cdots$	0.2
Δs	0.8	1.6	2.8		—
$\Delta s/\,\Delta t$	=2×2	=2×4	=2×7		2 t

$$\frac{ds}{dt} = v = 2t \qquad \rightarrow (1)$$

The graph is the figure 2.31.

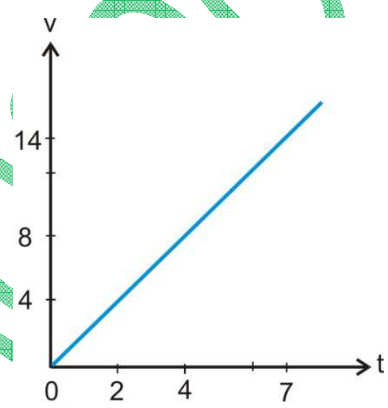


Figure 2.31

$$s = \int (2t)dt$$
$$s = t^2 + C \qquad \rightarrow (2)$$

Initially $s = 50$ at $t = 0$.

$$C = 50$$

The distance s as a function of time t is

$$s = t^2 + 50 \qquad \rightarrow (3)\,,\ \textbf{figure 2.32}$$

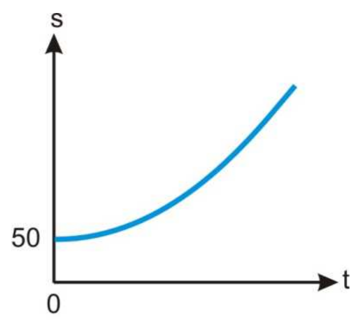


Figure 2.32

The distance of the car can be found at any time t, by eq.3.