

**BOOK 1**

# **CALCULUS**

**WITH APPLICATIONS**

**M. MAQSOOD ALI**

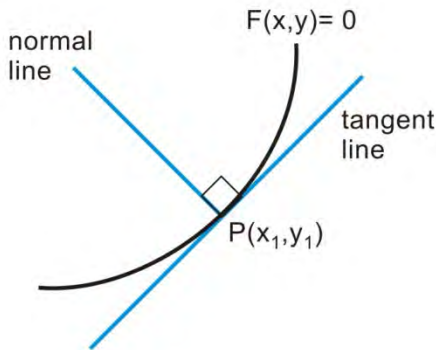
## Chapter 46

# TANGENT AND NORMAL LINE

A line touches the curve at a point is called **tangent line**. A line perpendicular to the tangent line at that point is called **normal line**.

## EQUATION OF TANGENT AND NORMAL LINE

A tangent is drawn to a curve  $F(x, y) = 0$  at a point  $P(x_1, y_1)$ .



### Equation of the tangent:

$$y - y_1 = m(x - x_1)$$

### Equation of the normal:

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

where  $m$  is the slope of the tangent at the point  $(x_1, y_1)$ . It can be found as

$$m = \left( \frac{dy}{dx} \right)_{(x_1, y_1)}$$

**Example 1:** Find the equation of tangent and normal to the curve  $2x^2 - 3y^2 = 6$  at the point  $(9, 2)$ .

**Solution:-**

$$\frac{d}{dx}(2x^2 - 3y^2) = \frac{d}{dx}(6) \quad (6)$$

$$\frac{dy}{dx} = \frac{2x}{3y}$$

Let  $m$  be the slope of the tangent at  $(9, 2)$ .

$$m = \left( \frac{dy}{dx} \right)_{(9,2)} = \frac{2 \times 9}{3 \times 2} = 3$$

Equation of Tangent:

$$y - 2 = 3(x - 9)$$

$$3x - y - 25 = 0$$

Equation of Normal:

$$y - 2 = \frac{-1}{3}(x - 9)$$

$$x + 3y - 15 = 0$$

## EXERCISE K-11

- (1) Find the slope of the curve  $y = 3x^2 - 6x$  at the point where  $x = 2$ .
- (2) Find the gradient of the curve  $y = 5\sqrt{x} - x^3 + 2$  at the point whose abscissa is 4.
- (3) The slope of the curve  $y = \frac{a}{\sqrt{x}} - 2\sqrt{x}$  at the point where  $x = 9$  is  $\frac{-5}{18}$ . Find the value of  $a$ .
- (4) The gradient of the curve  $e^{ax} + y = a$  at the point where  $x = 0$  is  $-1$ . Find  $a$ .
- (5) Find the values of  $a$  and  $b$ . The gradient of the curve  $y = ax^2 + b\sqrt{x} + 1$  at the point  $(4, 61)$  is 37.5.
- (6) The tangent to the curve  $y = e^{ax} + b$  is parallel to  $x$ -axis at the point  $(0, 6)$ . Find the values of  $a$  and  $b$ .
- (7) The tangent to the curve  $x^2 - 5xy^2 + a = by$  at the point where  $(2, 1)$  is perpendicular to the line  $2y = 34x + 7$ . Find the values of  $a$  and  $b$ .
- (8) The tangent to the curve  $\ln y + 2x^2 = 50$  at the point  $(-5, 1)$  is parallel to the line  $2y = ax - 5x + 2$ . Find the value of  $a$ .
- (10) The line  $12y = k - 3x$  is a normal to the curve  $y = \ln(3x^2 - 1) + x$ . Find the value of  $k$ .
- (11) A tangent to the curve  $y = ax^2 - 8x + 5$  at the point where  $x = -1$  passes through  $(1, 2)$ . Find the value of  $a$ .
- (12) A normal to the curve  $y = 2x^2 + kx - 5$  at the point where  $x = 1$  passes through the point  $(-1, 1)$ . Find the value of  $k$ .

- (13) A tangent to the curve  $y = \sqrt{x} - 2x^2 + 2$  at the point  $(a, b)$  is parallel to  $x$ -axis. Find the values of  $a$  and  $b$ .
- (14)  $y = 5 + (4 - x)^{-3}$  is the equation of the curve. A tangent parallel to  $y$ -axis is drawn at a point of the curve. Find the coordinates of the point.
- (15) A line  $3y = 5x + c$  is tangent to the curve  $y = (2 - x)^2 - 5$ . Find the value of  $c$ .
- (16) The tangents to the curve  $y = \frac{4x^2}{3} + 8x^2 - 9x + 6$  at two points are parallel to the line  $6x + 4y - 3 = 0$ . Find the coordinates of the points.
- (17) A normal to the curve  $y = x^3 - 5x^2 + 9$  at the point  $(-1, 3)$  form a triangle with the coordinate axes. Find the area of the triangle.
- (18) The tangent to the curve  $y = x^3 - 5x + 1$  at  $(-2, 3)$  cuts  $x$ -axis at point  $P$  and  $y$ -axis at point  $Q$ . Find the area of triangle  $POB$ , where  $O$  is the origin.

### M.C.Q's K-3

- (1) What is the equation of the tangent to the curve  $y = 3 - x^2$  at the point  $(1, 2)$ ?
- (a)  $y = 2 + 2(1 - x)$                       (b)  $y = 2 - 2(1 - x)$   
(c)  $y - 2 = 2(x - 1)$                       (d)  $y - 2 = -2(x + 1)$
- (2) What is the equation of the normal to the curve  $x^3 - y^2 = 0$  at the point  $(1, -1)$ ?
- (a)  $2(y - 1) = -3(x + 1)$               (b)  $2(1 - y) = 3(x - 1)$   
(c)  $2(y + 1) = 3(x - 1)$               (d)  $2(y + 1) = 3(1 - x)$
- (3) What is the equation of a tangent to the curve  $y^2 = x$ ?
- (a)  $y + 2 = 3(x - 2)$                       (b)  $y - 2 = -\frac{1}{4}(x - 4)$   
(c)  $y - 2 = \frac{1}{4}(x - 4)$                       (d)  $y - 4 = -\frac{1}{4}(x - 2)$

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