BOOK 1

CALCULUS

WITH APPLICATIONS

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Chapter 52

DISPLACEMENT, VELOCITY AND ACCELERATION

In this section we will discuss the motion of an object travelling in a straight line. The displacement of the particle depends on time t. If the displacement of an object from O at time t is s, the speed v and acceleration a can be found by the following methods.

$$v = \frac{ds}{dt}$$

and $$a = \frac{dv}{dt}$$

Note:

(1) s: the displacement of the object on a number line from O at time t.

(2) The direction of the motion of the object will be opposite after v = 0.

Explanation:

The displacement from O of an object at time t is given below.

$$s = 4t - t^2 \longrightarrow (1)$$

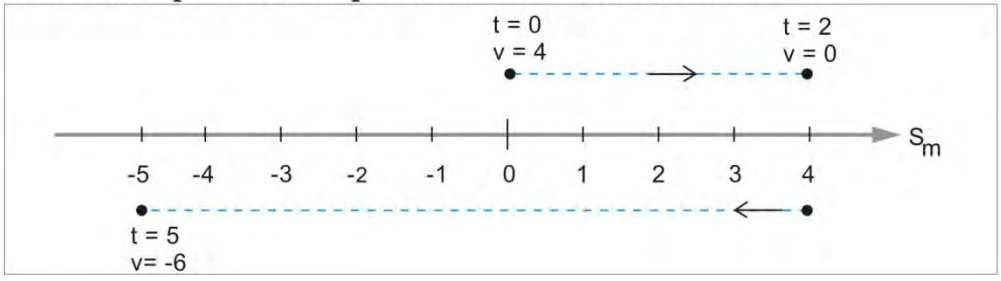
The speed of the particle at time t is

$$v = \frac{ds}{dt} = 4 - 2t \longrightarrow (2)$$

The displacement and velocity at each second is given below, where $0 \leq t \leq 5$.

v m/s	4	2	0	–2	–4	–6
t sec	0	1	2	3	4	5
s m	0	3	4	3	0	–5

The time, speed and displacement on a number line.



Total distance covered by the object in 5 seconds

$$x = 4 + (4 + 5)$$
$$= 13 \text{ m}$$

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$
$$= \frac{13}{5}$$
$$= 2.6 \text{ m/s}$$

[For more detail " application of integration".]

Example 1: A particle is moving in a straight line. The distance s, in meters, of the particle from a fixed point O, at time t is given

$$s = t^3 - 9t^2 + 15t + 3$$

Calculate:

(i) the distance of the particle from O at t = 0.
(ii) the values of t when the particle is at instantaneous rest.
(iii) the distances covered by the particle in 1, 5 and 7 seconds.
(iv) the average speed in 8 seconds.
(v) acceleration at t = 1 sec.

Solution:-

$$s = t^3 - 9t^2 + 15t + 3 \longrightarrow (1)$$

(i) Substituting t = 0 in (1), gives

$$S = 3m$$

(ii)

$$v = \frac{ds}{dt} = 3t^2 - 18t + 15 \longrightarrow (2)$$

Substitute v = 0 in (2)

$$3t^2 - 18t + 15 = 0$$
$$t^2 - 6t + 5 = 0$$
$$(t - 1)(t - 5) = 0$$
$$t - 1 = 0 \quad \text{or} \quad t - 5 = 0$$
$$t = 1 \text{ sec} \quad , \quad t = 5 \text{ sec}$$

The particle is at instantaneous rest at t = 1 sec and t = 5 sec.

EXERCISE K-17

(1) A particle travels in a straight line. Its distance s, in meters , from a fixed point O at time t is given by

$$s = t^2 - 2t.$$

Calcualte

(i) the value of t when the particle is at instantaneous rest.

(ii) the distance travelled by the particle in the time interval t = 0 to t = 4.

(iii) the average speed during the first 4 seconds.

(2) A particle travels in a straight line so that its distance s, in metres, from a fixed point O at time t is given by

$$s = 6t - t^2.$$

Calculate

(i) the value of t when the particle is at instantaneous rest.

(ii) the distance travelled by the particle in the time interval t = 0 to t = 7.

(iii) the average speed during the first 7 seconds.

(3) A particle travels in a straight line so that its distance s, in metres, from a fixed point O at time t is given by

$$S = 2t^2 - 8t + k.$$

Calculate

(i) the value of t when the particle is at instantaneous rest.

(ii) the displacement of the particle from O after 3 seconds is 1m. Find k.

(iii) calculate the total distance travelled by the particle in first 3 seconds.

(4) A particle travels in a straight line. Its distance s, in metres , from a fixed point O at time t is given by

$$s = 4t - t^2 + 3.$$

Calcualte

(i) the value of t when the particle is at instantaneus rest.

(ii) total distance travelled by the particle in first 5 seconds.

(5) A particle travels in a straight line so that its distance s, in metres, from a fixed point O at time t is given by

$$s = t^3 - 10.5t^2 + 30t - 4.$$

Calculate

(i) the values of t when the particle is at instantaneous rest.
(ii) total distance travelled by the particle in first 6 seconds.
(iii) average speed of the particle during the interval t = 0 to t = 6.
(iv) the acceeleration of the particle at t = 4 sec.

(6) A particle travels in a straight line so that its distance s, in metres, from a fixed point O in time t is given by

$$s = t^3 - 9t^2 + 15t.$$

Calculate

(i) the values of t when the particle is at instantaneous rest.
(ii) the distance travelled by the particle in first 7 seconds.
(iii) average speed of the particle during the interval t = 0 to t = 7.