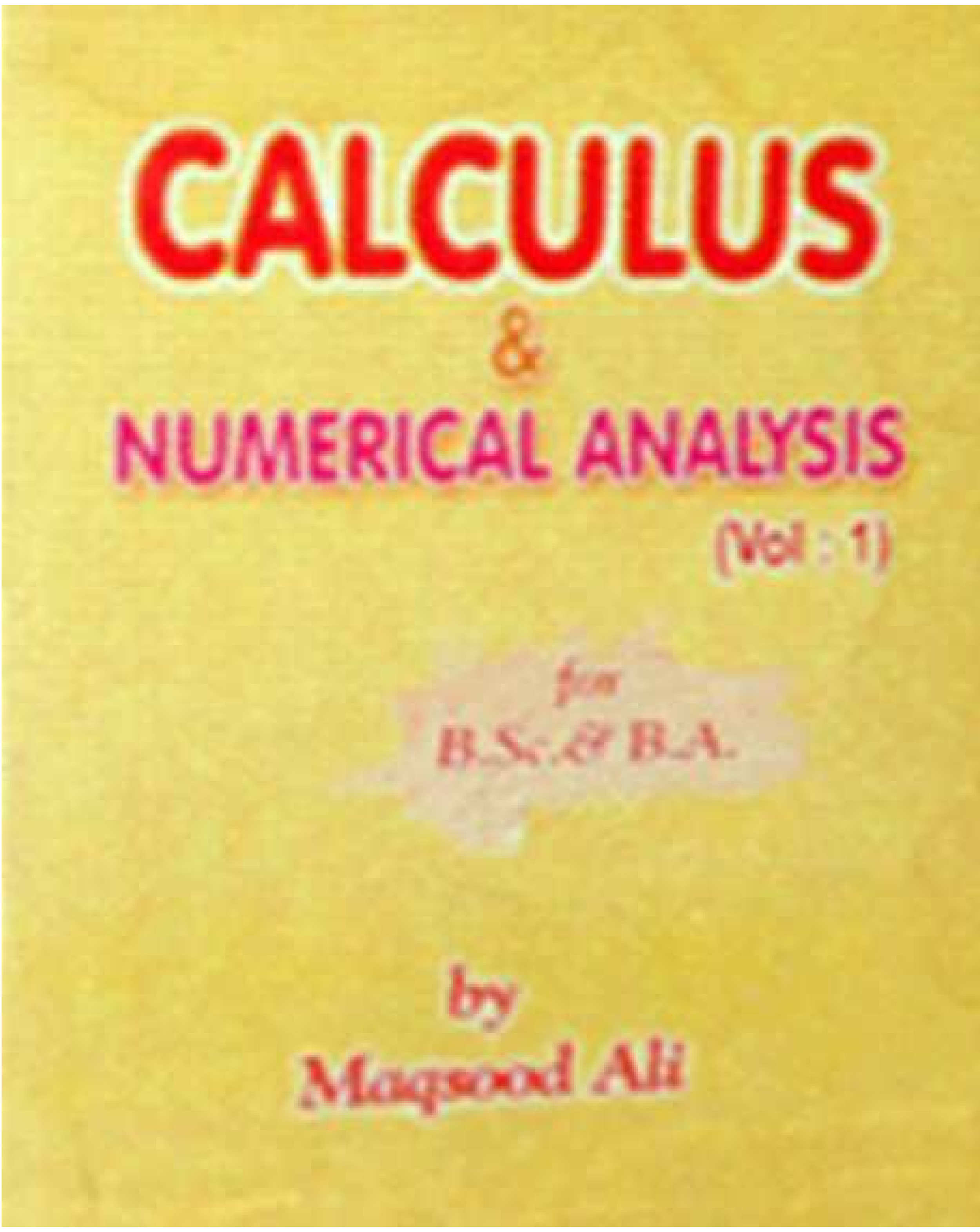
CALCULUS
&
NUMERICAL ANALYSIS
(Vol : 1)
for
B.Sc. & B.A.
by
Maqsood Ali

SECTION C

MULTIVARIABLES FUNCTIONS

In previous section, functions of one variable have been studied. Many phenomena depend on more than one variable. The study of multivariables functions is necessary to solve the problems in the managerial, social and life science etc. which depend one more than one variable.

SEVERAL VARIABLES FUNCTIONS:

Let S be the subset of R^n, $n = 2,3,4,\cdots,n$. A function f on S is a rule that assigns to each $(x_1, x_2, \cdots, x_n) \in S$ a unique number z, that is $z = f(x_1, x_2, \cdots, x_n)$, f is called functions of n variables.

Domain and Range:

If $z = f(x_1, x_2, \cdots, x_n)$ where $(x_1, x_2, \cdots, x_n) \in S$ then z is the value of f ordered n-tuples $(x_1, x_2, \cdots, x_n)$. The collection of all the values of f is called the range of f, while S is called the domain of S.

$x_1, x_2, \cdots, x_n$ are called indepentennt variables and z is called the dependent variable.

Real Valued Function of Two Variables:

Let S be the subset of R^2, so that S is the set of ordered pair (x, y). A function $f: S \to R$ is called a real valued function of two variables. Function f on S into R is a rule that assign to each pair (x, y) in S a unique number denoted by

$$z = f(x, y) \in R.$$

In this section we study limit, continuity, differentiability and derivability of real valued function of two variables.

THREE DIMENSIONAL COORDINATE SYSTEM:

We need three axis to sketch the graph of function of two variables. To understand three dimensional coordinates system suppose that you are standing in a corner of a room and your hands are touching two walls. Suppose that the corner where you are standing is the origin of three axes x, y and z. The positive x-axis is along the right wall at the floor, the positive y-axis is along the left wall at the floor and the positive z-axis is in the corner where the two walls meet, pointing up, see

Figure C-2.

