BOOK 1

CALCULUS

WITH APPLICATIONS

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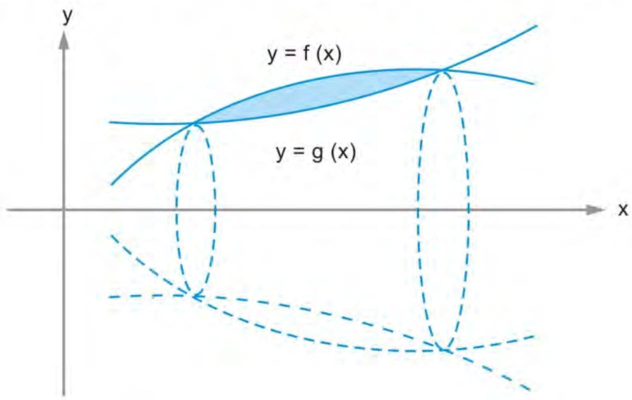
It rotates 360° about y-axis. The volume of the solid thus formed is

$$V = \int_a^b \pi\, x^2\, dy$$

$$\text{or} \qquad V = \int_a^b \pi \left[g(y)\right]^2 dy$$

(3) A shaded region enclosed by two curves y = f(x) and y = g(x).



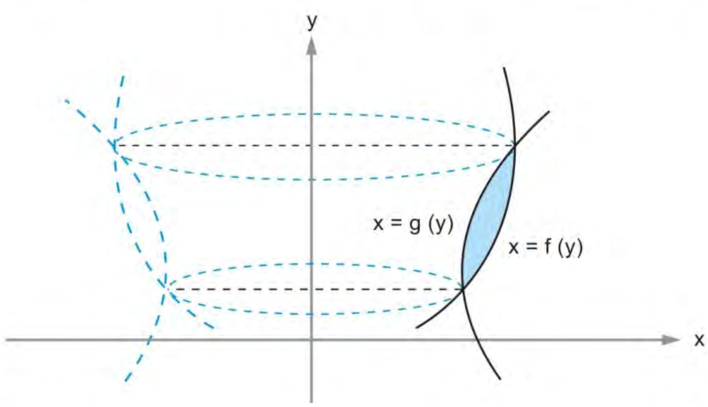
It rotates 360° about x-axis. The volume of the solid thus formed is

$$V = \int_a^b \pi \left[f(x)\right]^2 dx - \int_a^b \pi \left[g(x)\right]^2 dx$$

$$\text{or} \qquad V = \pi \int_a^b \left\{\left[f(x)\right]^2 - \left[g(x)\right]^2\right\} dx$$

where a and b are the abscissas of the points of intersection of the curves.

(4) A shaded region enclosed by two curves x = f(y) and x = g(y).



It rotates 360° about y-axis. The volume of the solid thus formed is

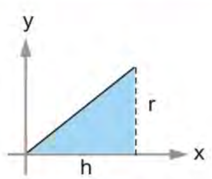
$$V = \int_a^b \pi \left[f(y)\right]^2 dy - \int_a^b \pi \left[g(y)\right]^2 dy$$

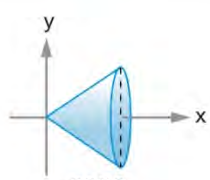
or $$V = \pi \int_a^b \left\{\left[f(y)\right]^2 - \left[g(y)\right]^2\right\} dy$$

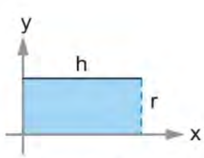
where a and b are the ordinates of the points of intersection of the curves.

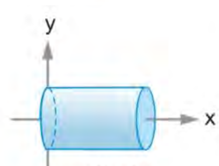
SOME PARTICULAR SOLIDS:

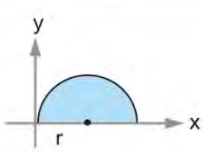
The rotation 360 ° about x-axis of shaded region enclosed by the indicated lines or curves form following solid.

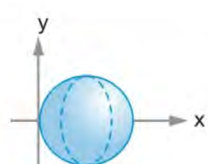
<table>
<tr><th>S.No.</th><th>Shaded Region</th><th>Solid</th><th>Volume</th></tr>
<tr><td>1</td><td>

straight line y = mx</td><td>

cone</td><td>$V = \frac{1}{3}\pi r^2 h$</td></tr>
<tr><td>2</td><td>

horizontal line y=r length h.</td><td>

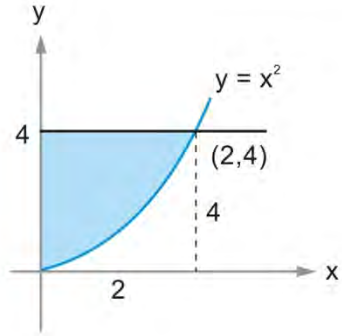
cylinder</td><td>$V = \pi r^2 h$</td></tr>
<tr><td>3</td><td>

semi circle of radius r</td><td>

sphere</td><td>$V = \frac{4}{3}\pi r^3$</td></tr>
</table>

<u>Volume of the Solid:</u>

The horizontal line y = 4 generates a cylinder of length 2 and radius 4.



