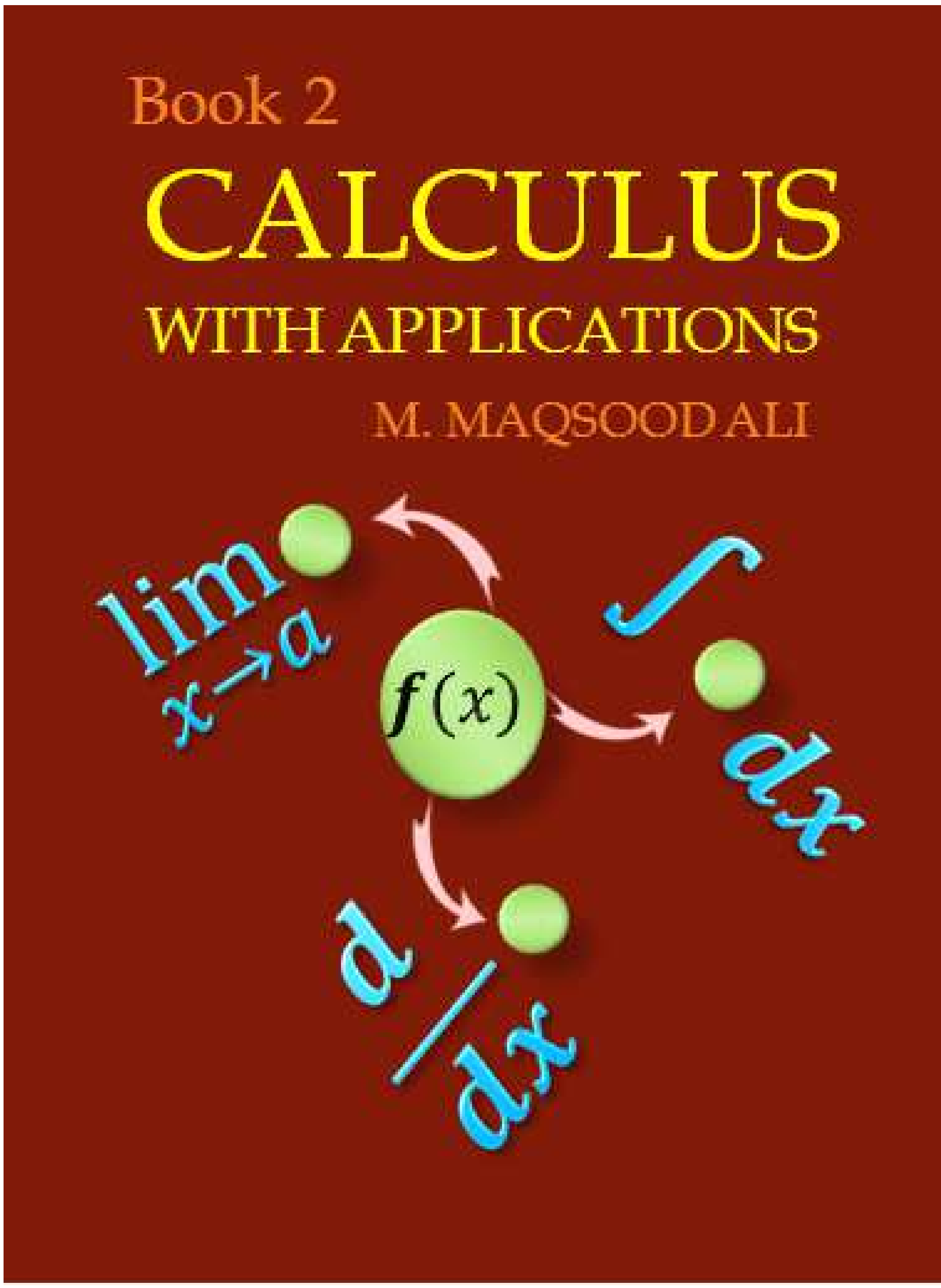
Book 2
CALCULUS
WITH APPLICATIONS
M. MAQSOOD ALI
lim x→a
f(x)
∫ dx
d/dx

