BOOK 1

CALCULUS

WITH APPLICATIONS

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Chapter 59

DISPLACEMENT , VELOCITY AND ACCELERATION

A particle is travelling in a straight line. The speed (v) of the particle depends on time (t). The acceleration (a) and the distance (s) from a fixed point O at time t can be found by the following method

$$a = \frac{dv}{dt}$$

and $$s = \int v\, dt$$

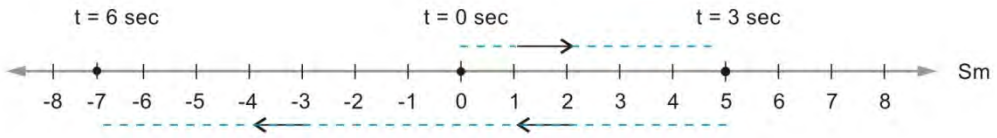
If the given acceleration (a) of the particle depends on time (t), then

$$v = \int a\, dt$$

and $$s = \int v\, dt$$

Explanation:

A particle is travelling with speed v = f(t). The displacement from O of the particle on a number line at time t = 0, 3, 6 seconds are shown in the diagram.



• : position of the particle on number line.

← and → : directions of motion of the particle.

s : displacement of the particle from O at time t.

[For more detail "Application of derivative"]

For example:The displacement s m, of a particle at t second is given below

$$s = 4t - t^2$$

we have to find the position of the particle at t = 0, 1, 2, 3, 4, 5 seconds and the total distance covered by the particle in 5 seconds.

Firstly, we find the time at which the speed of the particle is zero.

$$\text{Speed} = v = 4 - 2t$$

$$v = 0 \Rightarrow 4 - 2t = 0$$

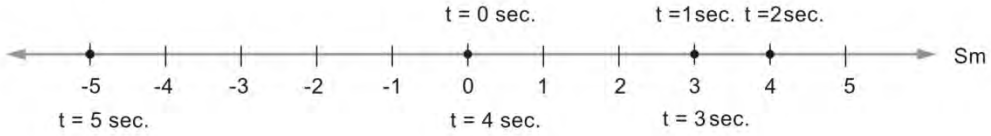
$$\Rightarrow \quad t = 2 \text{ sec}$$

$\therefore$ speed of the particle is zero at t = 2 sec.

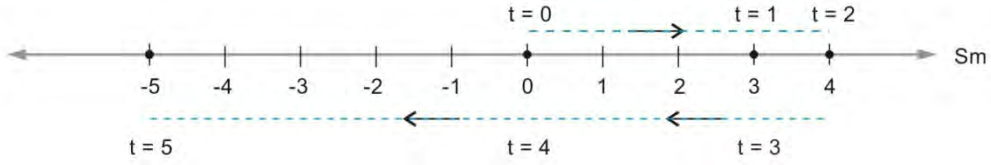
The displacement of the particle from O at t = 0, 1, 2, 3, 4, 5seconds.

t sec	0	1	2	3	4	5
S m	0	3	4	3	0	−5

Positions of the particle on a number line:



Displacemets from O:



Total distance covered by the particle in 5 seconds = 4 + 4 + 5

= 13m

(ii) $$v = \frac{ds}{dt} = 2t - \frac{1}{3}t^2$$

$$s = \int \left(2t - \frac{1}{3}t^2\right) dt$$

$$= t^2 - \frac{1}{9}t^3 + C \longrightarrow (2)$$

By putting t = 0, s = 0 in (2), we get

$$C = 0$$

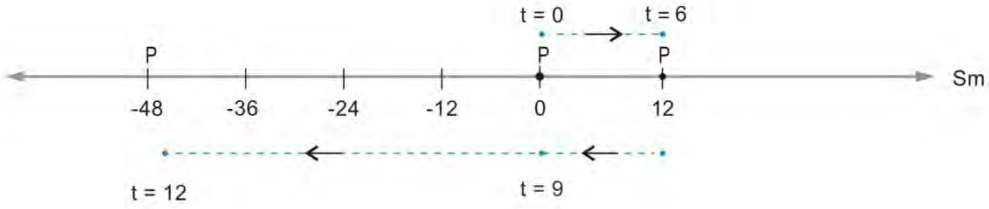
Substitute in equation (2)

$$s = t^2 - \frac{1}{9}t^3 \longrightarrow (3)$$

This equation represents the displacement of the particle in time t.

t	6	9	12
s	12	0	–48

Displacements on a number line:



Total distance in 6 seconds = 12m

Total distance in 9 seconds = 12 + 12 = 24m

Total distance in 12 seconds = 12 + 12 + 48 = 72m

(iii) $$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$= \frac{72}{12}$$

$$= 6 \text{ m/s}$$

(iv) Acceleration:

$$a = \frac{dv}{dt} = 2 - \frac{2}{3}t$$

acceleration at t = 6 sec

$$a = 2 - \frac{2}{3} \times 6 = -2 \text{ m/s}^2$$

acceleration at t = 9 sec

$$a = 2 - \frac{2}{3} \times 9 = -4 \text{ m/s}^2$$