$$V = \text{Volume of cylinder} - \int_0^2 \pi\, x^4\, dx$$

$$= \pi \times 4^2 \times 2 - \pi \left[\frac{x^5}{5} \right]_0^2$$

$$= 100.53 - 20.11$$

$$= 80.42 \quad \text{(correct to 4 significant figures)}$$

<u>Another Method:</u>

$$V = \int_0^2 \pi\, (4)^2\, dx - \int_0^2 \pi\, (x^2)^2\, dx$$

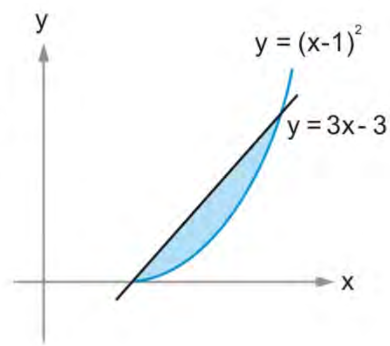
$$= 16\pi \int_0^2 dx - \pi \int_0^2 x^4\, dx$$

$$= 16\pi\, [x]_0^2 - \pi \left[\frac{x^5}{5} \right]_0^2$$

$$= 100.53 - 20.11$$

$$= 80.42$$

Example 3: Calculate the volume generated when the shaded region bounded by the curve $y = (x - 1)^2$ and the line $y = 3x - 3$ is rotated through 360° about the x-axis.



Solution:-

$$y = 3x - 3 \longrightarrow (1)$$

$$y = (x - 1)^2 \longrightarrow (2)$$

Points of Intersection:

Equations (1) and (2), give

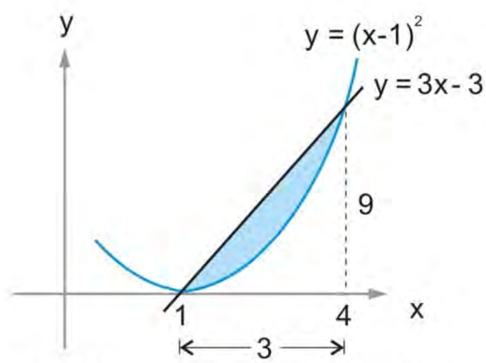
$$(x-1)^2 = 3x-3$$
$$x^2 - 5x + 4 = 0$$
$$(x-1)(x-4) = 0$$
$$x = 1, 4$$

Volume:

$$V = \pi \int_1^4 (3x-3)^2\,dx - \pi \int_1^4 (x-1)^4\,dx$$
$$= \pi \left[\frac{(3x-3)^3}{3 \times 3} \right]_1^4 - \pi \left[\frac{(x-1)^5}{5} \right]_1^4$$
$$= 3\pi \left[(x-1)^3 \right]_1^4 - \frac{\pi}{5} \left[(x-1)^5 \right]_1^4$$
$$= 3\pi\,(27) - \frac{\pi}{5}\,(243)$$
$$= 101.79 \quad \text{(correct to two decimal places)}$$

Another Method:

The line y = 3x – 3 generates a cone of radius 9 { $\because$ y = 3 × 4 – 3 = 9} and height 3.

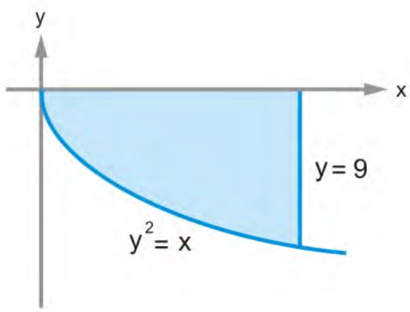


$$V = \text{Volume of cone} - \pi \int_1^4 (x-1)^4\,dx$$
$$= \frac{1}{3}\pi\,(9)^2 \times 3 - \pi \left[\frac{(x-1)^5}{5} \right]_1^4$$
$$= 254.47 - 152.68$$
$$= 101.79$$

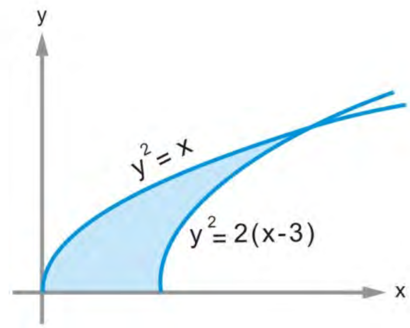
EXERCISE K-26

Calculate the volume generated when the shaded region shown in the following figures is rotated 360° about x-axis:

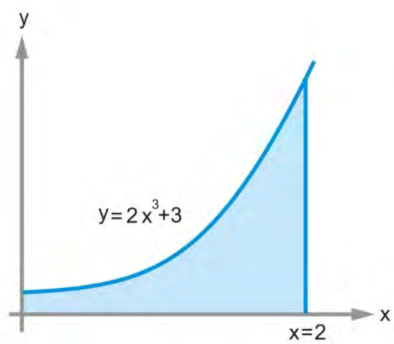
(1)



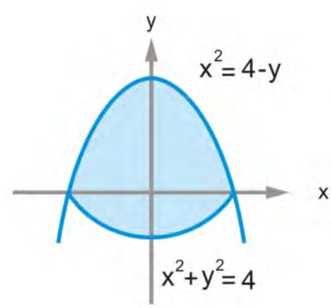
(4)



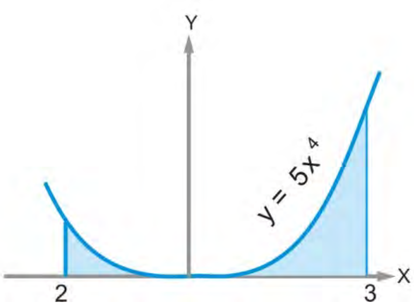
(2)



(5)



(3)



(6)