Chapter 2

FUNCTIONS FORMING

How, when and what function can be formed using the available information. This chapter is written particularly for this purpose. **How** does a function form? The functions are formed using the available data by drawing the data on a paper or according to the given conditions. **When** does a function form? It is very important to know when a function is formed. The function is formed when the data consist on many pages is to be recorded only by a function, to calculate the values which are missing in data, rate of change is needed, to calculate area under the curve etc. **What** function does form? There are a lot of functions such as linear, quadratic, cubic, rational, radical, exponential, trigonometric, logarithmic etc. It depends on the available information what function can be formed.

LINEAR FUNCTIONS:

An equation of straight line is called linear equation. The equation of straight line with slope m and y-intercept b is

$$y = mx + b$$

The graph of the equation is **figure 2.1.**

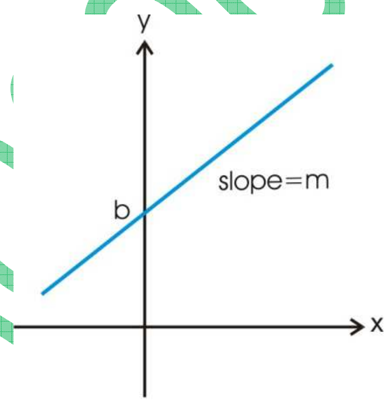

Figure 2.1

Putting $y = f(x)$, we have

$$f(x) = mx + b$$

f is a linear function of variable x , where m and b are constants.

m: rate of change in $f(x)$ with respect to x.

b: value of the function at $x = 0$.

FORMING LINEAR FUNCTIONS USING GIVEN CONDITIONS:

COST, REVENUE AND PROFIT FUNCTIONS:

$$P(x) = R(x) - C(x)$$

The graph is **figure 2.2**.

COST FUNCTION $C(x)$:

$$\text{Cost} = \text{variable cost} + \text{fixed cost}$$

$$C(x) = mx + b$$

where

$C(x)$: cost function

m: cost per unit

b: fixed cost (i.e rent , utilities etc.)

mx: variable cost =(cost per unit)(number of units)

REVENUE FUNCTION:

Revenue = (price per unit)(number of units)

$$R(x) = Px$$

where

$R(x)$: Revenue function

P: Price per unit

x: Number of units sell out at price P.

PROFIT FUNCTION:

$$P(x) = R(x) - C(x)$$

According to the **figure 2.2** loss and profit at $x = a$ and $x = b$ are respectively.

$$Loss = C(a) - R(a)$$
$$Profit = R(b) - C(b)$$

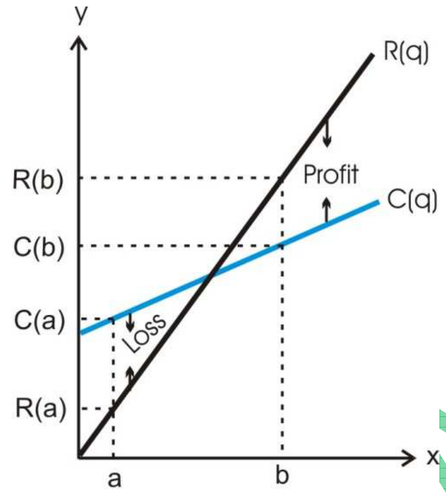

Figure 2.2

Example 2.1:

Business and Economics

A firm manufactures wall clocks. The production cost per unit is 15 dollars and the fixed cost for x clocks is 600 dollars.

(a) Find the cost to manufacture x wall clocks?

(b) The firm sells x wall clocks for 18 dollars per unit. Find the revenue.

(c) What is the profit the firm gets by selling 1000 clocks?

(d) How many clocks will the firm sell for no profit no loss?

(e) Draw the cost, revenue and profit functions.

