

**BOOK 1**

# **CALCULUS**

**WITH APPLICATIONS**

**M. MAQSOOD ALI**

## Chapter 47

## RATE OF CHANGE

The dependent variable  $y$  depends on the independent variable  $x$ .

Rate of change in  $y$  with respect to  $x = \frac{dy}{dx}$

**Example 1:** The area  $A$  of a circle of radius  $r$  is  $A = \pi r^2$ . Find the rate of change in  $A$  with respect to  $r$  when  $r = 3$ .

**Solution:-**

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

Rate of change in  $A$  with respect to  $r$  when  $r = 3$ .

$$\left( \frac{dA}{dr} \right)_{r=3} = 2\pi(3) = 6\pi$$

### RATE OF CHANGE WITH RESPECT TO TIME

Suppose that in an equation  $y$  depends of time  $t$ .

Rate of change in  $y$  with respect to  $t = \frac{dy}{dt}$

**Note:**

- (1) If the rate of change is **negative**, then the magnitude of  $y$  **decreases** as time **increases**.
- (2) If the rate of change is **positive**, then the magnitude of  $y$  **increases** as time **increases**.

**Example 2:** The volume of the water in a tank at time  $t$  is  $V = t^2 + 2$

Calculate the rate of increasing of  $V$  when  $t = 5$  seconds.

**Solution: -**

$$V = t^2 + 2$$

$$\frac{dV}{dt} = 2t$$

Rate of increasing of volume when  $t = 5$  sec.

$$\left( \frac{dV}{dt} \right)_{t=5} = 2(5) = 10 \text{ units/s}$$

## EXERCISE K-12

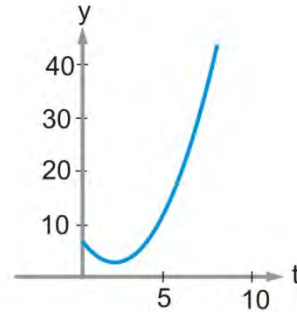
*Find the rate of change of  $y$  for the given value of  $t$ . Also state at these instant whether  $f$  is increasing or decreasing.*

(1)  $y = t^2 - 4t + 7$

(i)  $t = 1$

(ii)  $t = 5$

(iii)  $t = 8$



(2)  $y = t^3 - 3t^2 + 5$

(i)  $t = 3$

(ii)  $t = 1$

(iii)  $t = 2$

(3)  $f(y) = \sin^2 t$

(i)  $\frac{\pi}{3}$

(ii)  $\frac{2\pi}{3}$

(iii)  $t = \frac{\pi}{2}$

(4) Given  $y = t^2 - 3t + 5$ . The rate of increasing of  $y$  is 3. Find  $t$ .

(5) The distance,  $S$  m, of a particle at time  $t$  is given by  $S = 2t^3 - 3t^2 + 7$ . What is the speed of the particle at  $t = 3$  seconds.

(6) The distance,  $S$  m, of a particle at time  $t$  is given by  $S = \frac{1}{3}t^3 - \frac{3}{2}t^2 - 14t$ . Find  $t$  when the speed is decreasing at a rate of  $4\text{ms}^{-1}$ .

(7) The distance,  $S$  m, of a particle at time  $t$  is given by  $S = \frac{1}{12}t^4 - \frac{7}{6}t^3 + 5t^2$ . Find the instant when the deceleration is  $2\text{ms}^{-2}$ .

(8) The distance,  $S$  m, of a particle is given by  $S = t^3 - 5t^2 + 6$ . Find the speed and acceleration at  $t = 4$  seconds and  $t = 1$  second.

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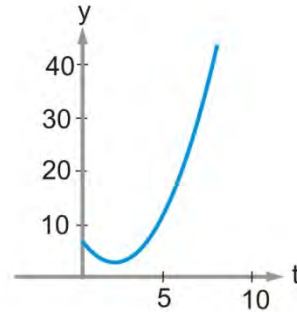
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(8) The distance,  $S$  m, of a particle is given by  $S = t^3 - 5t^2 + 6$ . Find the speed and acceleration at  $t = 4$  seconds and  $t = 1$  second.



- (9) The distance of a particle in metres at time  $t$  is  $x = 16(t - 5)^{5/4}$ . What is the acceleration in  $\text{m/s}^2$  of the particle at  $t = 21$  sec.
- (a)  $\frac{5}{8}$                       (b)  $-2.7$                       (c)  $40$                       (d)  $5$
- (10) Given that  $x = 25t - 50 + \frac{1}{2}(t - 2)^2$ ,  $x$  is the distance in metres of a particle at time  $t$ . At what time (in seconds) the speed of the particle is  $50 \text{ m/s}$ .
- (a)  $35$                       (b)  $10$                       (c)  $27$                       (d)  $25$
- (11) A car is running with speed  $v = 80 - 16t + 16t^{3/4}$ . What is the value of  $t$  when the deceleration of the car is  $13 \text{ m/s}^2$ .
- (a)  $(.3)^3 \text{ sec}$                       (b)  $64 \text{ sec}$ .                      (c)  $\sqrt[3]{4} \text{ sec}$                       (d)  $6 \text{ sec}$
- (12) An ice cream in the shape of cylinder is melting in such a way that the radius and height are always same. What is the rate of change in volume with respect to radius at  $r = 2$ ?
- (a)  $6\pi$                       (b)  $12\pi$                       (c)  $-12\pi$                       (d)  $\text{None}$
- (13) The height of a right circular cone is always twice the radius of the cone. The cone is melting. What is the radius of the cone when the rate of change in volume is  $9\pi$ .
- (a)  $\sqrt{3}$                       (b)  $-3$                       (c)  $3$                       (d)  $2$
- (14) The volume of a spherical balloon is increasing. What is the rate of change in volume with respect to radius at  $r = 2$ .
- (a)  $2\pi$                       (b)  $\pi$                       (c)  $4\pi$                       (d)  $16\pi$
- (15) The surface area of spherical balloon is decreasing. What is the radius of the balloon when the rate of change in surface area is  $16\pi$ .
- (a)  $2$                       (b)  $-2$                       (c)  $16$                       (d)  $-16$

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