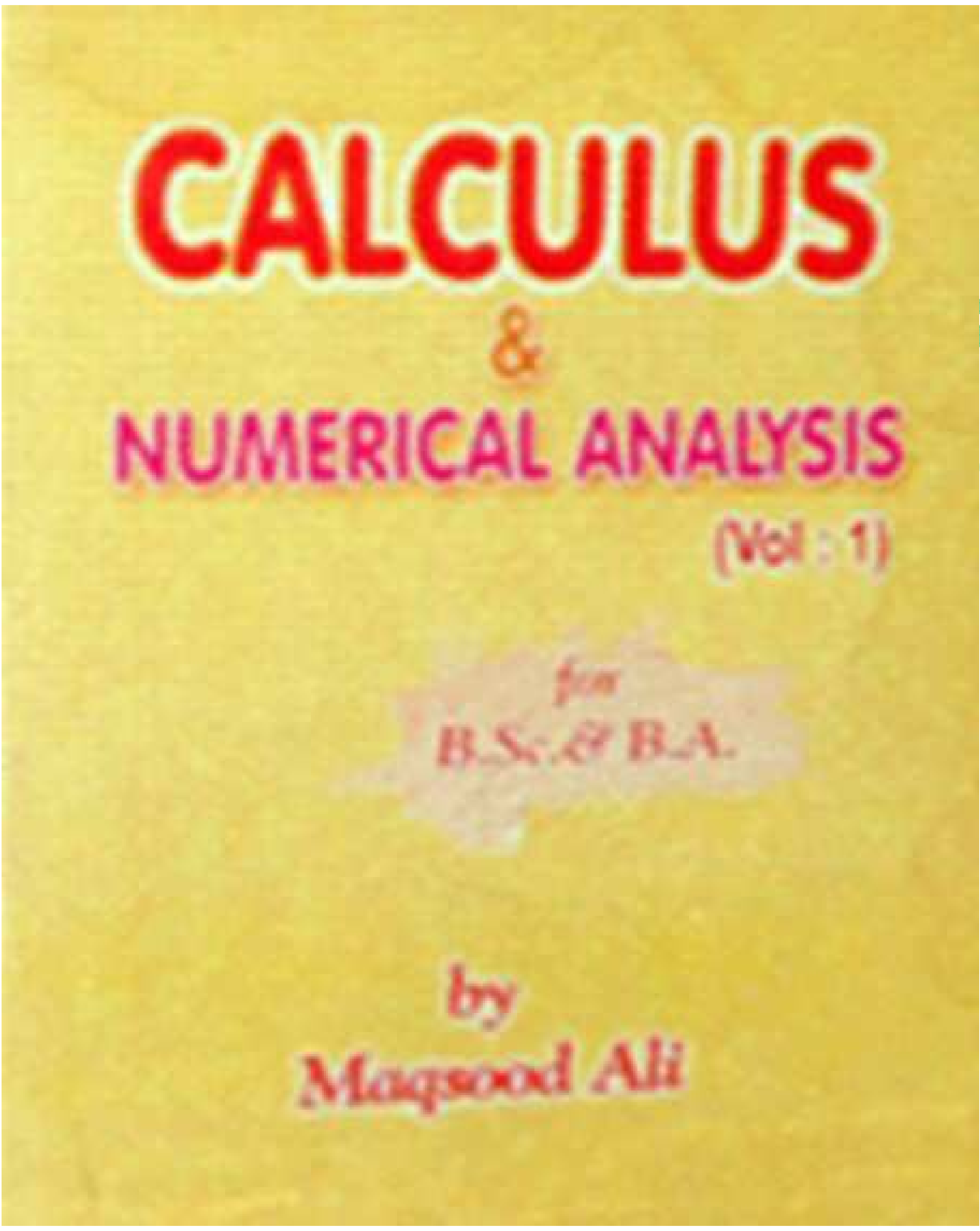
CALCULUS
&
NUMERICAL ANALYSIS
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for
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Q-D METHOD

Quotient Difference (Q.D) Method:

This is a vary general method to determine all the roots of a polynomial equation. This method gives approximate values of a polynomial equation. Exact values can be find by iterative method. For example by Newton's method, Secant method etc.

Suppose that $P_n(x)$ is nth degree polynomial equation.

$$P_n(x) = a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1} x + a_n = 0$$

To find the roots of the equation.Form an array of q and e terms. The first row of q's and second row of e's are calculated by the following method.