Solution

(a) Cost per unit $= m = \$\,15$,

Number of units$= x$

Fixed cost$= b = \$\,600$

The cost function is

$$C(x) = m\,x + b$$
$$C(x) = 15x + 600$$

The graph is **figure 2.3.**

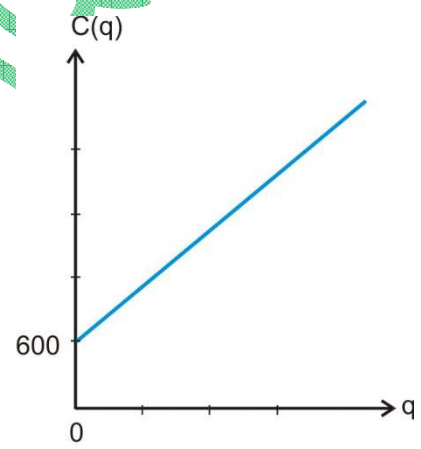

Figure 2.3

(b) Number of units sold = x

Price per unit = $p = \$\,18$

The revenue function is

$$R(x) = px$$
$$R(x) = 18\,x$$

The graph is **figure 2.4.**

(c) Number of units sold = $x = 1000$

The profit is

$$P(x) = R(x) - C(x)$$
$$= 18x - (15x + 600)$$
$$= 3x - 600$$

$x = 1000$

$$P(1000) = 3(1000) - 600$$
$$= 2400$$

Profit to sell 1000 wall clocks is 2400 dollars.

The graph is **figure 2.5.**

(d) $P(x) = 0$

$$P(x) = 3x - 600$$
$$0 = 3x - 600$$
$$x = 200$$

The firm will not get profit or lose to sell 200 wall clocks.

(e) The graphs of the cost, revenue and profit functions are shown in the figures 2.3, 2.4 and 2.5. The **figure 2.6** is the graph of cost and revenue function which shows the loss and profit.

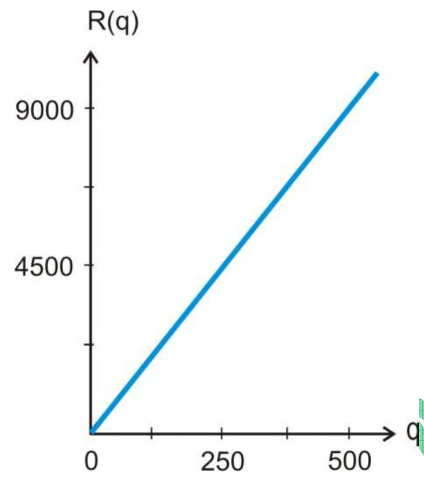

Figure 2.4

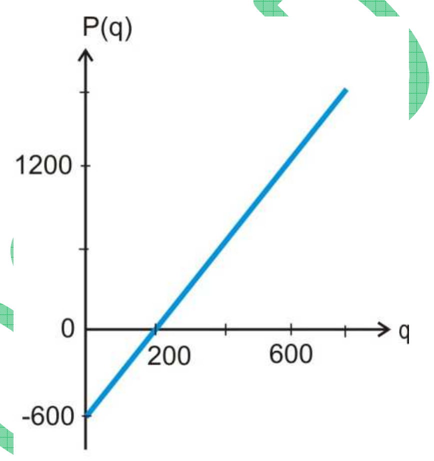

Figure 2.5

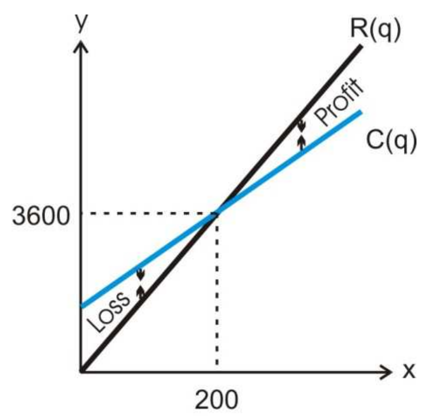

Figure 2.6

FUNCTIONS FORMING FROM THE GRAPHS:

It is very important for a mathematician or a student of mathematics to known the shapes of graphs of different functions. Often when the data is drawn on a paper to form a function the shape of the graph guide that which function can be form, for example, linear, quadratic, trigonometric etc.

