BOOK 1

CALCULUS

WITH APPLICATIONS

M. MAQSOOD ALI

Chapter 57

RATE OF CHANGE AND INTEGRATION

The rate of change in S with respect to t is given by

$$\frac{dS}{dt} = f(t)$$

The value of S at t = a can be found as

$$S = \int_{t=a} f(t)\, dt$$

Example 1: An empty tank is filling with water. The rate of increasing of the volume of water is given below

$$\frac{dV}{dt} = \frac{5}{(t+1)^{2/3}}.$$

What is the volume of the water in the tank at t = 26.

Solution:-

$$\frac{dV}{dt} = \frac{5}{(t+1)^{2/3}}$$

$$\int dV = 5\int (t+1)^{-2/3} dt$$

$$V = 15(t+1)^{1/3} + C \longrightarrow (1)$$

Substituting t = 0, V = 0 in (1), we get

$$C = 0$$

Substitute C = 0 in (1)

$$V = 15\,(t+1)^{1/3} \longrightarrow (2)$$

When t = 26

$$V = 15\,(26+1)^{1/3}$$
$$= 45 \quad \text{Ans.}$$

M.C.Q's K-11

(1) The rate of change of population is given by

$$\frac{dP}{dt} = \sqrt{P}.$$

What is the population at time t. Population of the area is 16, when t = 0.

(a) $4\sqrt{P} = t - 8$ (b) $4P = (t + 8)^2$

(c) $2\sqrt{P} = 8$ (d) $2P = t^2 + 64$

(2) The volume of the water in a tank is increasing at a rate

$$\frac{dV}{dt} = 2t + 1.$$

What is the volume of the tank at time t. The tank was empty, when t=0.

(a) $V = t^2 + t$ (b) $V = 2$

(c) $V = t^2$ (d) $V = 4t^2 + t$

(3) An empty tank is filling with water. The rate of increasing of volume is

$$\frac{dV}{dt} = \frac{1}{\sqrt{t}}.$$

What is the volume of the water in the tank at t = 9.

(a) 12 (b) $3/2$

(c) 9 (d) 6

(4) The cost of an object is increasing at a rate of

$$\frac{dC}{dt} = \frac{1}{2\sqrt{t}}$$

C = Rs. 200, when t = 0, where t is number of days.

What is the cost of the object, when t = 100 days.

(a) Rs.250 (b) Rs.210

(c) Rs.200.05 (d) Rs.205

(5) A particle is moving along a straight line with speed v. The displacement s = 0 when t = 0 and

$$v = 8 - 2t$$

How much distance is covered by the particle in 5 seconds.

(a) 10 (b) 40 (c) 15 (d) 32

Chapter 58

EQUATION OF CURVES

Case 1: The gradient dy/dx of the tangent to a curve at any point (x, y) on the curve is given by

$$\frac{dy}{dx} = g(x)$$

The equation of the curve can be found as

$$y = \int g(x)\,dx$$

Example 1: The gradient dy/dx of a tangent to a curve at any point (x, y) is $2x - 6$. The curve passes through the point (1, 5). Find the equation of the curve.

Solution:-

$$\frac{dy}{dx} = 2x - 6$$

$$y = \int (2x - 6)\,dx$$

$$y = x^2 - 6x + C \longrightarrow (1)$$

Substitute $x = 1$ and $y = 5$ in equation (1)

$$5 = 1 - 6 + C$$

$$C = 10$$

Substitute $C = 10$ in equation (1)

$$y = x^2 - 6x + 10$$

is the equation of the curve.

Case 2: The gradient dy/dx of a tangent to a curve is given by

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

The equation of the tangent can be found as

$$\int g(y)\,dy = \int f(x)\,dx$$

Example 2: The gradient of a tangent to a curve at any point (x, y) on the curve is $12x^2/y$ and the curve passes through the point (2, –9). Find the equation of curve.

(6) The gradient of the tangent to a curve at any point (x, y) on the curve is

$$\frac{3(x^2-1)}{y}$$

The curve is passes through the point (2, 5). Find the equation of the curve.

(7) The gradient of a tangent to a curve at any point (x, y) on the curve is

$$\frac{4x}{3y^2}$$

The curve passes through the point (1, 3). Find the equation of the curve.

(8) The gradient of the normal to a curve at any point (x, y) on the curve is

$$\frac{-x^2}{1+x^2}$$

The curve passes through the point (1, 2). Find the equation of the curve.

(9) A curve for which $\frac{dy}{dx} = -\frac{k}{x}$, where k is a constant. The curve passes through the point (1, 10) and the slope of the normal at that point is $\frac{1}{3}$. Find the equation of the curve.

(10) A curve for which $\frac{dy}{dx} = 2x + \frac{k}{x^3}$, where k is a constant, passes through the point (4, 15). The gradient of the normal to the curve at that point is $-\frac{2}{17}$. Find the equation of the curve.

M.C.Q's K-12

(1) The slope of the tangent to the curve at any point (x, y) is $4x^3 + 5$. The curve passes through the origin. What is the equation of the curve?

(a) $x^4 = 4 - 5x$ (b) $y - x^4 = 5x$

(c) $y = 12x^2$ (d) $y'' = 12x^2$

(2) The gradient of the tangent to the curve at any point (x, y) is $2e^{2x}$. The curve passes through (0, 1). What is the equation of the curve?

(a) $y = e^{2x} + 1$ (b) $\ln y = 2x - 3$

(c) $y = 4e^{2x} - 3$ (d) $\ln y = 2x$

(3) The gradient of the tangent to a curve at any point (x, y) on the curve is $\frac{3}{x^2}$. The point (1, 2) lies on the curve. What is the equation of the curve?

(a) $y = 5 - \frac{3}{x}$ (b) $y = \frac{-6}{x^3}$

(c) $y = \frac{-3}{x}$ (d) $xy = 3$

(4) The pint (1, 3) lies on a curve and the gradient at any point (x , y) on the curve is $3x^2$. What is the equation of the curve?

(a) $y = x^3$ (b) $y = 6x$

(c) $y = x^3 + 2$ (d) $y = 3x^3$

(5) The gradient of the normal to a curve at any point (x, y) is $-\frac{x^4}{6}$. The point (1, 3) lies on the curve. What is the equation of the curve?

(a) $y = \frac{-2x^3}{3}$ (b) $y = 5 - \frac{2}{x^3}$

(c) $y = \frac{91 - x^5}{30}$ (d) $y = \frac{x^3}{6}$

(6) The point (1, 3) lies on a curve and the gradient at any point (x, y) on the curve is $\frac{6}{x^4}$. What is the equation of the curve?

(a) $y = 5 - \frac{24}{x^5}$ (b) $y = \frac{-24}{x^5}$

(c) $y = \frac{-2}{x^3}$ (d) $y = 5 - \frac{2}{x^3}$