BOOK 1

CALCULUS

WITH APPLICATIONS

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Chapter 56

DIFFERENTIAL EQUATIONS

A differential equation consists of derivatives of dependent variable with respect to independent variables.

A differenti al equation involving ordinary derivatives $\left(\frac{dy}{dx}, \frac{dy}{dt}, \frac{dz}{dx} \text{ etc.}\right)$ is called ordinary differential equation.

A differenti al equation involving partial derivatives $\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial z}{\partial t} \text{ etc.}\right)$ is called partial differential equation.

Following equations are the examples of differential equations.

(1) $\frac{dy}{dx} + x = 9$

(2) $\frac{d^3y}{dx^3} + 2\frac{dy}{dx} = y$

(3) $\frac{d^2y}{dx^2} + 9x = 7$

(4) $\left(\frac{dy}{dx}\right)^3 - 3x\frac{dy}{dx} = 2y$

(5) $(x + 1)\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x$

ORDER OF DIFFERENTIAL EQUATION

The order of the highest derivative is called the order of differential equation.

Example 1:

(1) $\frac{d^3y}{dx^3} + x = \frac{d^2y}{dx^2}$; order = 3

(2) $\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^4 = y$; order = 2

DEGREE OF DIFFERENTIAL EQUATION

The power of the highest order derivative in the equation is called degree of the differential equation.

Example 2:

(1) $\frac{d^2y}{dx^2} + 2y = 3$; degree = 1

(2) $\left(\frac{d^2y}{dx^2}\right)^3 + 2\left(\frac{dy}{dx}\right)^5 = 9x$; degree = 3

(3) $\left(\frac{d^3y}{dx^3}\right)^2 + 5y\left(\frac{d^2y}{dx^2}\right)^4 = 9x\frac{dy}{dx}$; degree = 2

In this section we will discuss the differential equations of order 1 or 2 and degree 1.

EXERCISE K-27

Solve the following differential equations.

(1) $\frac{dy}{dx} = 5x^2 + 6x + 7$ (2) $\frac{dr}{dt} = 3t^5 + 9t^3$

(3) $\sin y \frac{dy}{dx} = x \cos y - \cos y$ (4) $x\frac{dy}{dx} = y + xy$

Solve the following differential equations.

(5) $\frac{d}{dx}(\cos x) = 5$; $x = \frac{\pi}{2}$

(6) $\frac{d}{dx}(x^2 + 2x) = 8x^3 + 1$; $x = 1$

(7) $\frac{d}{dx}(4x^3 + 7) = 0$; $x = -2$

(8) $x^2 y \frac{dy}{dx} = y^2 + 1$; $x = 1, y = 0$

(9) $y^2 e^{x-1}\frac{dy}{dx} = (y^3 + 3)^5$; $x = 0, y = 1$

(10) $y\frac{dy}{dx} = 2y^2\frac{dy}{dx} - 5x$; $x = 2, y = 1$

(8) Solve the following differential equation, where integral constant is zero.

$$x^{-2}\frac{dy}{dx} = 3\sqrt{1-y^2}$$

(a) $\frac{1}{2}\sqrt{1-y^2} = x^3$ (b) $\frac{1}{2\sqrt{1-y^2}} = x^3$

(c) $y = \sin x^3$ (d) $\sin^{-1} y = 9x$

(9) What is the solution of the differential equation

$$\frac{dy}{dx}(2x-1)^2 - 4 = 0$$

$y = 3$ when $x = 0$.

(a) $y = -1 - 4/(2x-1)$ (b) $y = 1 - 2/(2x-1)$

(c) $\frac{2}{(2x-1)^3}$ (d) $8(2x-1)^3 + 1$

(10) What is the solution of the differential equation

$$x^3\frac{dy}{dx} = 5\ ,\ y(1) = 5$$

(a) $x^2y = 8$ (b) $y = 2 - \frac{3}{x^2}$

(c) $y = 23 - \frac{18}{x^4}$ (d) $y = 8 - \frac{3}{x^2}$

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