Study the topic "Curve Fitting" from any book of Numerical Analysis to know how a curve can be drawn from the collected data.

TO FORM LINEAR FUNCTIONS DRAWING GRAPH OF THE COLLECTED DATA:

Many functions are formed by drawing graphs to use available data.

Example 2.2: **Business and Economics**

A firm manufactures toy cars. The production cost of 5000, 9000, 12000, 15000 and 20,000 units are 16800, 28800, 37800, 46800 and 61800 dollars respectively. Find the cost function.

Solution:

No. of toy car x	Total cost in dollars $C(x)$
5000	16800
9000	28800
12000	37800
15000	46800
20000	61800

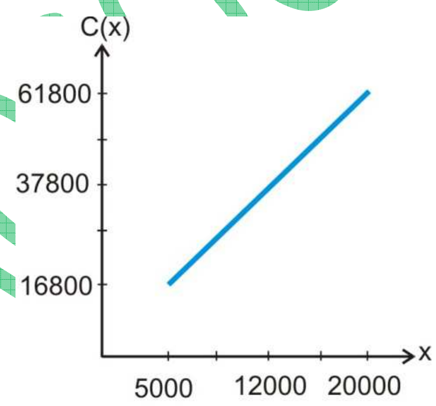

Figure 2.7

The graph is shown in the **figure** 2.7.

The graph is a straight line, which shows that the cost function must be linear.

$$m = \frac{61800-16800}{20000-5000} = 3$$

The cost function $C(x)$ is

$$C(x) = mx + b$$
$$C(x) = 3x + b$$

For the value of b, putting $x = 500$ and $C(x) = 16800$

$$16800 = 3(500) + b$$
$$b = 1800$$

The cost function is

$$C(x) = 3x + 1800$$

Example 2.3:

A man rents a car according to the conditions 80 cents per kilometer and 15 dollar. Find the rent function as a variable distance.

Solution:

Rent per kilometer= m = 80 cents = 0.8 dollars
Distance covered by the car = x kilometers
Fixed value = b = 15 dollars

$$R(x) = mx + b$$
$$R(x) = 0.8x + 15$$

is the rent function.

The graph of the revenue function is

figure 2.8.

Business and Economics

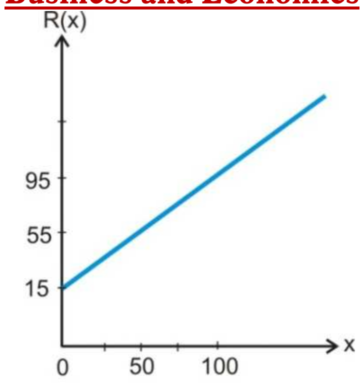

Figure 2.8

Example 2.4:

A school arranges a picnic for the students. The expenses are 5 dollars per head and 100 dollars bus rent. Find a cost function as a variable of number of students.

Solution:

Expanses per head= m = 5 dollars
Fixed cost= b =100 dollars
Let x be the number of student. The cost function is

$$C(x) = mx + b$$
$$C(x) = 5x + 100$$

Figure 2.9.

Business and Economics

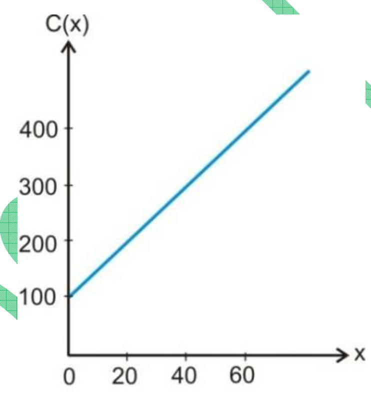

Figure 2.9

Example 2.5:

A man imports cooking oil. He imports oil for 90 cents per liter and also spends 2000 dollars as his office expenditure, find the cost function as a function of x liters oil.