First row of q's:

$$q_1 = -\frac{a_1}{a_0}, q_2 = 0, \cdots, q_n = 0$$

Second row of e's:

$$e_0 = 0 \text{ and } e_n = 0$$

$$e_1 = \frac{a_2}{a_1}, \quad e_2 = \frac{a_3}{a_2}, \cdots, \quad e_{n-1} = \frac{a_n}{a_{n-1}},$$

or $e_i = \dfrac{a_{i+1}}{a_i}$, $i = 1,2,3,\cdots,n-1$

The start of the array

e_0	q_1	e_1	q_2	e_2	$\cdots$	e_{n-1}	q_n	e_n
	$-\frac{a_1}{a_0}$		0		$\cdots$		0	
0		$\frac{a_2}{a_1}$		$\frac{a_3}{a_2}$	$\cdots$	$\frac{a_n}{a_{n-1}}$		0

A new row of q's: $q_i = e_i - e_{i-1} + q_i$

A new row of e's:

$$e_i = \left(\frac{q_{i+1}}{q_i}\right) e_i$$

There are three cases of e:

Case I: All roots are real and $a_n \neq 0$, $n = 0, 1, 2, \cdots, n$

$q_i$are the approximate values when $e_i = 0$

$$i = 1,2,3,\cdots,n-1.$$

Example 5:

Find all roots of the following equation.

$$x^4 + 31x^3 + 305x^2 + 1025x + 775 = 0$$

Solution:

$$x^4 + 31x^3 + 305x^2 + 1025x + 775 = 0$$

$a_0 = 1,\ a_1 = 31,\ a_2 = 305,\ a_3 = 1025,\ a_4 = 755$

e_0	q_1	e_1	q_2	e_2	q_3	e_3	q_4	e_4
	-31		0		0		0	
0		9.84		3.36		0.74		0
	-21.16		-6.48		-2.62		-0.74	
0		3.01		1.36		0.21		0
	-18.15		-8.13		-3.77		-0.95	
0		1.35		0.63		0.05		0
	-16.8		-8.85		-4.35		-1.00	
0		0.71		0.31		0.01		0
	-16.09		-9.25		-4.65		-1.01	
0		0.41		0.16		0.00		0
	-15.68		-9.5		-4.81		-1.01	
0		0.25		0.08		0.00		0
	-15.43		-9.67		-4.89		-1.01	
0		0.16		0.04		0.00		0
	-15.27		-9.79		-4.93		-1.01	
0		0.10		0.02		0.00		0
	-15.17		-9.87		-4.95		-1.01	
0		0.07		0.01		0.00		0
	-15.10		-9.93		-4.96		-1.01	
0		0.05		0.00		0.00		0
	-15.05		-9.98		-4.96		-1.01	
0		0.03		0.00		0.00		0
	-15.02		-10.01		-4.96		-1.01	
0		0.02		0.00		0.00		0
	-15.00		-10.03		-4.96		-1.01	
0		0.01		0.00		0.00		0
	14.99		-10.04		-4.96		-1.01	

Hence

$x_1 \cong q_1 = -14.99$, $x_2 \cong q_2 = -10.04$

$x_3 \cong q_3 = -4.99$, $x_3 \cong q_3 = -1.01$

Case II: A Pair of Complex Roots *and* $a_n \neq 0$:

In this case one of the e's will not converge to zero but will fluctuate in value. So we find a quadratic factor

$$x^2 - gx - h.$$

If e_j does not approach to zero, then

$$g = q_j + q_{j+1}$$

and $$h = -[q_j(above\ e_j).q_{j+1}(below\ e_j)]$$

Example 6:

Solve the equation $x^4 - 8x^3 + 27x^2 - 70x + 50 = 0$

Solution:

$$x^4 - 8x^3 + 27x^2 - 70x + 50 = 0$$

$$a_0 = 1\ ,\ a_1 = -8\ ,\ a_2 = 27, a_3 = -70\ ,\ a_4 = 50$$

e_0	q_1	e_1	q_2	e_2	q_3	e_3	q_4	e_4
	8		0		0		0	
0		−3.38		−2.56		−0.71		0
	4.62		0.79		1.88		0.71	
0		−0.58		−6.16		−0.27		0
	4.04		−4.79		7.77		0.98	
0		0.69		9.99		−0.03		0
	4.73		4.51		−2.25		1.01	
0		0.66		−4.98		0.01		0
	5.39		−1.13		2.74		1.00	
0		−0.14		12.08		0.00		0
	5.25		11.09		−934		1.00	
0		−0.30		−10.17		0.00		0
	4.95		1.22		0.83		1.00	
0		−0.07		−692		0.00		0
	4.88		−563		7.75		1.00	
0		0.08		9.53		0.00		0
	4.96		3.82		−1.78		1.00	
0		0.06		−4.44		0.00		0
	5.02		−0.68		2.66		1.00	
0		0.00		17.36		0.00		0
	5.02		16.67		−14.7		1.00	

Hence $x_1 \cong q_1 = 5.01$

$x_4 \cong q_4 = 1.00$

Since e_2 does not approach to zero, so two roots of the given polynomial equation are complex.

We find a quadratic factor $x^2 - gx - h$

$$g = q_2 + q_3 = 16.67 - 14.7 = 1.97$$

$$h = -q_2 \text{ (above } e_2) \, q_3 \text{ (below } e_2) = -(-0.68)(-14.7) = -9.996$$

$$x^2 - gx - h = 0$$

$$x^2 - 1.97x + 9.996 = 0$$

$$x = 0.985 \pm 3i$$

Hence the required roots are

$x_1 \cong 5.01, \quad x_2 \cong 0.985 - 3i, \quad x_3 \cong 0.985 + 3i, \quad x_4 \cong 1.00$

Case III: One of the Coeffcient in the polynomial is Zero:

1	1	0	−82	336	−360
		1	1	−81	225
	1	1	−81	255	−105
		1	2	−79	
	1	2	−79	1796	
		1	3		
	1	3	−76		
		1			
	1	4			
	1				

$$t^4 + 4t^3 - 76t^2 + 176t - 105 = 0$$

$$a_0 = 1\,,\ a_1 = 4\,,\ a_2 = -76\,,\ a_3 = 176\,,\ a_4 = -105$$

e_0	q_1	e_1	q_2	e_2	q_3	e_3	q_4	e_4
	-4		0		0		0	
0		-19		-2.32		-0.60		0
	-23		16.68		1.72		0.6	
0		13.78		-0.24		-0.21		0
	-9.22		2.66		1.75		0.81	
0		-3.98		-0.16		-0.02		0
	-13.2		6.48		1.89		0.83	
0		1.95		-0.05		-0.01		0
	-11.25		4.48		1.93		0.84	
0		-0.78		-0.02		0.00		0
	-12.03		5.25		1.95		0.84	
0		-0.34		-0.01		0.00		0
	-11.69		4.89		1.96		0.84	
0		-0.14		0.00		0.00		0
	-11.83		5.03		1.96		0.84	
0		0.06		0.00		0.00		0
	-11.77		4.97		1.96		0.84	
0		-0.02		0.00		0.00		0
	-11.79		4.95		1.96		0.84	
0		-0.01		0.00		0.00		0
	-11.78		4.94		1.96		0.84	

$x_1 \cong 1 + q_1 = 1 - 11.78 = -10.78$

$x_2 \cong 1 + q_2 = 1 + 4.96 = 5.96$

$x_3 \cong 1 + q_3 = 1 + 1.96 = 2.96$

$x_4 \cong 1 + q_4 = 1 + 0.84 = 1.84$

EXERCISE D-3

Solve the following equations by Q-D method.

(1) $x^4 - 28x^3 + 263x^2 - 980x + 1170 = 0$

(2) $x^4 - 21x^3 + 131x^2 - 255x + 40 = 0$

(3) $x^4 - 6x^3 - 7x^2 - 32x + 20 = 0$

(4) $x^4 + 20x^3 + 116x^2 + 208x + 60 = 0$

(5) $x^4 - 11x^3 + 37x^2 - 96x - 220 = 0$

(6) $x^4 + 9x^3 - 67x^2 - 270x + 400 = 0$

(7) $x^4 - 12x^3 - 134x^2 - 72x + 160 = 0$

(8) $x^3 - 13.5x^2 + 48x - 22 = 0$

(9) $x^3 - 6.3x^2 + 10.6x - 8.6 = 0$

(10) $x^4 - 12x^3 - 125x^2 + 250 = 0$

(11) $x^4 - 11x^3 + 13x^2 - 700 = 0$