BOOK 1

CALCULUS

WITH APPLICATIONS

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Chapter 48

RELATED RATE OF CHANGE

The variabl x and y are connected by an equation. If dy/dx is the rate of change of x with respect to time t, the rate of change of y with respect to t is given by

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

Example 1: Variables x and y are connected by the equation

$$y = 5x^3 - 2x$$

x is increasing at the rate of 6 units/s, find the rate of increasing of y when x= 2.

Solution:-

$$y = 5x^3 - 2x$$

$$\frac{dy}{dx} = 15x^2 - 2$$

$$\left(\frac{dy}{dx}\right)_{x=2} = 15(2)^2 - 2 = 58$$

Rate of increasing of y when x = 2

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$= 58 \times 6$$

$$= 348 \text{ units/s}$$

Example 2: The pressure y and the volume x are connected by the equation

$$y = \frac{10}{x}$$

Given that x is changing at the rate of 5 units/s, find the rate of change of y when x = 4 units.

Solution:-

$$y = \frac{10}{x}$$

$$\frac{dy}{dx} = \frac{-10}{x^2}$$

$$\left(\frac{dy}{dx}\right)_{x=2} = \frac{-10}{2^2} = \frac{-10}{4} = \frac{-5}{2}$$

The rate of change of y when x = 4 units.

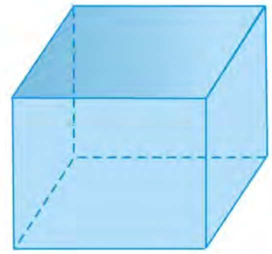
$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$= \frac{-5}{2}(4)$$

$$= -10 \text{ units/s}$$

The pressure y decreases at a rate of 10 units/s.

(7) The volume of a cube of ice is decreasing at a rate of 5.4 $cm^3 s^{-1}$. Find the rate of change in the length of a side, when the side is 6 cm.

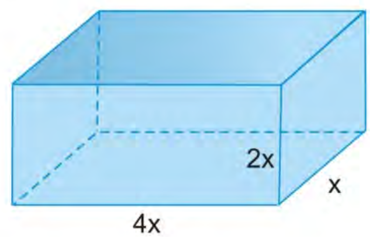


(8) A rectangular ice block of sides x, 2x and 3x melting at a rate of 8 cubic feet per minute. The initial volume of the block is 216 cubic feet. Calculate the rate of decreasing of x, when

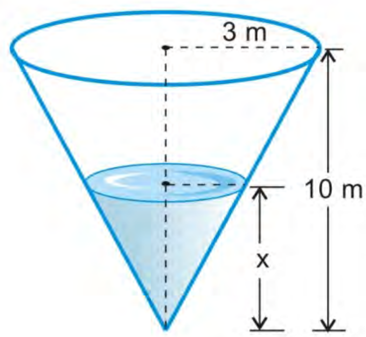
(a) x = 1.5 feet

(b) V = 64 cubic feet

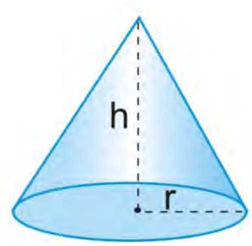
(c) t = 11 3/8 minutes



(9) The radius and height of a conical tank are 3m and 10m respectively. The tank is being filled with water at a rate of 9 π cubic meter per minute. Calculate the rate of rising of x, when x = 5m.

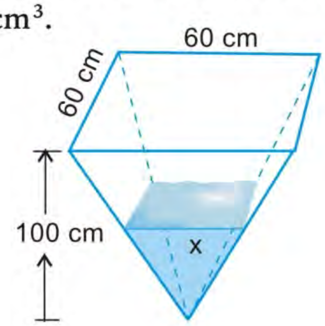


(10) The height of a ice cream cone is always three times the radius of the cone. The cone is melting at a rate of 10 cm^3 per minute. Calculate the rate of decreasing of radius, when r = 5 cm.

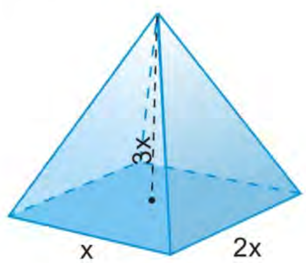


(11) A tank has the shape of the inverted pyramid as shown in the figure. The base of the pyramid is a square of lenght 6 cm and height 100 cm.

Show that the volume of the pyramid is $V = \dfrac{3x^3}{100} cm^3$. The depth x increases at a rate of 2 cm s^{-1}. Calculate the rate of increase of volume of water, when V = 30 cm^3.



(12) A pyramid is melting at a rate of 18 cm ^{3} s^{-1}. If x and 2x are the sides of the base and 3x is the height of the pyramid.



Calculate the rate of decreasing of x, when

(a) x = 2 cm

(b) V = 250 cm^3

(9) $V = \frac{4}{3}\pi r^3$ is the volume of a sphere of radius r. The rate of increase of volume and radius are 256 cm^2/s and $\frac{4}{\pi}$ cm/s respectively at r = ______.

(a) $\frac{8}{\sqrt{\pi}}$ (b) 1 (c) 2 (d) 4

(10) The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. The rate of increasing of radius is 2 cm s^{-1}. The rate of increases of volume is _____ $cm^3 s^{-1}$, when r = 3 cm.

(a) 4π (b) 72π (c) 36π (d) 5

(11) $V = \pi r^2 h$ is the volume of a cylindrical ice cream of radius r and height h. The height of the cylinder is always three times the radius.The rate of decreasing of volume is 10 $cm^3 s^{-1}$. The rate of decreases of radius is ______ cm s^{-1} when $r = \frac{1}{3}$.

(a) $\frac{10}{\pi}$ (b) π (c) $\frac{1}{\pi}$ (d) 2π