Solution:

Cost per liter = m = 90 cents = 0.9 dollars
Fixed cost = b = 2000 dollars
Let x be the volume of oil in liters. The cost function is

$$C(x) = mx + b$$
$$C(x) = 0.9x + 2000$$

Figure 2.10

Business and Economics

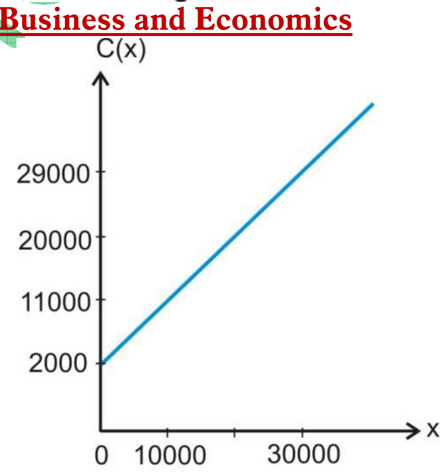

Figure 2.10

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DEMAND AND SUPPLY FUNCTIONS:

Marketing studies help to form demand functions. A demand function expresses the relationship between the price of a product and the quantity of the product that will sell. According to the law of demand the quantity demanded will increase as price decrease and vice versa.

Economists and businessmen use demand and supply functions to decide the price of the product. A supply function expresses the relationship between the price of a product and the quantity manufactures are willing to produce at that selling price.

The point of intersection of the graphs of demand and supply functions is called market Equilibrium and the price corresponding to the point of intersection is the equilibrium price, **figure 2.13**.

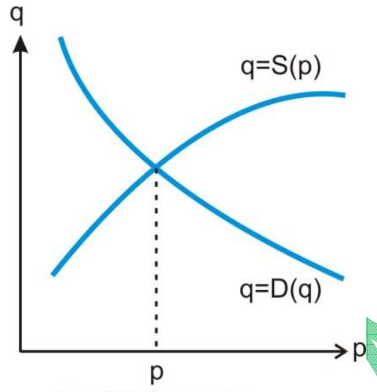

Figure 2.13

Example 2.7: **Business and Economics**

According to the market survey the consumers will buy 4600 tooth paste tubes per month if the price is ¢ 170 and 3600 if the price is ¢ 220. The manufacturer is willing to supply 3800 if the price is ¢ 180 and 4300 if the price is ¢ 230. Assuming that the demand and supply functions are linear, find the equilibrium price and demand for the market.

Solution: Demand function:

$$m = \frac{3600 - 4600}{220 - 170} = -20$$

The demand function as a variable of price is

$$q - 4600 = -20(p - 170)$$
$$q = -20\,p + 8000$$

Supply function:

$$m = \frac{4300-3800}{230-180} = 10$$

The supply function as a variable of price is

$$q - 3800 = 10(p - 180)$$
$$q = 10\,p + 2000$$

For market equilibrium

$$-20\,p + 8000 = 10\,p + 2000$$
$$P = 200$$

So that $q = 4000$

Equilibrium price = $p = 200$

Equilibrium demand and supply= $q = 4000$

The graph is the **figure 2.14**.

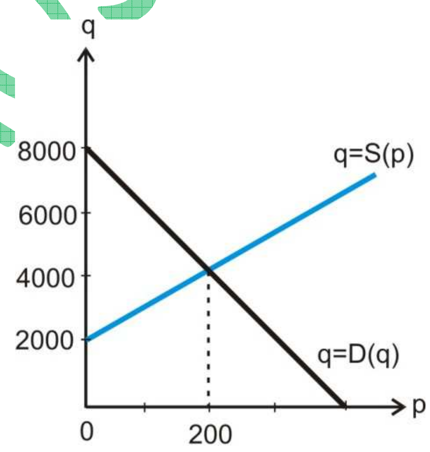

Figure 2.14

FUNCTIONS FORMING FROM A FORMULA:

Many formulas consist of more than one unknown values but one variable function such as $f(x)$ depends on only one variable that is x and some constants. A formula may be converted into a function by putting all unknown values except one which is the variable of the function.