A point in three dimensional coordinates system is determined by a triple (x_0, y_0, z_0).

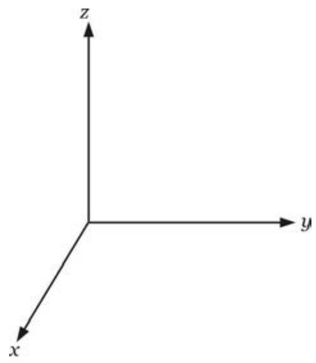

Figure C-2

For example a ball is placed on the floor at the point $(3,5)$ **figure C-2B**. If the unit of measurement is metre then you can get the ball by walking 3m along xz-walls and then 5m parallel to yz-wall. Therefore $(3,5)$ is the exact position of the ball because it is in the xy-plane.
Now the ball is placed 10m high directly over the point $(3,5)$, so the coordinates $(3,5)$ is not showing the exact position of the ball.

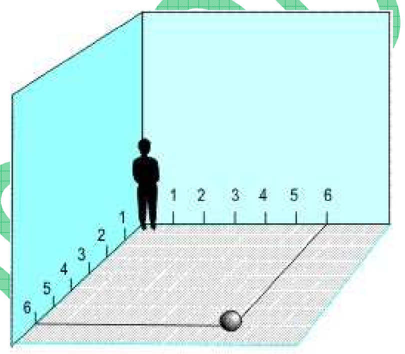

Figure C-2 B

So we cannot get the ball by walking 3m along xz-wall and then 5m parallel to yz-wall, we must go 10m high also from the floor to get the ball, so the point $(3,5,10)$ shows the exact position of the ball.

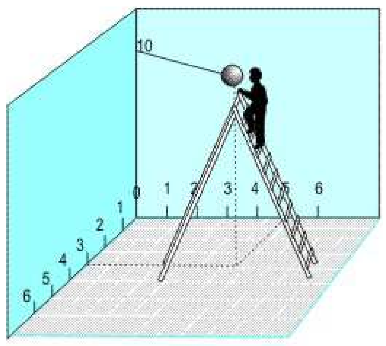

Figure C-2 C

DEFINITIONS FOR TWO VARIABLES FUNCTION

1- Interior Point:

If $f: R^2 \to R$ then a point $(a, b) \in R^2$ in a region R_1 in the xy-plane is an interior point of R_1 if it is the centre of the disk that lies entirely in R_1 as shown in **figure C-3**. The interior points of a region, as a set, make up the interior of the region.

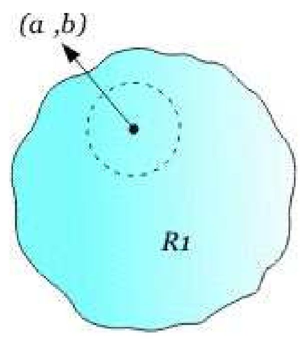

Figure C-3

2- Boundary Point:

If $f: R^2 \to R$ then a point $(a, b) \in R^2$ is a boundary point of R_1 if every disk centered at (a, b) contain points that lies outside of R_1 as well as points that lie in R_1. The boundary point itself need not belong to R_1. The region's boundary points make up its boundary.

Figure C-4

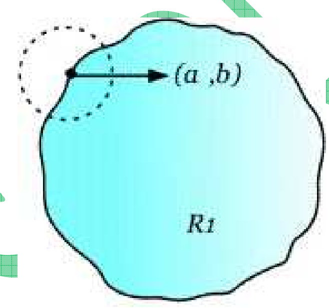

Figure C-4

3- Open Disk:

A disk is open if it consists entirely of interior points. If r is the radius and (a, b) centre of the disk then the set of all points of open disk is

$$\{(x, y) | (x-a)^2 + (y-b)^2 < r^2\}$$

Figure C-5

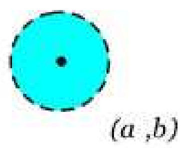

Figure C-5

4- Closed Disk:

A disk is closed if it contains all interior points and all of its boundary points. If r is the radius and (a, b) centre of the disk the set of all points of the closed disk is

$$\{(x, y) | (x-a)^2 + (y-b)^2 \le r^2\}$$

Figure C-6

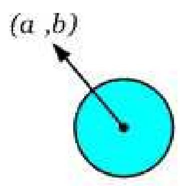

Figure C-6

5- Neither Open nor Closed Disk:

Some regions or some disks are neither open nor closed. An open disk consists some but not all of its boundary points is called neither open nor closed disk.

6- Bounded and Unbounded Region:

A region in the plane is bounded if it lies inside a disk of fixed radius.

A region is unbounded if it does not lies inside a disk of fixed radius.

7- Neighborhood:

If $(a, b) \in R^2$, then neighbourhood of (a, b) is an open disk with (a, b) as its centre. If for $\delta \in R^+$ or $\delta > 0$, then open disk

$$\{(x, y) \in R^2 / |x - a| < \delta \ \ and \ \ |y - b| < \delta\}$$

is a neighborhood of (a, b), where δ is called the radius of open disk (or neighborhood).

8- Deleted Neighborhood:

If $(a, b) \in R^2$, then deleted neighbourhood of (a, b) is an open disk with (a, b) as its centre. For $\delta \in R^+$ or $\delta > 0$, then open set

$\{(x, y) \in R^2 | (x, y) \neq (a, b) \ \ where \ |x - a| < \delta \ \ and \ \ |y - b| < \delta\}$

is a deleted neighborhood of (a, b), which consists of all interior points of an open disk of radius δ except from centre (a, b).

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