SIMPLE INTEREST (FORMULA):

Simple interest is the interest that is computed solely on the amount of the principal.

Formulae: (i) $I = Prt$ (ii) $A = P(1 + nr)$

I : Simple interest , r : interest rate per time period

P : Principal , n : Number of time periods

Example 2.8:

Business and Economics

Find the total amount as a function of time period for an invested amount \$ 6000 at 20% per annum.

(a) Find the amount at the end of 5 years.

(b) Find the amount at the end of 8 years.

Solution:

$P = \$\,6000$, $r = 20\% = 0.2$

$$A = P(1 + n\,r)$$
$$A(n) = 6000(1 + 0.2\,n)$$
$$A(n) = 1200\,n + 6000 \quad , n \in \mathbb{N}$$

is the total amount as a function of time periods n. **Fig. 2.15**.

(a) Total amount at the end of 5 years.

$A(5) = 1200(5) + 6000 = 12000$ dollars

(b) Total amount at the end of 8 years.

$A(8) = 1200(8) + 6000 = 16800$ dollars

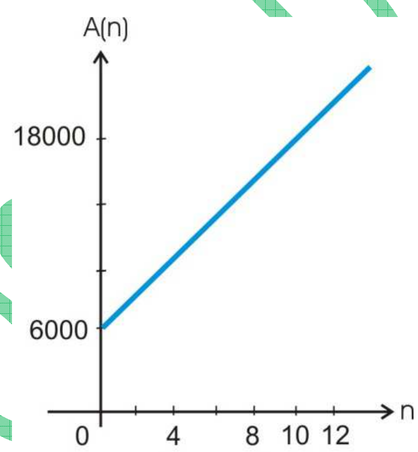

Figure 2.15

Example 2.9:

Business and Economics

Find total amount A as a function of interest rate per time period for a principal \$ 800 in 5 years.

(a) What is the amount at 20% per annum?

(b) What is the amount at 15% per annum?

Solution:

$P = 800$, $n = 5$

$$A = P(1 + n\,r)$$
$$A(r) = 800(1 + 5r)$$
$$A(r) = 4000r + 800 \quad , \quad 0 < r < \infty$$. **Fig. 2.16**.

(a) $r = 20\% = 0.2$

$A(0.2) = 4000(0.2) + 800 = 1600$ dollars

(b) $r = 15\% = 0.15$

$A(0.15) = 4000(0.15) + 800 = 1400$ dollars

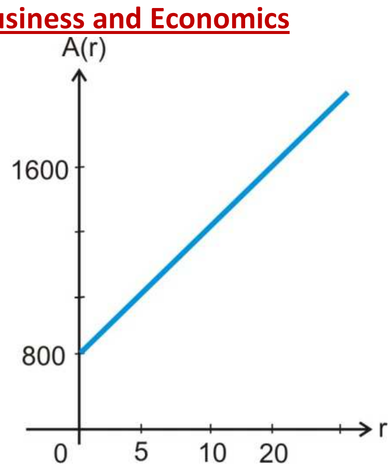

Figure 2.16

RATIONAL FUNCTIONS:

A function of the form

$$f(x) = \frac{p(x)}{q(x)}$$

is a rational function such that $p(x)$ and $q(x)$ are polynomials (or polynomial functions).

Business and Economics

Example 2.10:

A man imports cooking oil. He imports oil 90 cents per liter and 2000 dollars his office expenditure. Find cost per liter oil as a function of volume of oil in liters.

What is the cost per liter oil if he imports oil

(a) 1000 liters (b) 2000 liters (c) 4000 liters?

Solution:

The cost function for x liters oil is

$$f(x) = \frac{0.9x + 2000}{x}$$

$$f(x) = 0.9 + \frac{2000}{x}$$

Figure 2.17.

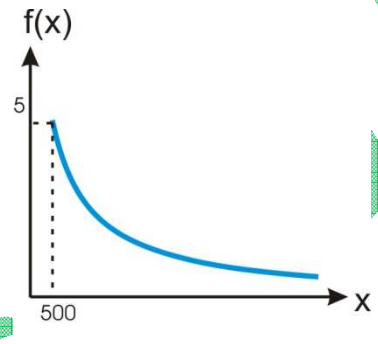

Figure 2.17

$$(a)\ f(1000) = 0.9 + \frac{2000}{1000} = 2.9 \text{ dollars per litre}$$

$$(b)\ f(2000) = 0.9 + \frac{2000}{2000} = 1.9 \text{ dollars per litre}$$

$$(c)\ f(4000) = 0.9 + \frac{2000}{4000} = 1.4 \text{ dollars per litre}$$

Social Science

Example 2.11:

A school arranges a picnic for the students. The expanses are 5 dollars per head and 100 dollars bus rent. Find per head cost as a function of number of students.

What is the cost per head for number of students

(i) 20 (ii) 40 (iii) 50 ?

Solution:

Cost function for x students.

$$C(x) = 5x + 100$$

Per head cost function is

$$f(x) = \frac{5x + 100}{x}$$

$$f(x) = 5 + \frac{100}{x}$$

Figure 2.18.

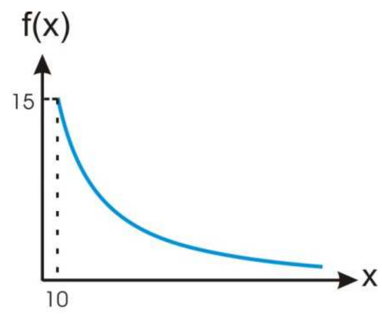

Figure 2.18

$(i)\ \ f(20) = 5 + \dfrac{100}{20} = 10\ \text{ dollars per head}$

$(ii)\ \ f(40) = 5 + \dfrac{100}{40} = 7.25\ \text{ dollars per head}$

$(iii)\ f(50) = 5 + \dfrac{100}{50} = 7\ \text{ dollars per head}$

Example 2.12:

Business and Economics

There are two type of cost to export wheat. The fare cost and storage cost. The fare cost is 30 cents per kilogram and storage cost is a fraction of 700 dollars and the mass of wheat in kilogram. Find the cost function.

Draw the graph to find the value of x for minimum cost.

Solution:

Suppose that x kilogram wheat is exported.

Per kg fare cost $= m = 30$ cent $= 0.3$ dollars

Storage cost $= 700/x$ dollars

The cost function is

$$C(x) = 0.3x + \frac{700}{x}$$

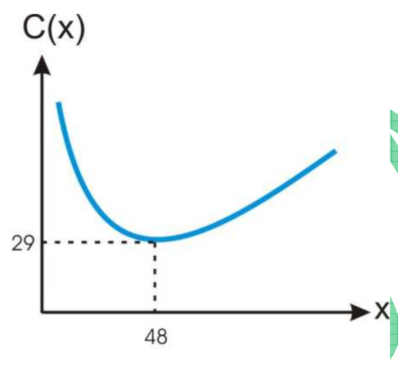

Figure 2.19

Graph of the function is **figure 2.19**.

Example 2.13:

Business and Economics

The package of a marriages garden for less than or equal to 200 person is 8 dollars per head and 400 dollars fixed charges. If the number of person lie between 200 and 500, the package is 8 dollars per head and a fraction of 200 and $(0.02x - 2)$. Find the cost function.

Draw the graph of the function.

Solution:

The cost function for less than and eequal to 200 person.

$$C_1(x) = 8x + 200$$

The cost function for number of person between 200 and 500.

$$C_2(x) = 8x + \frac{200}{0.02x - 2}$$

So that cost function can be written as

$$C(x) = \begin{cases} 8x + 200 & \text{for} \quad 0 < x < 20 \\ 8x + \dfrac{200}{0.02x - 2} & \text{for} \quad 200 \le x < 500 \end{cases}$$

The graph is figure